Analysis of Stress and Strain Distribution in the Artery Wall Consisted of Layers with Different Elastic Modulus and Opening Angle*

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Bovine thoracic aorta is stiffer in the inner wall than in the outer, and its opening angle is larger in the inner layer than in the outer. A model for mechanical analysis of such a heterogeneous artery wall was developed. The wall was assumed to be made of thin, incompressible, homogeneous, and isotropic layers having different elastic properties and opening angle. Stress and strain distributions in the wall were calculated using the opening angle and stress-strain relationship measured in thin sliced layers of bovine thoracic aortas. Stress distribution was uniform under a physiological condition if elastic properties and the opening angle were set uniform. Stress distribution was not uniform under any condition when the material heterogeneity was introduced. Such non-uniformity was reduced if heterogeneity in the opening angle was considered. The opening angle may be higher in the inner wall to compensate stress concentration caused by the material heterogeneity.

Key Words: Biomechanics, Material Testing, Elasticity, Residual Stress, Blood Vessel Wall, Stress/Strain Analysis

1. Introduction

Blood vessel walls have been assumed to be homogeneous in most of the studies on the mechanical analysis of arteries. Arterial walls are, however, not homogeneous. Some papers have suggested that the stiffness of the arterial wall is different in the radial position[1],[2]. Using bovine carotid arteries, von Maltzahn et al.[3] showed that the adventitial layer is stiffer than the medial layer. Yu et al.[4] performed a bending test on the porcine aorta to find that the Young's modulus of intima-media layer is about 10 times higher than that of adventitial layer. Similar results have been obtained in the rat aorta[5]. Recently, Ohashi et al.[6] measured local elastic modulus in the wall of bovine thoracic aorta with a micropipette aspiration technique[7] to find that elastic modulus was 162 kPa near the inner wall and it decreased almost linearly to the outer wall, and was almost zero in the outer wall. Quantitative image analysis showed that relative content of collagen to smooth muscle cell was much higher near the inner wall than the outer[8]. Matsumoto et al.[9] sliced the bovine thoracic aorta in the circumferential direction into 4 layers, measured stress-strain relationship of each layer, and found similar results to Ohashi et al.

Arteries can be heterogeneous in terms of residual stress also. If you cut a ring-like short segment of an artery radially, then the ring opens up to form an arc, indicating compressive and tensile residual stresses in the inner and outer walls, respectively. Conventionally, this configuration has been assumed to be stress-free and the opening angle, i.e., the angle subtended by the two radii drawn from the midpoint of the inner wall arc to the inner tips has been used as an index of residual stress[10]. However, Vossoughi et al.[11] showed that the opening angle is not uniform. They cut a ring-like segment of a bovine thoracic

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aorta radially and then sliced it into inner and outer rings to show that the inner ring opened up more, and the outer ring almost closed. Similar result were observed in bovine carotid arteries by Greenwald et al. These results suggest that arteries are heterogeneous with respect to residual stress and strain and opened-up configuration is not stress-free. Thus, we need another approach for stress and strain analysis in the artery wall taking such heterogeneity into account.

In this study, we have established a method to calculate the stress and strain distribution in the artery wall having heterogeneities in material properties and residual stress, and studied the effects of such heterogeneities on stress and strain distribution in the bovine thoracic aortas.

2. Model of Heterogeneous Artery Wall

Figure 1 shows the coordinates and variables used in the present analysis. We focused only on circumferential components for simplicity. The arterial wall is assumed to be made of thin layers having different elastic properties and opening angle. Four states were considered: no load, opened-up, stress-free, and in vivo states. The no load state is the state in which the ring-like segment is. When you cut the ring radially, it opens up. This state was taken as the opened-up state, which had been used as the stress-free state in conventional theory assuming homogeneity of the wall. In this study, the stress-free state was taken as the state in which the opened-up specimen is separated into layers which are stress-free. The in vivo state is the state in which the artery wall is pressurized and stretched in the longitudinal direction as in its physiological condition.

The following assumptions were made: 1) aortic wall consists of N thin, incompressible, homogeneous, and isotropic layers having different initial elastic modulus $E(n)$; 2) each layer is a thin-walled cylinder having equal thickness, $\delta r$, inner radius $r_i(n)$, and outer radius $r_o(n)$ in the no load state; 3) it springs open upon radial cutting and slicing to form a stress-free arc whose opening angle, inner and outer radii are $a^*(n)$, $r^*(n)$ and $r^*(n)$, respectively; 4) strain is continuous, i.e., strain in the outer wall of layer $n$ is equal to that in the inner wall of layer $n+1$; 5) $r_i = r_i(1)$, $r_o = r_o(N)$, $\theta^*(n) = \pi - \alpha^*(n)$, $E(n)$, and $N$ are given.

2.1 Residual strain and stress

Strain in the stress-free state is obtained by comparing the configurations in the no load and stress-free states. In this study, they were calculated from the no load radii, the opening angle, and the initial elastic modulus of each layer. The no load radii $r_i(n)$ and $r_o(n)$ are obtained from inner and outer radii ($r_i = r_i(1)$ and $r_o = r_o(N)$, respectively) of a ring-like segment as follows:

$$r_o(n) = r_i(n+1) = r_i + n \delta r = r_i + \frac{n}{N} (r_o - r_i). \quad (1)$$

Incompressibility of wall material yields:

$$\pi(r_o(n)^2 - r_i(n)^2) = \theta^*(n)(r_o(n)^2 - r_i(n)^2). \quad (2)$$

Strain in the form of stretch ratio in the outer wall of a layer is equal to that in the inner wall of its outer adjoining layer:

$$\lambda^*(n) = \frac{2 \pi r_o(n)}{2 \theta^*(n) r_i^*(n)} = \frac{\pi r_o(n)}{\theta^*(n) r_i^*(n)} = \lambda(n+1) \frac{\pi r_o(n+1)}{\theta^*(n+1) r_i^*(n+1)}$$
\[
\frac{\pi r_c(n)}{\theta^*(n+1)r^*(n+1)} = 0
\]
(3)
\[
\therefore \theta^*(n)r^*(n) = \theta^*(n+1)r^*(n+1)
\]
(3')

All of the radii in the no load state \(r(n)\) and \(r_c(n)\) had been obtained from Eq. (1). Those in the stress-free state \(r^*(n)\) and \(r^*_c(n)\) are obtained by applying Eqs. (2) and (3') alternately for a radius \(r^*_1(1)\). The radius \(r^*_1(1)\) is determined considering that the sum \(F\) of circumferential force in each layer is zero in the no load state:
\[
F = \sum\sigma_n(n) \cdot \partial r = \sum E(n)(\lambda_n(n)-1) \cdot \partial r = 0
\]
\[
\Rightarrow \sum E(n)(\lambda_n(n)-1) = 0
\]
(4)
where \(\sigma_n(n)\) is the mean stress of layer \(n\), and \(\lambda_n(n)\) is the mean stretch ratio in the layer \(n\):
\[
\lambda_n(n) = (\lambda(n) + \lambda_c(n))/2.
\]
(5)

We assumed linear elasticity to calculate residual stress because residual strain are relatively low (< 20%). The residual strain in layer \(n\) in the form of stretch ratio \(\lambda_n(n)\) can be obtained from Eq. (3), and the residual stress \(\sigma_n(n)\) as
\[
\sigma_n(n) = E(n)(\lambda_n(n)-1).
\]
(6)

2.2 Strain and stress in the opened-up state

The opened-up state is conventionally assumed to be stress-free. This is not the case if you consider heterogeneity in the residual stress. Stress and strain in this state are obtained by comparing the configurations in the opened-up and stress-free states. Stretch ratio in layer \(n\) in the opened-up state is given by definition as
\[
\lambda(n) = \frac{\partial r_c(n)}{\theta^*(n) r^*(n)}, \quad \lambda(n) = \frac{\partial r_c(n)}{\theta^*(n) r^*(n)}
\]
(7)

Incompressibility of wall material yields:
\[
\pi (r_c(n)^2 - r(n)^2) = \theta (r_c(n)^2 - r(n)^2).
\]
(8)

From the integrity of the wall:
\[
r(n) = r_c(n+1).
\]
(9)
The sum of circumferential force \(F\) and the sum of moment \(T\) in each layer is zero in this state:
\[
F = \sum E(n)(\lambda(n)-1)(r_c(n) - r(n)) = 0,
\]
(10)
\[
T = \sum E(n)(\lambda(n)-1)(r_c(n) - r(n)) \cdot r_c(n) = 0
\]
(11)
where
\[
\lambda(n) = (\lambda(n) + \lambda_c(n))/2
\]
(12)
and
\[
r_c(n) = (r(n) + r_c(n))/2
\]
(13)

All of the radii in the opened-up state can be obtained from Eqs. (8) and (9) for given \(r_c\) and \(\theta\). The stretch ratios are then obtained from Eqs. (7) and (12), and

\[
N, \text{number of layers; } r_i \text{ and } r_o \text{, inner and outer radii in the no load state; } \sigma^*(n), \text{ opening angle in layer } n; \ E(n), \text{ initial elastic modulus in layer } n(4); \ r^*_1(1), \text{ inner radius of layer 1 in the stress-free state; } r(1), \text{ inner radius of layer 1 in the opened-up state; } \theta, \text{ central angle of the opened-up configuration. Negative radius indicates the opening angle}>180°.
\]

the stresses \(\sigma_n(n)\) as
\[
\sigma_n(n) = E(n)(\lambda_n(n)-1)
\]
(14)
assuming linear elasticity. Stresses and strains obtained for \(r^*_i\) and \(\theta\) satisfying Eqs. (10) and (11) were taken as the solution.

2.3 Strain and stress in the in vivo state

In vivo radii were obtained for various circumferential and axial stretch ratios (\(\lambda_c\) and \(\lambda_s\), respectively) by assuming the incompressibility:
\[
R(n-1) = R_c(n) = \sqrt{R_0^2 - r^2 - r^2_c(n)}
\]
(15)
where the circumferential stretch ratio was defined as \(\lambda_c = R_c/R_0\) and the longitudinal stretch ratio as the ratio of physiological length \(L\) to no load length \(l\), i.e., \(\lambda_s = L/l\). In vivo strains were obtained in the form of stretch ratios from Eqs. (3) and (5) by substituting these in vivo radii with no load radii. In vivo stress corresponding to each strain was obtained by referring to the stress-strain curves obtained for each layer.

All of these calculations were performed on a spreadsheet of Excel 2001 for MacOS (Microsoft Corp.) with a macro of our own making.

2.4 Inputs to the model

The number of layers for calculation \(N\) was taken as 50. No load dimensions and opening angles of each layer were obtained in the present study for bovine thoracic aortas and shown in the next section. As for the initial elastic modulus of each layer \(E(n)\), data obtained by Ohashi et al.\(^{10}\) were used. The stress-strain curves measured in the bovine thoracic aortas sliced into 4 layers\(^{17}\) were used to obtain in vivo stress. Thus, in vivo strain and stress were evaluated in 4 layers, while other parameters in 50. Actual values used for the analysis are later summarized in Table 1.

Five tubular segments of the bovine thoracic aortas were obtained from a local slaughter house and stored in a physiological saline solution at 4°C until the experiment which was performed within 24 hours. After loose connective tissues were removed carefully, a ring-like specimen having the axial length of about 7 mm was cut out from each segment. After the measurement of its inner and outer radii in the no load state, \( r_i \) and \( r_o \), respectively, the specimen was cut longitudinally to measure its opening angle \( \alpha \). It was then laid flat and cut into halves. Then each of the halves were sliced into 4 layers (layer 1 being the innermost) having thickness of about 1 mm with a trimming blade (No. 260, Feather) with a special jig. The opening angle of the half ring \( \alpha^* \) was measured again for each layer and was converted to that of the whole ring \( \alpha^* \) as \( \alpha^* = 2\alpha^* - \pi \). Each layer was then weighed to obtain its relative volume assuming that their specific gravity was the same. Then their radial position in the wall, i.e., the relative distance between the center of each layer and the lumen was calculated. All of the specimens were manipulated in the physiological saline at room temperature.

4. Results

Figure 2 summarizes the opening angle of each layer along with that of the whole wall. The opening angle was higher near the inner wall and decreased linearly as you go from the inner to the outer wall. The difference was significant between layer 1 and 4 \( (P<0.02, \text{ Student's } t\text{-test}) \). The opening angle of the whole wall was 139±21° (mean±SEM, \( n=5 \)) and was very close to the average of all data (141°). By applying linear regression as the first step, we estimated that the opening angle was 226° in the inner wall and 27° in the outer. Mean inner and outer radii were 11.48±0.42 and 17.45±0.30 mm (mean±SEM, \( n=5 \)), respectively.

Data used for the analysis are summarized in Table 1 along with the cases studied. The number of layers was taken as 25 to obtain results insensitive to \( N \) based on a pilot calculation with \( N=25 \) and 40. No load dimensions \( r_i \) and \( r_o \), and the opening angle \( \alpha^* \) were obtained in the present analysis. Local elastic modulus \( E(n) \) was adopted from Ohashi et al.\(^4\). Radii \( r_i^*(1) \) and \( r_o^*(1) \) and central angle \( \theta \) were determined for each case to satisfy Eqs. (4), (10), (11). Calculation was performed in three cases A-C. In the case A (hetero E, hetero \( \alpha \)), \( E(n) \) was assumed to be 162 kPa at the inner wall and to decrease linearly to 0 kPa at the outer. The opening angle \( \alpha^*(n) \) was assumed to change linearly as shown in the above. In the case B (hetero E, homo \( \alpha \)), the distribution of \( E(n) \) was the same as in the case A and \( \alpha^*(n) \) was assumed to be 143°, the opening angle of the opened-up ring calculated for case A, for all layers. In the case C (homo E, homo \( \alpha \)), \( E(n) \) was assumed to be 96 kPa, mean initial elastic modulus of the whole wall obtained from a tensile test and \( \alpha^*(n) \) to be 143° for all layers. The case C is the conventional case.

Figure 3 shows residual strain distributions in the bovine thoracic aorta obtained for three cases. The distribution of residual strain was almost linear and its average was zero for the homogeneous case (case C). The distribution was also almost linear in the case B, but the curve shifted upwards because the inner wall is assumed to be stiffer than the outer. The strain distribution became curvilinear when the inhomogeneity of the opening angle was considered (case A). The strain decreased slightly in the inner and outer walls and increased slightly in the mid-wall region. The change in strain was less than 0.02 except...
Fig. 4 Intramural distribution of residual stress in the bovine thoracic aorta obtained for the three cases.

Fig. 5 Intramural distributions of stress and strain in the opened-up ring of the bovine thoracic aorta. Those in the unopened ring, i.e., residual components are also shown for comparison.

near the outer wall.

Residual stress distributions corresponding to Fig. 3 are shown in Fig. 4. In the case C, the distribution was almost linear and its average was zero, similar to the strain. In contrast, the distribution was very much different from the strain in the case B. It was larger in the inner wall and zero in the outer wall because the elastic modulus is largest in the inner wall and zero in the outer. In the case A, the distribution was almost similar to that in the case B. The stress is a little bit larger in the inner wall and mid-wall region, similar to the change in the strain from the case B to A.

Intramural distribution of strain and stress in the opened-up state are shown in Fig. 5 for the case A. Residual strain and stress are also shown for comparison. Strain and stress were not completely released in the opened-up configuration when heterogeneity in the residual strain was considered, which did not happen in other cases. As imagined from the increase in the opening angle in the inner wall and the decrease in the outer by the slicing, the strain in the opened-up state was compressive near the inner and outer walls and tensile in the mid-wall region. The stress distribution was more complicated than the strain distribution as happened in the residual strain and stress in the case A (Figs. 3 and 4).

In vivo strain and stress distributions are shown in Fig. 6 for the cases A and B. They were calculated for longitudinal stretch ratio \( \lambda_0 \) of 1.2 and circumferential stretch ratio \( \lambda_0 \) of 1.2, 1.4, and 1.6. In the case B, the strain was almost uniform at \( \lambda_0 = 1.6 \) due to the residual strain. Contrary to the strain, the stress did not become uniform in any conditions because of the marked heterogeneity in the mechanical properties of the bovine aorta. Compared to those for the case B, the strain in the case A decreased slightly in the inner and outer walls and increased slightly in the mid-wall region, which was similar modulation to those observed in the residual strain and stress (Figs. 3 and 4). When the strain distribution was converted to stress distribution, the effect of such modulation became remarkable. High stress concentration in the sub-intimal region observed in the case B attenuated in the case A. The ratio of stress in the sub-intimal region to that in the inner half of the wall decreased from 1.14 to 1.01 with the heterogeneity of the opening angle in a typical condition (\( \lambda_0 = 1.6 \) and \( \lambda_0 = 1.2 \)).

5. Discussion

Although there have been a lot of studies on stress and strain analysis of vascular walls, studies taking wall heterogeneity into account are very few and most of the studies on the heterogeneity were performed on two-layered models. In this study, we have
developed a new method to obtain the stress and strain distributions in the artery wall in which the mechanical properties and residual strain change gradually in the radial direction. The present method is based on a widely-used spreadsheet with a macro and it takes only several minutes to obtain solutions on PowerBook G4 (Apple Computer Inc.). Thus, the present method can be used widely and easily for the study of blood vessel wall mechanics. (The authors are happy to distribute the sheet upon request.)

The heterogeneity in the residual strain was first pointed out by Vossoughi et al. using a ring-like segment of the thoracic aorta obtained from a 1500 lb cow. The opening angle measured in Fig. 2\(^a\) was 49° for whole wall and 109° and 10° for inner and outer rings, respectively. Although statistical comparison is difficult for the paucity of their data, the opening angle obtained in the present study seems a little larger than theirs. One of the possible reasons for this difference would be the difference in the specimen thickness or the longitudinal length of the ring-like segment: their specimen was 3 mm long and ours about 7 mm. If the specimen is too long, residual stress in the longitudinal direction might not be released fully, which increases residual stress in the circumferential direction and thus causes the increase in the opening angle. Although it has been reported that the opening angle is insensitive to the longitudinal length of the segment in case of the rabbit thoracic aorta\(^{16}\), it might be necessary to examine whether this is the case for bovine thoracic aortas.

Since Vaiashnav and Vossoughi\(^{14}\) and Fung et al.\(^{15}\) suggested the importance of residual stress in the artery wall mechanics, there have been many papers suggesting that stress and/or strain distribution is uniform in a physiological state\(^{11,12,14-16}\). However, the stress distribution in the bovine thoracic aorta is not uniform as shown in Fig. 6 due to its material heterogeneity. The stress concentration appeared near the inner wall. This is an interesting finding because it may suggest that strain but not stress is maintained at a constant level in a physiological condition. The stress concentration in the inner wall attenuated when considering the heterogeneity in the residual stress. The larger opening angle near the inner wall may be the adaptation of the bovine aortic wall to reduce the imbalance of the stress distribution.

There are some limitations to this study. Deformation in the axial direction is not considered in the present model. Arterial walls deform not only in the plane perpendicular to the blood vessel axis but also in the axial direction when cut radially and sliced into layers. Although the in-plane deformation is dominant on the circumferential strain and stress, it would be necessary to extend the model for 3D deformation of the specimen. Linear elasticity was assumed when obtaining the residual stress and stress in the opened-up specimen, although the maximum of the residual strain was over 15%. In case of rabbit thoracic aorta, tensile stress at 20% strain estimated from initial elastic modulus was 20% lower than the actual stress obtained from a pressure-diameter test (unpublished observation). In vivo stress was obtained only for 4 layers. To improve the analysis, we need to increase the number of sliced layers whose elastic modulus and opening angle are measured for the analysis. The slicing have been done manually at this moment, and 4 to 5 layers are maximum for bovine thoracic aortas. The use of slicing apparatus such as a micro slicer is necessary to increase the number of slices.

In summary, to obtain stress and strain in the heterogeneous wall, we formulated a new stress/strain analysis method. It is confirmed that the bovine thoracic aorta is heterogeneous both in elastic modulus and in residual strain. The higher elastic modulus near the inner wall caused high stress concentration in the inner wall even though the opening angle of the whole wall was considered. Instead, the larger opening angle near the inner wall reduced such concentration. The opening angle may be higher in the inner wall to compensate stress concentration caused by the material heterogeneity.

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