Analysis of Bubbly Lubrication in Journal Bearings*

Song CHOI** and Kyung Woong KIM***

The effect of air bubbles evenly distributed in lubricating oil on the bearing performances is analyzed theoretically. The Reynolds equation for the bubbly lubricant in a steady-state and isothermal condition is solved with an iterative numerical method. Surface tension and the radius of bubble are taken into account in the analysis. It is assumed that bubbles move along streamlines and do not split apart or coalesce. The density and the viscosity of air-oil mixture are treated as functions of oil film pressure and the volume fraction of air. Numerical results show that the load carrying capacity of a journal bearing increases as bubbles in the supplied lubricant become smaller or the surface tension is getting larger. The load carrying capacity increases as the volume fraction increases up to a critical volume fraction that gives maximum load carrying capacity. Beyond the critical volume fraction, it decreases as the volume fraction increases.

Key Words: Bubbly Lubrication, Mixture, Volume Fraction, Surface Tension, Density, Journal Bearing

1. Introduction

When oil is supplied to lubricating parts, air can be entrained into the oil in the form of small bubbles. These bubbles make the rheology of air-oil mixture different from pure oil. Viscosity and density are important properties of mixtures that affect the characteristics of bearings. These properties are dependent on the volume fraction that is defined as the volume ratio of air to total mixture. Changes in the density of mixtures by changing volume fraction affect the compressibility of mixtures. It is noticeable that the viscosity of mixtures can be higher than that of pure oil. Theoretical models and experimental results have shown the fact that the viscosity of the mixture increases as the volume fraction increases. Einstein⁽¹⁾ studied theoretically on the viscosity of two-phase fluids, and showed that the viscosity of mixtures is higher than that of pure liquid. That

model was verified experimentally for fluid-solid particle mixtures⁽²⁾. Taylor⁽³⁾ extended the study to fluid-fluid mixtures which can be applied to air-oil mixtures. Hayward⁽⁴⁾ showed experimentally that, in low volume fraction range, the viscosity of air-oil mixtures increases as the volume fraction increases.

In the bubbly lubrication, the most important factor for the bearing performance is the viscosity of bubbly lubricant. There are some contradicting reports on the effect of the viscosity of mixtures in bearings and squeeze film dampers. According to some reports^{(5),(6)}, the load carrying capacity of bearings does not show significant change as the volume fraction changes. However, other papers^{(7),(8)} argued that the amount of air entrained in lubricant affects the load carrying capacity or the pressure distribution of a bearing significantly. These results come from using different models and governing equations. Tonder⁽⁵⁾ ignored the effect of viscosity considering it to be small. He only considered the effect of density change, that is, compressibility. The load carrying capacity was almost unchanged compared with non-bubbly case. Abdel-Latif et al.⁽⁶⁾ used a model of viscosity in THD analysis of a thrust bearing, which considers the effect of shear stress. There was no appreciable rise in the load carrying capacity with rising bubble content. Chamniprasart et al.⁽⁷⁾ did not

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** Senior Research Engineer, Digital Appliance Research Laboratory, LG Electronics Inc., 327-23, Gasan-dong, Keumchun-gu, Seoul 153-802, Korea. E-mail: csong@lge.com

*** Professor, Department of Mechanical Engineering, KAIST, 373-1 Kusung-dong, Yusong-gu, Taejeon, 305-701, Korea. E-mail: taeho@kaist.ac.kr

use an effective viscosity model for the mixture lubricant. The viscosity of oil itself was used, but the Reynolds equation was changed to accommodate a mixture lubricant based on the continuum theory. The conclusion was that the amount of air entrainment in the lubricant affects the bearing pressure in a significant way. However they did not pay much attention to the load carrying capacity itself. Nikolajsen^{(8),(9)} presented an analytical model for the viscosity and the density of aerated oil. The model suggests the overall viscosity model considering both the reduction of viscosity due to addition of negligible air viscosity and the increase in viscosity due to the increase in shear force against surface tension by the deformation of bubbles. From the parametric studies the model explains the increase in viscosity with the increase in the volume fraction of air. He concluded that highly aerated lubricating oil was predicted to increase the load carrying capacity of journal bearings by a factor of two due to doubling of the effective oil film viscosity.

In squeeze film dampers, Feng and Hahn^{(10),(11)} showed that the damper film forces of bubbly lubricant were greater than those of non-bubbly lubricant. Other researches^{(12)–(14)} reported that the increase in volume fraction reduces damper performance. Diaz and San Andres⁽¹³⁾ suspected the validity of the viscosity models showing increase in the viscosity with the increase in the volume fraction for large shear stress and bubble sizes in squeeze film dampers. They proposed the viscosity model showing the decrease of the viscosity with the increase in the volume fraction for squeeze film dampers⁽¹⁴⁾.

The discrepancy of the effect of the volume fraction on the viscosity between bearings and squeeze film dampers can be thought to come from the difference of shear stress and ratio of bubble size to the film thickness and so on. In the dampers the compressibility affects more than the viscosity. The cause of the disagreement of the effect of volume fraction on the load carrying capacity will be the difference of viscosity, density model and operation range. From the physical reasoning, the load carrying capacity will increase up to a certain volume fraction with the increase in volume fraction by the dominant effect of increased viscosity, but after attaining maximum it will decrease by the strong effect of compressibility of bubbles. Prior researches lack the analysis of the effect of the volume fraction of bubbly lubricant on the load carrying capacity itself.

The purpose of this paper is to analyze the relationship between the load carrying capacity and volume fraction of mixtures especially in journal bearings taking the effect of both viscosity and density

change due to volume fraction into account.

Nomenclature

- h : film height (m)
- p : pressure (N/m²)
- p_a : atmospheric pressure (N/m²)
- p_i : inside pressure of a bubble (N/m²)
- r_0 : bubble radius at maximum film thickness (m)
- r : bubble radius in the bearing (m)
- $\bar{r}$: non-dimensional bubble radius
- x, y, z : rectangular coordinate system (m)
- u, v, w : velocity in x, y, z direction (m/s)
- C : radial clearance in the bearing (m)
- H : non-dimensional film height
- L : bearing length (m)
- P : non-dimensional pressure
- R : bearing radius (m)
- S_o : Sommerfeld number $\left(= \frac{\eta_i L R^3 \omega}{\pi W_i C^2} \right)$
- V_a : volume of air (m³)
- V_i : volume of oil (m³)
- W_t : load (N)
- W_ξ : load in ξ -direction (N)
- W_ϵ : load in ϵ -direction (N)
- X, Y, Z : non-dimensional rectangular coordinate system
- β : volume fraction
- β_0 : volume fraction at maximum film thickness
- β_{cr} : critical volume fraction
- ϵ : eccentricity ratio
- ζ, ξ : coordinates
- η : viscosity (Pa·s)
- η_i : viscosity of oil (Pa·s)
- $\bar{\eta}$: non-dimensional viscosity
- θ : angular coordinate
- ρ : density of mixture (kg/m³)
- ρ_i : density of oil (kg/m³)
- ρ_a : density of air (kg/m³)
- $\bar{\rho}$: non-dimensional density
- ρ^* : non-dimensional density of air
- σ : surface tension (N/m)
- σ^* : non-dimensional surface tension
- ω : shaft rotation speed (rad/s)
- Λ : bearing number $\left(= \frac{6\eta_i \omega}{p_a} \left(\frac{R}{C} \right)^2 \right)$
- Φ : attitude angle $\left(= \tan^{-1} \left(\frac{W_\xi}{W_\epsilon} \right) \right)$
- Subscripts
 - l : oil
 - a : air
 - 0 : state at the location of maximum film thickness

2. Theory

2.1 Reynolds equation

Figure 1 shows the coordinate system used for the analysis of a journal bearing. The ξ coordinate is directed to maximum film thickness. The θ and x coordinates start from the maximum film thickness along the direction of shaft rotation.

The following assumptions are used to reduce the Navier-Stokes equations and the continuity equation to the Reynolds equation.

Bubbles of the same size which is small enough compared with film thickness are evenly distributed in the supply lubricant, move along streamlines, and do not split apart or coalesce in a bearing. Pressure is constant through the fluid film. The thickness of fluid film is very small compared to the bearing size. There is no slip at the boundaries. Lubricant behaves as a Newtonian fluid. Flow is laminar, and inertia terms are neglected. Fluid density and viscosity are constant across the film. Liquid part of lubricant is incompressible. Steady-state and isothermal condition is assumed also.

The Reynolds equation for mixtures becomes:

$$\frac{\partial}{\partial x} \left[\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right] = R\omega \frac{\partial}{\partial x} \left[\frac{\rho h}{2} \right] \quad (1)$$

where the viscosity and the density of the lubricant mixture is to be expressed as a functions of the volume fraction.

Boundary Conditions used are:

$$\frac{\partial p}{\partial \theta} = 0, \quad p = p_a \quad \text{at} \quad \theta = \theta_r \quad (2)$$

$$p = p_a \quad \text{at} \quad \theta = 0, \quad \theta_r \leq \theta \leq 2\pi \quad (3)$$

Fig. 1 Coordinate system and force components in a journal bearing

$$p = p_a \quad \text{at} \quad y = 0, \quad y = L \quad (4)$$

2.2 Viscosity, density and volume fraction

Volume fraction β is defined as:

$$\beta = \frac{\text{volume of air}}{\text{volume of air and oil}} = \frac{V_a}{V_i + V_a} \quad (5)$$

The volume fraction affects the viscosity and the density of mixtures directly. In this paper Hayward's model is used for the viscosity model, because it is one of the most popular experimental models for air-oil mixtures. According to the Hayward's model the viscosity of the lubricant is written as follows:

$$\eta = \eta_i (1 + 1.5\beta) \quad (6)$$

To get the density of mixtures, the relation between bubble radius and pressure should be known. The simplified Rayleigh-Plesset equation⁽¹⁵⁾ gives the relation between the inside pressure of a bubble p_i and the surrounding pressure p as follows:

$$p_i = p + 2 \frac{\sigma}{r} \quad (7)$$

Using Eq.(7) and the simple gas law, the following relation between bubble radius r and the lubricant pressure p is obtained:

$$\frac{p + 2 \frac{\sigma}{r}}{p_0 + 2 \frac{\sigma}{r_0}} = \frac{V_{a0}}{V_a} = \left(\frac{r_0}{r} \right)^3 \quad (8)$$

Not only the radius of a bubble and but also the volume fraction vary according to the pressure, and the relation between the volume fractions at pressures p and p_0 is expressed as follows:

$$\beta = \frac{V_a}{V_a + V_i} = \frac{1}{1 + \frac{V_i}{V_a}} = \frac{1}{1 + \frac{1 - \beta_0}{\beta_0} \left(\frac{r_0}{r} \right)^3} \quad (9)$$

The density of air-oil mixture is:

$$\begin{aligned} \rho &= \frac{V_i \rho_i + V_a \rho_a}{V_i + V_a} = \beta \rho_a + (1 - \beta) \rho_i \\ &= \beta \rho_{a0} \left(\frac{p + 2 \frac{\sigma}{r}}{p_0 + 2 \frac{\sigma}{r_0}} \right) + (1 - \beta) \rho_i \end{aligned} \quad (10)$$

2.3 Non-dimensionalization

For the advantages of parametric study and calculation accuracy, equations to be solved to get the film pressure distribution are non-dimensionalized. Following non-dimensional parameters are employed to non-dimensionalize Eqs.(1), (6), (8), (9) and (10).

$$\begin{aligned} X &= \frac{x}{R}, \quad Y = \frac{y}{L}, \quad \bar{\rho} = \frac{\rho}{\rho_i}, \quad P = \frac{p}{p_a}, \\ H &= \frac{h}{C} = 1 + \varepsilon \cos X, \quad \bar{\eta} = \frac{\eta}{\eta_i}, \quad \Lambda = \frac{6\eta_i \omega}{p_a} \left(\frac{R}{C} \right)^2, \\ \sigma^* &= \frac{\sigma}{p_a r_0}, \quad \bar{r} = \frac{r}{r_0}, \quad P_0 = \frac{p_0}{p_a}, \quad \rho^* = \frac{\rho_{a0}}{\rho_i} \end{aligned}$$

Governing equations in non-dimensional form

become :

$$\frac{\partial}{\partial X} \left[\frac{\bar{\rho} H^3}{12 \bar{\eta}} \frac{\partial P}{\partial X} \right] + \left(\frac{R}{L} \right)^2 \frac{\partial}{\partial Y} \left[\frac{\bar{\rho} H^3}{12 \bar{\eta}} \frac{\partial P}{\partial Y} \right] = A \frac{\partial}{\partial X} \left[\frac{\bar{\rho} H}{2} \right] \quad (11)$$

$$\bar{\eta} = 1 + 1.5\beta \quad (12)$$

$$\frac{P + 2 \frac{\sigma^*}{\bar{r}}}{P_0 + 2\sigma^*} = \left(\frac{1}{\bar{r}} \right)^3 \quad (13)$$

$$\beta = \frac{1}{1 + \frac{1 - \beta_0}{\beta_0} \left(\frac{1}{\bar{r}} \right)^3} \quad (14)$$

$$\bar{\rho} = \beta \rho^* \left(\frac{P + 2 \frac{\sigma^*}{\bar{r}}}{P_0 + 2\sigma^*} \right) + (1 - \beta) \quad (15)$$

Boundary Conditions used are expressed in non-dimensional form as :

$$\frac{\partial P}{\partial X} = 0, \quad P = 1 \quad \text{at} \quad X = X_r \quad (16)$$

$$P = 1 \quad \text{at} \quad X = 0, \quad X_r \leq X \leq 2\pi \quad (17)$$

$$P = 1 \quad \text{at} \quad Y = 0, \quad Y = 1 \quad (18)$$

2.4 Numerical calculations

Input data are σ^* , β_0 , ρ^* , ε , A , P_0 , and L/R . To get the non-dimensional pressure distribution of a bearing, Eqs.(11) - (15) are solved simultaneously by using the following iterative method.

Step 1: Assign initial non-dimensional pressure distribution of a bearing.

Step 2: Get the non-dimensional radius distribution from Eq.(13).

Step 3: Obtain the non-dimensional volume fraction distribution from Eq.(14).

Step 4: Get the non-dimensional viscosity distribution by Eq.(12).

Step 5: Calculate the non-dimensional density distribution by Eq.(15).

Step 6: Renew the non-dimensional pressure distribution by Eq.(11). Finite difference method using SOR (Successive Over-Relaxation) is used to solve Eq.(11).

Step 7: Repeat Step 2~Step 6 until the convergence criterion of pressure is satisfied.

Distributions of P , β , $\bar{\eta}$, $\bar{r}$, and $\bar{\rho}$ in the journal bearing are obtained as results of the convergence. Load carrying capacity and attitude angle are obtained to analyze the static characteristics of the journal bearings lubricated by bubbly oil.

The following constant values are used in the calculations unless otherwise noted.

$$P_0 = 1, \quad \sigma^* = 0.07, \quad \rho^* = 0.0014, \\ L/R = 1.0, \quad \varepsilon = 0.6, \quad \beta_0 = 0.2, \quad A = 36$$

3. Results and Discussion

Figure 2 shows the non-dimensional pressure profiles of the mid-plane of a bearing for an incom-

Fig. 2 Comparison of non-dimensional pressure profiles at the mid-plane of a bearing with and without bubbles

pressible non-bubbly case and for an compressible bubbly case of initial volume fraction $\beta_0 = 0.2$. It can be seen that maximum non-dimensional pressure for $\beta_0 = 0.2$ is higher than that of a non-bubbly case, and the location of the maximum non-dimensional pressure shifts towards downstream of bearing. This phenomenon affects stability in the case of tilting pad bearings as Tonder reported⁽⁵⁾.

When non-dimensional pressure of lubricant is varying from 1 to over 9, bubbles shrink to about a half of initial size in radius as can be seen from the non-dimensional pressure profiles of Fig. 2 and Fig. 3 (a). Because bubble size changes in a bearing, volume fraction is also modified as modeled in Eq. (14).

In Fig. 3(b), the volume fraction decreases to a sixth of the volume fraction of supply lubricant at the position of maximum pressure. Figure 3(c) shows that non-dimensional density increases about 20% higher than the initial non-dimensional density of the atmospheric pressure at the position of maximum pressure. Non-dimensional viscosity of mixture is shown in Fig. 3(d). As approaching the high-pressure zone of journal bearings, the viscosity of mixtures is lowered.

Initial bubble size and surface tension of bubbly lubricant also affect the load carrying capacity of bearings. Figure 4 shows the effect of non-dimensional surface tension for the constant value of volume fraction of the supplied lubricant. Even though the effect of non-dimensional surface tension is not so big, it can be seen from the figure that the supply lubricant with larger non-dimensional surface tension gives smaller Sommerfeld number, which means high load carrying capacity. Larger non-dimensional surface tension means smaller bubble size or stronger

Fig. 3(a) Contour map of non-dimensional bubble radius $\bar{r}$ in a bearing

Fig. 3(b) Contour map of volume fraction β

Fig. 3(c) Contour map of non-dimensional mixture density $\bar{\rho}$

Fig. 3(d) Contour map of non-dimensional viscosity $\bar{\eta}$

Fig. 4 Effect of non-dimensional surface tension σ^*

surface tension as can be seen from $\sigma^* = \frac{\sigma}{p_a r_0}$ and Eq. (8).

Figure 5(a), (b), (c) show the effect of initial volume fraction on the load carrying capacity for the value of $\epsilon=0.2$, 0.4, and 0.6. The load carrying capacity does not change monotonously with the increase in volume fraction. That is different from the result of the literature⁽⁸⁾ that shows monotonous increase of the load carrying capacity with the increase in volume fraction. There is a “critical volume fraction” β_{cr} that gives the minimum Sommerfeld number or the maximum load carrying capacity for each eccentricity ratio. At the condition used for Fig. 5(a), (b), (c), the increment of load carrying capacity at the critical volume fraction is about 10% compared with non-bubbly case. At low eccentricity ratio, for example $\epsilon=0.2$, the critical volume fraction is 0.1, and the value of volume fraction β_e , which gives the same

Fig. 5(a) Effect of initial volume fraction β_0 ($\epsilon=0.2$)

Fig. 5(b) Effect of initial volume fraction β_0 ($\epsilon=0.4$)

Fig. 5(c) Effect of initial volume fraction β_0 ($\epsilon=0.6$)

Sommerfeld number as the non-bubbly case, is 0.33. This means that up to the volume fraction of 0.33, the load carrying capacity is higher than that of non-bubbly case. For $\epsilon=0.6$ the critical volume fraction increases above 0.2, and β_e is about 0.6 which is greater than that of $\epsilon=0.2$. Beyond the value of β_e , the Sommerfeld number becomes larger than that of non-bubbly case, that is, smaller load carrying capacity than that of non-bubbly case. From these results, it can be concluded that at high eccentricity ratio, the

Fig. 6 Critical volume fraction β_{cr} vs. eccentricity ε

load carrying capacity of the bubbly lubricant is greater than that of non-bubbly case in a fairly wide range of volume fraction. However at low eccentricity ratio, β_e is small, that is, increase in volume fraction can lower the load carrying capacity more easily. Attitude angle also varies remarkably at low eccentricity ratio. Therefore more attention should be paid for the air not to be entrained too much at the operation condition of low eccentricity ratio.

The existence of the critical volume fraction for mixtures can be explained by the relationship between the viscosity increase and compressibility increase. In a low volume fraction range, the effect of the viscosity of air-oil mixture is more remarkable than that of compressibility. However, in high volume fraction, the effect of compressibility of excessive air predominates. As a result, the critical volume fraction comes to exist.

Nikolajsen's model⁽⁸⁾ shows that the load carrying capacity of mixtures increases monotonously by two times than pure oil as the volume fraction increases. Nikolajsen's model overestimates the effect of volume fraction on the viscosity of mixtures, because it does not show the existence of the critical volume fraction.

The critical volume fraction β_{cr} changes according to the eccentricity ratio ε and the ratio of bearing length to bearing radius L/R . Small β_{cr} means that the compressibility takes effect easily with increase in volume fraction, while large β_{cr} means that the viscosity affects strongly with increase in volume fraction than the compressibility does. Figure 6 shows that the critical volume fraction increases as the eccentricity ratio increases for a fixed Λ . That means the viscosity takes effect more with increase in volume fraction than the compressibility does. As for the effect of L/R , larger β_{cr} is obtained for larger L/R at high ε .

Figure 7(a), (b), (c) show the relations between the critical volume fraction and bearing

Fig. 7(a) Critical volume fraction β_{cr} vs. bearing number of mixture Λ ($\varepsilon=0.2$)

Fig. 7(b) Critical volume fraction β_{cr} vs. bearing number of mixture Λ ($\varepsilon=0.4$)

Fig. 7(c) Critical volume fraction β_{cr} vs. bearing number of mixture Λ ($\varepsilon=0.6$)

number Λ . $\beta_{cr}=1$ means that load carrying capacity increases monotonously as the volume fraction increases up to 1.0. This result, in the given Λ , comes from that the effect of viscosity is more dominant than that of compressibility up to $\beta=1$. For a fixed ε , large Λ gives less β_{cr} than small Λ does, that is, the

Fig. 8 Critical volume fraction β_{cr} vs. non-dimensional surface tension σ^*

compressibility takes effect more with increase in volume fraction than the viscosity does.

From Fig. 8, it can be known that the critical volume fraction β_{cr} tends to increase slightly as the non-dimensional surface tension σ^* increases. Moreover the critical volume fraction β_{cr} is higher for higher value of L/R . For large Λ , which is general use for bearings, the effect of ϵ and L/R for β_{cr} is dominant, however the effect of Λ and σ^* is trifling.

4. Conclusion

The effect of bubbly lubricant on the static performance of journal bearings is analyzed theoretically. Surface tension, and volume fraction were considered as parameters. The results presented in this paper have shown that:

1. The load carrying capacity increased about 10% compared with non-bubbly case for the sample analyses of this paper. However excessive entrainment of air can generally lower the load carrying capacity than that of liquid lubricant itself. Attitude angle decreases with the increase in volume fraction.
2. As non-dimensional surface tension increases, that is, as initial bubble size becomes smaller or surface tension increases, the load carrying capacity of journal bearings increases.
3. There can be a critical volume fraction that gives maximum load carrying capacity. Below the critical volume fraction the load carrying capacity increases with increase in volume fraction, but beyond the critical volume fraction it decreases. For some cases the critical volume fraction becomes 1.0, that is, the load carrying capacity increases monotonously with the increase in the volume fraction.
4. The critical volume fraction increases as the

eccentricity ratio or the non-dimensional surface tension increases, however it decreases as the bearing number increases. Moreover it also depends on the ratio of bearing length to bearing radius.

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