A Data Conversion Technique for Accurate Interferometric Testing of Spherical Surfaces*

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This paper presents a Cartesian to polar conversion technique based on a polynomial interpolation. The method is used for the processing of data from the two-orientation method for measuring spherical surfaces with an interferometer. The interferometer output consists of points on a square-grid coordinate system, which are then converted to a polar-grid coordinate system. The profile height data of the points on the polar-grid are calculated from a polynomial neighborhood analysis of points on the square-grid. Since the converted point on the polar-grid does not need to be at the center of the points in the analysis, profiles along concentric circles near to the edge of the test surface can also be calculated. A simulation of the polynomial and conventional 4-point techniques is carried out to verify the process. The comparison shows that the polynomial technique has produce a higher data conversion accuracy. Experiments on a phase-measuring interferometer were also conducted to verify the approach.

Key Words: Measurement, Accuracy, Sensor, Profile, Spherical Surface, Error Separation, Data Conversion, Square Grid, Polar Grid, Two-orientation Method

1. Introduction

Interferometers are widely used for the testing of optical surfaces⁽¹⁾⁻⁽³⁾ and are especially suited to analyzing spherical surfaces, because it is a simple process to fabricate accurate reference surfaces with existing machining techniques. Large spherical surfaces can also be easily tested⁽³⁾.

In principle, the profile of a test surface is calculated from the interferogram formed by the interference of wavefronts from the test and reference surfaces. The form accuracy of the reference surface influences the measurement accuracy of the test surface profile. In the case of high-precision measurement, it is required to remove the influence of the form error of the reference surface.

Many efforts have been made for this purpose, such as the well-known three-flat method^{(4),(5)} and Elssner's method⁽⁶⁾. In the three-flat method, multiple

measurements of three flat surfaces combined in pairs at different positions are carried out. In the Elssner's method, measurements are of two flats carried out at three positions—the basic position, the rotated position (rotated through 180° about the optical axis) and the cat's eye position. These methods, which are typically employed for the calibration of interferometers in national standard institutes or research laboratories, are complicated and time consuming⁽⁷⁾. Hence, it is difficult to apply them to calibrate interferometers for common usage.

In contrast to these complex techniques, the two-orientation method⁽⁸⁾⁻⁽¹⁰⁾ for the measurement of cross-sectional profiles along concentric circles on a flat surface is a simple process. The authors have shown the possibility of applying this method to the measurement of spherical surfaces⁽¹¹⁾ and have been investigating the characteristics of this method in spherical surface profile measurement. The influence of form errors along concentric circles on the reference surface can be removed by a differential output of two data sets obtained before and after a small rotation of the test surface based on the principle of the two-probe method⁽¹²⁾⁻⁽¹⁴⁾.

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The data processing in the two-orientation method requires for the interference phase data to be distributed about a polar-grid. Since the phase-measuring (CCD) devices employed in most interferometers are conventional square-grid detectors, it is necessary to convert the square-grid data to the polar-grid data. Errors in the conversion procedure may influence the accuracy of the profile measurement by the two-orientation method. In this paper, a polynomial interpolation based technique is presented to improve the accuracy of the data conversion. The algorithm is discussed and some experimental results are provided.

2. The Data Conversion Technique

2.1 The two-orientation method for measurement of spherical surfaces⁽¹¹⁾

The two-orientation method (see Fig. 1) is used to remove the influence of the form error of the reference surface. Assume that the output from the interferometer is points on a polar-grid. The output along the i th concentric circle before and after a small rotation $\Delta\varphi$ of the test surface can be expressed by Eqs. (1) and (2).

$$m_{i1}(\theta) = f_i(\theta) + \varepsilon_i(\theta) \quad (1)$$

$$m_{i2}(\theta) = f_i(\theta - \Delta\varphi) + \varepsilon_i(\theta), \quad i=1, 2, \dots, Nc \quad (2)$$

Where $f_i(\theta)$ and $\varepsilon_i(\theta)$ represent the cross-sectional profiles along the i th concentric circle on the test surface and the corresponding form error of the reference surface, respectively. θ represents the polar coordinate of the data point. Nc is the number of concentric circles on the test surface.

The influence of the form error of the reference surface along the circle can be removed in a differential output of Eqs. (1) and (2). The differential output is then Fourier transformed and surface profiles along the concentric circles are determined by a phase operation based on the digital filter technique⁽¹⁵⁾⁻⁽¹⁸⁾.

The differential output can be expressed as:

$$m_{idc}(\theta) = m_{i1}(\theta) - m_{i2}(\theta)$$

Fig. 1 Principle of the two-orientation method

$$= f_i(\theta) - f_i(\theta - \Delta\varphi), \quad i=1, 2, \dots, Nc \quad (3)$$

Taking the Fourier transform of Eq. (3) gives:

$$M_{idc}(n) = H(n)F_i(n), \quad i=1, 2, \dots, Nc \quad (4)$$

Where $M_{idc}(n)$ and $F_i(n)$ are the Fourier transforms of $m_{idc}(\theta)$ and $f_i(\theta)$, respectively. n is the spatial frequency number. $H(n)$ is called the transfer function, which is defined as:

$$H(n) = 1 - \exp(jn\Delta\varphi) \quad (5)$$

Apparently, $F_i(n)$ can be calculated from Eq. (4). The profile $f_i(\theta)$ of the test surface can then be obtained by Fourier inverse transform of $F_i(n)$.

2.2 The data-conversion technique

The CCD output from interferometers (see Fig. 2) is XY data in a Cartesian coordinate system. As shown above, however, data along concentric circles are needed for the two-orientation method. Therefore, it is necessary to perform a data conversion to obtain the desired polar-grid data.

The 4-point technique based on linear interpolation is the simplest method for data conversion⁽¹⁹⁾. As shown in Fig. 3, profile heights of four neighboring data points in the CCD square-grid are used to calculate the profile height at point P of the polar-grid based on a linear interpolation as follows.

$$f_p = \kappa_1 S_{i,j} + \kappa_2 S_{i+1,j} + \kappa_3 S_{i,j+1} + \kappa_4 S_{i+1,j+1} \quad (6)$$

Where $S_{i,j}$, $S_{i+1,j}$, $S_{i,j+1}$, $S_{i+1,j+1}$ are interferometer outputs of points on the square-grid. κ_1 , κ_2 , κ_3 , κ_4 are

Fig. 2 XY CCD output data

Fig. 3 4-point data conversion technique

Fig. 4 Polynomial data conversion technique

weighted coefficients corresponding to the distances between point P and neighboring points.

Use of a linear interpolation technique means that data conversion errors will occur, thereby introducing errors into the profile evaluation result by the two-orientation method. Another technique, in which 16 neighboring data points around the point P are used for a second-order interpolation, has also been proposed⁽²⁰⁾. Although this technique improves the data conversion accuracy to some extent, errors still arise in the data conversion procedure when high-order components are included in the interferometer outputs. Furthermore, because the point P needs to be at the center of the neighboring data points, the circles near to the edge of the test surface can not be resampled, thereby reducing the measurement area.

The proposed technique is based on a polynomial interpolation to improve the accuracy of the data conversion. In the technique, which is referred to as the polynomial technique, the profile height of point P on the polar-grid is obtained by an n th-order polynomial interpolation as shown in Eq. (7)

$$f_p = g_{ij} \sum x^i y^j \quad (i=0, 1, 2, \dots, n; j=0, 1, 2, \dots, n) \quad (7)$$

Where x and y are the X and Y coordinates of point P on the polar-grid. g_{ij} is the polynomial coefficient and n is the order of polynomial interpolation.

The polynomial coefficient g_{ij} is calculated firstly by the data points on the square-grid around point P, in which the least square approximation method is employed to obtain an optimal solution. As a result, the approximate profile height data at point P (see Fig. 4) is obtained from Eq. (7) using the known X , Y coordinates of point P. Besides the higher accuracy of data conversion, another advantage of this technique is that point P does not need to be at the center of the points on the square-grid. Hence, circles near to the edge of the test surface can be calculated.

3. Simulation

Data conversion errors in the 4-point and

Fig. 5 Cross-sectional profiles along concentric circles on a test surface

Fig. 6 Input cross-sectional profile along a circle

polynomial techniques, as well as the corresponding profile evaluation errors in the two-orientation method, were compared by computer simulation.

The form error profile of the test surface was assumed to be a parabola with some high-frequency components along concentric circles.

The concentric circles (see Fig. 5) were numbered as $i=1, 2, \dots, N_c$ from the center to the outermost circle.

The simulation conditions were as follows:

- 1) Radius of curvature of test optics: 519 mm
- 2) Number of test circles: 96 (radius: 48 mm)
- 3) Number of sampling points N_r at each circle: 350
- 4) Sampling interval θ_s : $2\pi/350$
- 5) Rotary angle $\Delta\phi$: $29\theta_s$
- 6) Profile of the test surface along concentric circle: (see Fig. 6)

The simulation steps were as follows:

a) The data conversion was carried out to obtain the desired polar-grid profile height data from the square-grid output data.

b) Profile height data along each circle on the test surface was used in the two-orientation method, from

Fig. 7 Data conversion errors

Fig. 8 PV value of data conversion errors with respect to circle number

which an accurate profile of the test surface was obtained.

Considering the height resolution of the interferometer used in the experiments, random errors with a Peak-to-Valley (PV) value of ± 0.5 nm were added to $m_{i1}(\theta)$ and $m_{i2}(\theta)$.

The data conversion errors at the outermost circle are shown in Fig. 7. The order of the polynomial interpolation was set to 3. The data conversion errors of the 4-point and polynomial techniques were 20 nm (PV) and 0.8 nm (PV), respectively. The data conversion accuracy of the polynomial technique was approximately 25 times higher than that of the 4-point technique. Figure 8 shows the PV values of the data conversion errors with respect to the circle number for both techniques over the entire measurement range. We can see that the data conversion error of the 4-point technique increased with the radius of the concentric circle. On the contrary, the data conversion error of the polynomial technique was less than 1 nm for all circles.

The evaluation errors of the test surface profiles of the 96th circle by the two-orientation method using data from the 4-point and polynomial techniques are shown in Fig. 9. Figure 10 shows the evaluation errors of the corresponding reference surface profiles. As can be seen in these figures, the profile evaluation errors based on the polynomial technique were approximately 10 times smaller than those of the 4-

Fig. 9 Residual error of test surface profiles of the 96th circle

Fig. 10 Residual error of corresponding reference surface profiles of the 96th circle

point technique.

These simulation results confirmed the applicability of the polynomial data conversion technique.

4. Experiment

A ZyGo phase-measuring interferometer (GPI-XP)⁽²¹⁾ was used in the experiment. Figure 11 shows the interferometer output data for the test spherical surface. The PV value of the output was approximately 4λ ($2.5\mu\text{m}$), which is a combination of the profile of the spherical test surface and the form error of the reference surface.

The aperture of the test optics was 100 mm and the radius of curvature was 519 mm. There were 96 concentric circles with an equal interval. Figure 12 shows the data-conversion results of the outermost (96th) circle by the 4-point and polynomial techniques. The difference is also shown in the figure. Here we assume that the accuracy of the polynomial technique is significantly higher than that of the 4-point technique based on the simulation results. As can be seen in Fig. 12, the error, which is composed of high-frequency components, has a PV value of approximately 13 nm.

Using the converted polar-grid data of the interferometer output, the profiles of the test surface along concentric circles were evaluated by the two-orientation method without the influence of the form error of the reference surface. Figure 13 shows the evaluation

Fig. 11 Interferometer output

Fig. 12 Results of data conversion (outermost circle)

Fig. 13 Measured cross-sectional profiles of test surface (outermost circle)

results of the cross-sectional profile along the 96th circle. The PV value was approximately $1.1 \mu\text{m}$. The difference between the profiles evaluated with the converted data by the 4-point and polynomial techniques is shown in Fig. 14. The PV value of the difference was approximately 8 nm . Figure 15 shows the corresponding evaluation results of the form error of the reference surface. The PV value was approximately 25 nm . The difference between the evaluated form errors evaluated with the converted data by the 4-point and polynomial techniques is shown in Fig. 16. The difference was approximately 18 nm . The differences shown in Figs. 14 and 16 can be considered to be the errors caused in the data conversion by the lower-accuracy 4-point technique.

5. Conclusions

(1) A technique to improve the accuracy of the data conversion for interferometric testing of spheri-

Fig. 14 Compared result of test surface profile (outermost circle)

Fig. 15 Measured form errors of reference surface (outermost circle)

Fig. 16 Compared result of reference surface profiles (outermost circle)

cal surfaces based on a polynomial interpolation has been presented.

(2) A computer simulation was performed to compare the polynomial and conventional 4-point techniques and the data conversion errors at the outermost circle were 0.8 nm and 20 nm , respectively. Furthermore, the data conversion error of the 4-point technique increased with the radius of the concentric circles, whilst that of the polynomial technique was less than 1 nm for all circles. These results verified the effectiveness of the polynomial technique.

(3) Using data converted by the polynomial technique, a spherical test surface with a curvature of 519 mm was measured by the two-orientation method on a ZyGo phase-measuring interferometer. The maximum cross-sectional profile error along concentric

circles on the test surface was approximately 1100 nm. The maximum cross-sectional form error along concentric circles on the reference surface was approximately 25 nm. The difference between the polynomial technique and the 4-point technique was also confirmed.

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