Model-Following Controller Based on Neural Network for Variable Displacement Pump*

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The variable displacement axial piston pump (VDAPP) is inherently nonlinear, time variant and subjected to load disturbance. The controls of flow and pressure of VDAPP are achieved by changing the swashplate angle. The swashplate actuators are controlled by an electro-hydraulic proportional valve (EHPV). It is reasonable for swashplate angle of a VDAPP to employ neural network based on adaptive control. In this study, the nonlinear model of the VDAPP with a three-way electro-hydraulic proportional valve is proposed, and a neural network model-following controller is designed to control the swashplate swivel angle. The time response for the swashplate angle is analyzed by simulation and experiment, and a favorable model-following characteristic is achieved. The proposed neural controller can conduct nonlinear control in VDAPP, enhance adaptability and robustness, and improve the performance of the control system.

**Key Words**: Variable Displacement Axial Piston Pump, Proportional Valve, Neural Network, Model-Following Adaptive Control

1. Introduction

The VDAPP has been used in industrial and heavy-load systems for years. They can transmit large specific powers with high efficiency and low cost. Their flow rate can be varied and controlled by changing the swashplate angle. The swashplate actuators are controlled by an EHPV, which consists of a proportional solenoid direct attached to a three-way spool valve. The valve spool is positioned by a proportional solenoid and directs controlled flow to the swashplate actuator.

The EHPV requires closed loop control to improve the valve performance, and a linear PD controller is always used as a valve spool controller. The valve spool position, picked up by a transducer, is a feedback signal to valve spool controller.

There are two actuators with different piston areas to drive the swashplate. One is the control actuator with larger piston area; the other is the bias actuator with smaller piston area. The three-way spool valve meters flow into the control actuator for driving the swashplate counterclockwise, and meters flow out from the control actuator to drive the swashplate clockwise. It is benefit to stabilize swashplate angular response and with fail-safe by this way.

Several studies have been conducted in an attempt to investigate and improve the dynamics and control of axial piston pumps. Yamaguchi and Ishikawa(1) examined the effects of load pressure on pump dynamic using a simplified form of the swashplate equation of motion, and investigated the characteristic of the two-way servo valve control mechanism used in both a swashplate type of axial piston pump and a three-way valve control for the bent-axis type piston pump. Mack et al.(2) examined the feasibility of interfacing a microcomputer to a variable displacement pump for the purpose of controlling flow rate and providing pressure compensation for the pump action. Akers and his colleagues(3)(4) looked at the pressure transient in the context of moments acting on the swash plate, and applied optimal control the-
ories to design a pressure regulator for an axial piston pump with single-stage electro-hydraulic servo valve. Inoue\textsuperscript{59} extends Akers' model to include the dynamic characteristic. Schoneau et al.\textsuperscript{60} built on the results of Zeiger and Akers\textsuperscript{59} to include dynamics of pump actuation. Due to the complex and nonlinear nature of pumping dynamics, previous researchers have simplified the pump dynamics by linear approximation, and few of them have developed control strategy.

In industrial application, a PD controller without adaptability is usually applied for pump control. And a dip switch is usually applied for adjusting the parameters of the PD controller for different pressure load conditions. But there are two defects by this way: (1) The dip switch is manual and disabled to be adjusted automatically and continuously, so that only several load pressure conditions are suitable for the designed parameters of PID controller. The best performance of pump could not be achieved. (2) Because the controller lacks adaptability to keep steady response when the control surrounding is changed, sometimes it causes overshoot of pressure response.

It is reasonable for a variable displacement pump control system, which is inherently nonlinear, time variant and complicated dynamics, to employ adaptive control. But the conventional adaptive control techniques are usually based on system model parameters. The unavailability of the accurate pump model leads to a cumbersome design approach. And real-time implementation is often difficult and sometimes not feasible because of using a large number of parameters in these adaptive schemes.

In recent years, the neural network controllers have successfully been put into use in various fields owing to their capability of self-learning and adaptability. Tremendous studies of neural network controller have been conducted to dynamic systems. Psaltis et al.\textsuperscript{7} proposed the general learning architecture, populated the input space of the plant with training samples so that the network can interpolate for intermediate points. Another control scheme, the specialized learning architecture, learns by directly evaluating the accuracy of the network, that is, the error between the actual and desired output of the plant is used to update the connective weights in the network. In this sense, the controller learns continuously, and hence it can control plants with time-varying characteristics.

There are two strategies to facilitate the specialized learning, one being direct control and the other indirect control. In the former, the plant can be viewed as an additional but no modifiable layer of the neural network. The latter, which has been used in many applications\textsuperscript{60\textsuperscript{-110}}, is a two-step process including identification of dynamics of plant and control.

In the indirect control strategy, a sub-network (called "emulator") is required to be trained before the control phase, and the quality of the trained emulator is crucial to the controlling performance. It is therefore very important that the data sets for training the emulator must cover a sufficiently large range input and output pairs, but it is very possible that the future behaviors in on-line control may outside the range that was used during the emulator's training, the backpropagation through the emulator fails, causing poor or even unstable control performance.

The direct control strategy can avoid this problem if a priori qualitative knowledge or Jacobian (the partial derivative of plant output to input) of the plant is available. But it is usually difficult to approximate the Jacobian of an unknown plant. Zhang and Sen\textsuperscript{11} presented a direct neural controller for on-line industrial tracking control application, and a simple sign function applied to approximate the Jacobian of a ship track keeping dynamics. Results from a nonlinear ship course-keeping simulation were presented, and the on-line adaptive control has been successfully achieved. Lin et al.\textsuperscript{12} used the same way to design on-line trained neural network model-following controller to control the rotor position of an ultrasonic motor. An accurate tracking response can be obtained by random initialization of the weights and biases of the network, owing to the powerful online learning capability. For increasing the speed of convergence, Lin and Wa\textsuperscript{12\textsuperscript{-14}} proposed the $\delta$ adaptation law to increase the on-line learning rate of the weights and overcome the problem of approximating the Jacobian of an unknown plant. They designed a neural network controller with the $\delta$ adaptation law for PM synchronous servo motor drive, and preserved a favorable model-following characteristic under various operating conditions. Lin et al.\textsuperscript{1,12} have proved the convergence of the learning algorithm with the $\delta$ adaptation law.

Recently neural network controllers have also applied to electro-hydraulic servo system\textsuperscript{12,13}. But a system identifier is always applied in their direct control strategy, which makes real-time implementation become difficult. In this study, a direct neural controller with specialized learning architecture is proposed for the pump swashplate angle control, characterized by the simplicity of its structure and real-time implementation.

In this study, the nonlinear model of the variable displacement pump with three-way EHPV is established, and a neural network model-following control-
2. Dynamical Model of the VDAPP System

The schematic drawing and circuit of VDAPP are shown in Fig. 1. Its physical parameters are defined in Table 1. While the pump not rotating and the control system is pressure less, the swashplate is held in position +100% by the preload spring in bias actuator. In the static condition (swashplate angle command value equals actual swashplate angle value), the spool of the proportional valve is in its central position.

When the spool displacement variation $\Delta X_T$ ($\Delta X_T > 0$ is defined) is exerted from center to right, the proportional valve allows meter-out flow from the control actuator to tank, and the swashplate swivels clockwise. According to Appendix A1, The exerted flow, linearized about the null point, can be expressed as

$$\Delta Q_t = K_w \Delta X_T + K_t \Delta P_t$$  \hspace{1cm} (1)

where $\Delta P_t$ is the pressure variation of control actuator.

According to Appendix B1 the flow continuity into the control actuator is expressed as

$$\Delta P_t = \frac{1}{C_{16}} \left( D_5 \frac{d\alpha_0}{dt} - \Delta Q_t \right)$$  \hspace{1cm} (2)

where $C_t = \frac{V_t}{\beta_t}$.

Substituting Eq. (1) into Eq. (2) gives

$$\Delta P_t = \frac{1}{C_{16} + K_t} \left[ D_5 \frac{d\alpha_0}{dt} - K_w \Delta X_T \right]$$  \hspace{1cm} (3)

The continuity equation of the pump volume, shown in Appendix A3, is achieved

$$\Delta P_t = \frac{1}{C_{16}} \left( d\alpha_0 \frac{d\alpha_0}{dt} - C_t \Delta P_t - \Delta Q_t - \Delta Q_b \right)$$  \hspace{1cm} (4)

where $C_t = \frac{V_t}{\beta_t}$, $V_t =$ volume of discharge system of pump, $d\alpha_0 \frac{d\alpha_0}{dt} =$ pump output flow to discharge system, $C_t \Delta P_t =$ total pump leakage, $\Delta Q_t =$ flow into the load system, and $\Delta Q_b =$ where the swashplate angle is regulated by actuators with internal supply, which is partial flow of pump output flow must be supplied to the bias actuator.

According to Akers and Lin(4) however the symmetric actuators condition of VDAPP is described, for

![Hydraulic circuit](image)

![Schematic drawing of a typical VDAPP](image)

**Fig. 1 A typical VDAPP**

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$[m$^3$/MPa$^{-1}$]</td>
<td>Total pump leakage coefficient</td>
</tr>
<tr>
<td>$D_5$[m$^3$/radian]</td>
<td>Volumetric displacement of control cylinder due to swashplate angle</td>
</tr>
<tr>
<td>$D_5$[m$^3$/min]</td>
<td>Volumetric displacement of bias cylinder due to swashplate angle</td>
</tr>
<tr>
<td>$s_5$[m/s$^{-1}$]</td>
<td>Specific volumetric displacement of pump</td>
</tr>
<tr>
<td>$\alpha$[degrees]</td>
<td>Swashplate inertia</td>
</tr>
<tr>
<td>$K_w$[m$^3$/MPa$^{-1}$]</td>
<td>Total flow-pressure coefficient of the control valve</td>
</tr>
<tr>
<td>$K_h$[radians/mol]</td>
<td>Coefficient of swashplate torque due to pressure</td>
</tr>
<tr>
<td>$K_g$[m$^3$/mol]</td>
<td>Flow gain of control valve</td>
</tr>
<tr>
<td>$K_s$[radians/mol]</td>
<td>Proportional solenoid constant</td>
</tr>
<tr>
<td>$K_r$[radians/mol]</td>
<td>Coefficient of swashplate torque due to swashplate angle</td>
</tr>
<tr>
<td>$K_p$[radians/mol]</td>
<td>Coefficient of swashplate torque due to swashplate angular velocity</td>
</tr>
<tr>
<td>$K_u$[radians/mol]</td>
<td>Gain of PWM driver</td>
</tr>
<tr>
<td>$K_l$[radians/mol]</td>
<td>Neural controller output multiplied gain</td>
</tr>
<tr>
<td>$K_m$[radians/mol]</td>
<td>Gain of speed displacement sensor</td>
</tr>
<tr>
<td>$M_s$[radians/mol]</td>
<td>Spool mass</td>
</tr>
<tr>
<td>$K_s$[radians/mol]</td>
<td>Damping coefficient of spool</td>
</tr>
<tr>
<td>$K_p$[radians/mol]</td>
<td>Stiffness coefficient of spool</td>
</tr>
<tr>
<td>$P_s$[MPa]</td>
<td>Output pressure of pump</td>
</tr>
<tr>
<td>$P_f$[MPa]</td>
<td>Pressure of control cylinder</td>
</tr>
<tr>
<td>$V_t$[liters]</td>
<td>Volume of discharge system of pump</td>
</tr>
<tr>
<td>$\beta$[MPa$^{-1}$]</td>
<td>Bulk modulus of fluid</td>
</tr>
<tr>
<td>$\alpha$[radians/s]</td>
<td>Angular rotational speed of pump</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Proportional constant of valve controller</td>
</tr>
<tr>
<td>$C_v$</td>
<td>Derivative constant of valve controller</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Proportional constant of pump controller</td>
</tr>
<tr>
<td>$K_v$</td>
<td>Derivative constant of pump controller</td>
</tr>
<tr>
<td>$\alpha_z$</td>
<td>Swashplate angle</td>
</tr>
</tbody>
</table>
VDAPP in this study with asymmetric actuator, the relationships between torque and pressure differential across the pump; swashplate angle; and angular velocity of the swashplate are derived

\[ T = K_0 P_2 + J_s \alpha_2 + K_0 \alpha_2 + K_{0a} \alpha_2 \]  \hspace{1cm} (5)

The pressure variation of bias actuator is defined as \( \Delta P_a \). There is no actuator valve to control the flow of bias actuator so that the pressure variation of bias actuator is similar to the pressure variation of pump output pressure i.e. \( \Delta P_a = \Delta P_s \). So that according to Appendix C, we have

\[ \Delta P_s \Delta P_a = J_s \Delta \alpha_2 + K_{0s} \Delta \alpha_2 + K_{0a} \Delta \alpha_2 + K_{0d} \Delta P_3 \]  \hspace{1cm} (6)

where the torque exerted on the swashplate is balanced by the actuators, which have spring and pressure forces acting on them.

Substituting Eq. (3) into Eq. (6) gives

\[ \Delta P_s = \frac{D_1}{C_{is} + K_c} (D_1 \Delta \alpha_2 + K_{0s} \Delta \alpha_2 + K_{0d} \Delta P_3) \]

\[ = J_s \Delta \alpha_2 + K_{0s} \Delta \alpha_2 + K_{0d} \Delta P_3 \]  \hspace{1cm} (7)

If neglecting the effect of compressibility i.e. \( C_i \approx 0 \), the swashplate angle dynamics can be obtained as follows

\[ \Delta \alpha_2 = \frac{(D_1 - K_c) \Delta P_3 D_1}{K_{0s} K_c} \frac{D_1}{C_{is} + K_c} + D_1 K_c \Delta X_r / K_c \]

\[ J_s \Delta \alpha_2 + K_{0s} \Delta \alpha_2 + K_{0d} \Delta P_3 \]  \hspace{1cm} (8)

When the displacement variation of spool \( \Delta X_r < 0 \) is exerted from center to left, the EHPV allows meter-in flow into the control actuator, then the swash plate swivels counterclockwise. In this condition, assume the EHPV meter-in flow gain to be \( K_s \), the EHPV meter-in flow-pressure coefficient to be \( K_i \), spring coefficient to be \( K_c \), and damping coefficient to be \( K_d \).

According to Appendix A, The meter-in flow of control actuator is

\[ \Delta Q_s = -K_{0d} \Delta X_r + K_i (\Delta P_3 - \Delta P_1) \]  \hspace{1cm} (9)

where \( \Delta P_1 \) is the pressure variation of control actuator. According to Appendix B, the flow continuity into the control actuator is expressed by

\[ \Delta P_3 = \frac{1}{C_{is}} (\Delta Q_s + D_1 \Delta \alpha_2) \]  \hspace{1cm} (10)

Substituting Eq. (9) into Eq. (10) gives

\[ \Delta P_3 = \frac{1}{C_{is} + K_c} [D_1 \Delta \alpha_2 - K_{0d} \Delta X_r + K_i \Delta P_3] \]  \hspace{1cm} (11)

According to Appendix C the continuity equations of the pump volume is achieved

\[ \Delta P_3 = [d_s \omega_2 \Delta \alpha_2 - D_1 \Delta Q_s + C_i \Delta P_3 - \Delta Q_s - \Delta Q_s] \frac{1}{C_{is}} \]  \hspace{1cm} (12)

where \( C_i = V_c / \beta_c \), \( V_c \) = volume of discharge system of pump, \( d_s \omega_2 \Delta \alpha_2 \) = pump output flow to discharge system, \( C_i \Delta P_3 \) = total pump leakage, \( \Delta Q_s \) = flow into the load system, \( \Delta Q_s = D_1 \Delta \alpha_2 \) and \( \Delta Q_s = D_1 \Delta \alpha_2 \), where the swashplate angle is regulated by actuators with internal supply, which is partial flow of pump output flow must be supplied to the control actuator.

Substituted Eq.(11) into Eq.(6) gives

\[ \Delta P_s \Delta P_a = -D_1 \frac{D_1}{C_{is} + K_c} [D_1 \Delta \alpha_2 - K_{0d} \Delta X_r + K_i \Delta P_3] \]

\[ + K_i \Delta P_3 \]  \hspace{1cm} (13)

neglecting the compressibility i.e. \( C_i \approx 0 \), the swashplate angle dynamics can be obtained as follows

\[ \Delta \alpha_2 = \frac{(D_1 - K_c) \Delta P_3 D_1}{K_{0s} K_c} \frac{D_1}{C_{is} + K_c} + D_1 K_c \Delta X_r / K_c \]

\[ J_s \Delta \alpha_2 + K_{0s} \Delta \alpha_2 + K_{0d} \Delta P_3 \]  \hspace{1cm} (14)

If \( D_1 / D_2 = 1/2 \), \( K_r < D_2 \) and neglect the spring force effect, then in static condition, leading to \( P_3 D_1 + P_3 D_2 \approx 0 \), and \( P_1 \approx 1/2 P_3 \); the pressure difference of the meter-in side is approximately the same as which of the meter-out side condition, leading to \( K_s \approx K_0 \), \( K_i \approx K_r \). Equation (8) can be expressed the swashplate angle dynamics in both \( \Delta X_r > 0 \) (clockwise) and (counter-clockwise) condition. And \( \Delta Q_s = 2 \Delta Q_s \), Eqs. (4) and (12) can be expressed by

\[ \Delta P_3 = \frac{1}{C_{is}} [d_s \omega_2 \Delta \alpha_2 - C_i \Delta P_3 - \Delta Q_s - \Delta Q_s] \]  \hspace{1cm} (15)

When the spool displacement of EHPV is small (less than 3 mm), then the proportional solenoid can be linearized, the force developed on the solenoid due to electrical current input for operation near null point can be expressed as

\[ \Delta F_s = K_r \Delta \alpha \]  \hspace{1cm} (16)

Applying Newton's second law, we obtain

\[ \Delta F_s = M_r \Delta X_r + B_r \Delta X_r + K_0 \Delta X_r + \Delta F_r \]  \hspace{1cm} (17)

where \( M_r \) is effective mass of the spool, \( B_r \) is damping coefficient of the spool, \( K_r \) is the spool spring stiffness, \( \Delta F_r \) is total friction force of the spool.

Omit \( \Delta F_r \), Eq. (10) is substituted by Eq. (9), the spool dynamics can be expressed as

\[ \Delta X_r + \left[ \frac{B_r}{M_r} \right] \Delta X_r + \left[ \frac{K_r}{M_r} \right] \Delta X_r = \left[ \frac{K_r}{M_r} \right] \frac{1}{M_r} \Delta \alpha \]  \hspace{1cm} (18)

Equation (12) can be modeled by

\[ \Delta X_r + 2 \omega_n \Delta X_r + \omega_n^2 \Delta X_r = \left[ \frac{K_r}{M_r} \right] \omega_n^2 \Delta \alpha \]  \hspace{1cm} (19)

or

\[ \Delta X_r \frac{\Delta \alpha}{\Delta t} = \left[ \frac{K_r}{M_r} \right]^2 \omega_n^2 + 2 \omega_n \Delta X_r \]  \hspace{1cm} (20)

The PWM driver with a voltage to current gain \( K_s \) is applied to provide sufficient current to drive the EHPV. In industrial application, a linear PD controller with the proportional constant \( C_r \) and derivative constant \( C_d \) is usually used as a valve spool controller to improve the performance of valve spool position control, and the valve spool displacement command is denoted as \( X_c \). The integrated VDAPP system of the PD controller, PWM driver, EHPV defined by Eq. (20) and VDAPP defined by Eqs. (8) and (14) can be obtained and shown in Fig. 2.

A PD controller or P controller shown in Fig. 3 (a) is usually applied to control the whole closed loop
3. Design of Neural Network Controller

Cybenko\cite{18} has shown that one hidden layer with sigmoidal function is sufficient to compute arbitrary decision boundaries for the outputs. Although a network with two hidden layers may give better approximation for some specific problems, de Villiers et al.\cite{19} has demonstrated that networks with two hidden layers are more prone to fall into local minima. Furthermore the more CPU time is needed, so that, in this study, a network with single hidden layer is applied.

Another consideration is the right number of units in a hidden layer. Lippmann\cite{20} has provided comprehensive geometrical arguments and reasoning to justify why the maximum number of units in a single

hidden layer should equal to $M(N+1)$, where $M$ is the number of output units and $N$ is the number of input units. Zhang et al.\cite{21} have tested different numbers of units of the single hidden layer. It was found that a network with three to five hidden units is often enough to give good results.

The proposed neural network has three layers with two units in the input layer, one unit in the output layer and five units in the hidden layer. The $\alpha_c$, $\alpha_e$ and $\alpha_r$ denote the required command input, output of the reference model and the output of the controlled plant respectively. The two inputs of the network are the error and its derivative between $\alpha_e$ and $\alpha_r$.

The reference model can be designed by a standard second order transfer function, the damping ratio and natural frequency can be defined based on the physical specification of VDAPP. The neural control system for VDAPP is shown in Fig. 3 (b).

The adaptation law applied to approximate the error of back propagation is described by the following equations.

The proposed direct neural controller is shown in Fig. 4, including: the hidden layer (subscript "j"), output layer (subscript "k"), and input layer (subscript "i"). The input signal is multiplied by gains $K_i$, $K_o$ at the input layer, in order to be normalized within $-1$ and $+1$. A tangent hyperbolic function is used as the activation function of the nodes in the hidden and output layers, the number of units in hidden layer equals to $J$, the number of units in input layer equals to $I$, and the number of units in output layer equals to $K$, the net input to node $j$ in the hidden layer is:

$$net_j = \sum(W_{ij}O_i) + \theta_j \quad i = 1, 2, \ldots, I, \quad j = 1, 2, \ldots, J$$

(21)

the output of node $j$ is:

$$O_j = f(net_j) = \tanh(\beta \cdot net_j)$$

(22)

where $\beta > 0$, the net input to node $k$ in the output
layer is
\[ \text{net}_k = \sum (W_{kj} \cdot \text{O}_j) + \theta_k \quad j=1, 2, \ldots, J, \quad k=1, 2, \ldots, K \]  
(23)

the output of node \( k \) is
\[ \text{O}_k = f(\text{net}_k) = \tanh (\beta \cdot \text{net}_k) \]  
(24)

The output \( \text{O}_k \) of node \( k \) in the output layer is treated as the control input \( u_p \) of the system for a single-input and single-output system. As expressed equations, \( W_{kj} \) represent the connective weights between the input and hidden layers and \( W_{kj} \) represent the connective weights between the hidden and output layers. \( \theta_k \) and \( \theta_k \) denote the bias of the hidden and output layers, respectively.

For the \( N \)th sampling time, the error function is defined as
\[ E(N) = \frac{1}{2} (\alpha(N) - \alpha(N))^2 = \frac{1}{2} e(N)^2 \]  
(25)

where \( \alpha(N) \) and \( \alpha(N) \) denote the outputs of the reference model and the outputs of the controlled plant at the \( N \)th sampling time, respectively. The weights matrix is then updated during the time interval from \( N \) to \( N+1 \).

\[ \Delta W(N) = W(N+1) - W(N) \]
\[ = -\eta \frac{\partial E(N)}{\partial W(N)} + \alpha \cdot \Delta W(N-1) \]  
(26)

where \( \eta \) is denoted as learning rate and \( \alpha \) is the momentum parameter. The gradient of \( E(N) \) with respect to the weights \( W_{kj} \) is determined by
\[ \frac{\partial E(N)}{\partial W_{kj}} = \sum \frac{\partial E(N)}{\partial \text{net}_k} \frac{\partial \text{net}_k}{\partial W_{kj}} = \delta_k \text{O}_j \]  
(27)

where \( \delta_k \) is defined as
\[ \delta_k = \frac{\partial E(N)}{\partial \text{O}_k} = \frac{\partial \alpha}{\partial \text{O}_k} \cdot \frac{\partial \text{O}_k}{\partial \text{net}_k} = \frac{\partial \text{net}_k}{\partial \text{net}_k} = \beta(1 - \text{O}_k^2) \quad n=1, 2, \ldots, K \]  
(28)

where \( \partial \alpha / \partial \text{O}_k \) is difficult to be evaluated due to the uncertainty of the plant.

However, for a single-input and single-output control system (i.e., \( n=1 \)), the sensitivity of \( E(N) \) with respect to the network output \( \text{O}_k \) can be approximated by a linear combination of the error and its derivative according to the \( \delta \) adaptation law as below
\[ \frac{\partial E(N)}{\partial \text{O}_k} = K_\text{e}e_\text{e} + K_\text{r} \frac{\partial e_\text{e}}{\partial t} \]  
(29)

where \( K_\text{e} \) and \( K_\text{r} \) are positive constants. Similarly, the gradient of \( E(N) \) with respect to the weights, \( W_{ji} \) is determined by
\[ \frac{\partial E(N)}{\partial W_{ji}} = \frac{\partial E(N)}{\partial \text{net}_j} \frac{\partial \text{net}_j}{\partial W_{ji}} = \delta_i \text{O}_j \]  
(30)

where \( \delta_i \) is defined as
\[ \delta_i = \frac{\partial E(N)}{\partial \text{net}_j} = \frac{\partial \text{net}_j}{\partial \text{net}_k} \frac{\partial O_k}{\partial D_{O_k}} \frac{\partial O_k}{\partial \text{net}_k} = \sum \delta_k W_{km} \beta(1 - O_j^2) \quad m=1, 2, \ldots, J \]  
(31)

The weight-change equations on the output layer and the hidden layer are
\[ \Delta W_{ij}(N) = -\eta \frac{\partial E(N)}{\partial W_{ij}(N)} + \alpha \cdot \Delta W_{ij}(N-1) \]
\[ = -\eta \delta_i \text{O}_j + \alpha \cdot \Delta W_{ij}(N-1) \]  
(32)

\[ \Delta W_{kr}(N) = -\eta \frac{\partial E(N)}{\partial W_{kr}(N)} + \alpha \cdot \Delta W_{kr}(N-1) \]
\[ = -\eta \delta_k \text{O}_r + \alpha \cdot \Delta W_{kr}(N-1) \]  
(33)

where \( \delta_i \) and \( \delta_k \) can be evaluated from Eqs. (24) and (21). The connective weights in the neural network are updated during the time interval from \( N \) to \( N+1 \).

\[ W_{kr}(N+1) = W_{kr}(N) + \Delta W_{kr}(N) \]  
(34)

\[ W_{ij}(N+1) = W_{ij}(N) + \Delta W_{ij}(N) \]  
(35)

A tangent hyperbolic function is used as the activation function, so that the neural network controller output \( u_p \) evaluated from Eq.(17) is between \(-1\) and \(+1\), which is multiplied by the scaling factor \( K_o \) to be the proportional valve spool displacement command, so that the plant is defined to be the integrated system of the proportional valve closed loop control system, VDAPP and scaling factor \( K_o \). The weights and biases is initialized randomly in the interval between \(+0.5\) and \(-0.5\), and updated by Eqs.(34) and (35).

4. Numerical Simulations

The plant model is described by Eq.(8), (14) and (15). The data used for the VDAPP are shown in Table 2, reference to Ackers model[10]. The maximum swashplate angle is 0.2618 rad that means the swashplate is held at position 100%, the swashplate angle is measured by a hall sensor with maximum output voltage of 10 V corresponding to maximum swashplate angle condition. The swashplate angle command is denoted as \( \alpha \) between 0 V and 10 V. The maximum spool displacement of proportional valve is limited in 3 mm, so that the EHPV can be linearized.

A PD controller with the parameters \( C_r=1 \) and \( C_o=0.005 \) is designed to control the position of spool with 0 overshoot and 1 ms time constant. The PWM driver \( K_s=1 \) amp/V. The output of the network is multiplied by \( K_o=3 \) V and converted by a D/A converter to be the spool position command \( X_o \) within \(-3 \) V and \(+3 \) V corresponding to spool displacement \(-3 \) mm and \(+3 \) mm respectively i.e. \( K_o=1 \) V/mm. The volume of discharge system of pump is 10 liters. And the bias spring effect is neglected.

Both the neural network model-following controller and the P controller with \( K_{rc}=1 \) are simulated to control the swashplate angle. The standard second order transfer function with damping ratio \( \xi=1 \) and natural frequency \( \omega_n=200 \) rad/s is designed to be the reference model for the proposed neural network controller, as follow
\[ \frac{A_{rc}}{A_{rc}} = \frac{40000}{s^2 + 4000s + 40000} \]  
(36)
Table 2  Data used for the VDP and EHPV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_l$</td>
<td>$1 \times 10^{-6}$</td>
<td>(m$^2$/MPa s)</td>
</tr>
<tr>
<td>$D_1$</td>
<td>$300 \times 10^{-5}$</td>
<td>(m$^2$/rad)</td>
</tr>
<tr>
<td>$D_2$</td>
<td>$150 \times 10^{-6}$</td>
<td>(m$^2$/rad)</td>
</tr>
<tr>
<td>$d_p$</td>
<td>$35.38 \times 10^{-6}$</td>
<td>(m$^2$/rad$^2$)</td>
</tr>
<tr>
<td>$J_r$</td>
<td>$3.1 \times 10^{-5}$</td>
<td>(kg m$^2$)</td>
</tr>
<tr>
<td>$K_o$</td>
<td>$0.6312$</td>
<td>(m$^2$/m/s)</td>
</tr>
<tr>
<td>$K_C$</td>
<td>$3.727 \times 10^{-5}$</td>
<td>(m$^2$/MPa s)</td>
</tr>
<tr>
<td>$K_p$</td>
<td>$5.2$</td>
<td>(N m/MPa)</td>
</tr>
<tr>
<td>$K_T$</td>
<td>$6$</td>
<td>(N/amp)</td>
</tr>
<tr>
<td>$K_a$</td>
<td>$35$</td>
<td>(N m/rad)</td>
</tr>
<tr>
<td>$K_o$</td>
<td>$13$</td>
<td>(N m/(rad/s))</td>
</tr>
<tr>
<td>$K_s$</td>
<td>$1$</td>
<td>(amp/V)</td>
</tr>
<tr>
<td>$K_v$</td>
<td>$3$</td>
<td>(V)</td>
</tr>
<tr>
<td>$K_\eta$</td>
<td>$1$</td>
<td>(V/mm)</td>
</tr>
<tr>
<td>$K_v$</td>
<td>$20$</td>
<td>(N/m)</td>
</tr>
<tr>
<td>$M_p$</td>
<td>$0.02$</td>
<td>(Kgm)</td>
</tr>
<tr>
<td>$B_p$</td>
<td>$2$</td>
<td>(N/(m/s))</td>
</tr>
<tr>
<td>$V_s$</td>
<td>$10$</td>
<td>(liter)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1000$</td>
<td>(MPa)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$150$</td>
<td>(rad/s)</td>
</tr>
</tbody>
</table>

Table 3  MSE tests for variations of $K_o$ and $K_T$

<table>
<thead>
<tr>
<th>$K_T=0.1$</th>
<th>$K_T=6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_o$</td>
<td>MSE of $e_t$ MSE of $e_t$</td>
</tr>
<tr>
<td>0.005</td>
<td>0.053472</td>
</tr>
<tr>
<td>0.008</td>
<td>0.053808</td>
</tr>
<tr>
<td>0.01</td>
<td>0.053434</td>
</tr>
<tr>
<td>0.012</td>
<td>0.052636</td>
</tr>
<tr>
<td>0.013</td>
<td>0.053807</td>
</tr>
</tbody>
</table>

The learning rate $\eta=0.1$ and $\beta=0.5$ are chosen for simulation. The maximum error is 10 V and the maximum error differential is less than 200 V, so that the error and its differential are multiplied by $K_T=0.1$ and $K_T=0.01$ respectively for normalization, and the parameters $K_o$ is chosen properly by some MSE tests which are listed in Table 3.

The maximum error between the reference model output and plant output is 0.13 V by simulation. The error need to be normalized, so that $K_o=6$ is chosen. The $K_T=0.1$ is chosen properly by some MSE tests which are listed in Table 3.

Assume the pump is in the static condition and the output pressure of the pump is regulated to 2 MPa by a pressure control valve, so that the pump load pressure keeps constant in control process ($\Delta P=0$).

The time responses to a step input of $\Delta \alpha=20\%$ (2 V) for the swashplate angle and spool displacement are determined and shown in Fig. 5, the output of the reference model is shown by dash line, and a favorable model-following characteristic is achieved. Fig. 6 shows the output of neural controller is adjusted to be positive and negative rapidly, it means the neural controller can enhance the performance of spool, however the traditional controller can’t provide this performance.

Assume the load pressure can be varied but lim-
5. Experiments

The Rexroth A10VSO variable displacement axial piston pump is applied in our experiment. The output of neural controller between $-1$ and $+1$ is multiplied by $K_o=8 \text{ V}$, then converted to be the analog voltage between $-8 \text{ V}$ and $+8 \text{ V}$ by a 12 bits bipolar D/A converter. The analog signal is the spool displacement command of EHPV system. And two sets of A/D converter are used to converter the output voltage of hall sensor to numerical digital for real time control and data acquisition.

A hydraulic cylinder, directional control valve and pressure control valve are mounted at the pump outlet. The volume of discharge system of pump is fixed at 10 liters. The actuators of swashplate are supplied with internal supply. Let the pump outlet pressure is controlled by the pressure control valve, the pump outlet pressure is prior adjusted at 0.1 MPa, and the relief pressure is adjusted at 1 MPa.

Both the P controller with fine-tuning and the proposed neural controller are used to control the VDAPP system. The proposed neural controller uses a linear second order reference model with $\xi=1$, $\omega_n=$
\[ \Delta \alpha = 0.0628 \text{ rad with Rexroth SYDE1 controller} \]

(b) \[ \Delta \alpha = 0.0638 \text{ rad with neural controller} \]

Fig. 8 Time response for the swashplate angle

100 rad/s is designed as below:

\[ \frac{\Delta \alpha}{\Delta \alpha} = \frac{0.0952Z - 0.0861}{Z^2 - 1.8096Z + 0.8187} \quad (37) \]

The maximum command input \( \alpha \) is 10 V, and the maximum output of the hall sensor is 10 V, then the two input signals of the network must be normalized in the input layer by multiplying \( K_1 \) and \( K_2 \) respectively. The learning rate \( \eta = 0.03 \), sampling time \( T = 1 \text{ ms} \) is chosen.

The experiment results of time responses for the swashplate angle are shown in Fig. 8. The stable time responses of \( \alpha = 0.0628 \text{ rad (2 V)} \) are achieved in both the P controller and the neural controller condition. Figure 9(a) shows the swashplate angular response of \( \alpha = 0.0314 \text{ rad (1 V)} \) with randomly initialized weights between \(-0.5 \text{ and } +0.5\). The neural controller preserves a favorable model-following characteristic under pressure variation condition. Because the volume of discharge system of pump is fixed so that when the swashplate angle increases, the pump outlet flow will increase and the pump load pressure will be increased but limited at 1 MPa by the pressure control valve. This is shown in Fig. 9(b), the pressure is varied from 0.1 MPa to 1 MPa i.e. \( \Delta P \approx 0.9 \text{ MPa} \) in control process. Figure 10(a) shows the better model-following characteristic of swashplate angle achieved, because of the prior training of \( \alpha = 0.0314 \text{ rad has been accomplished and the weights of network have had better initial weighting coefficients. The proposed neural controller can provides more steady and faster response. The model-following characteristic of swashplate angle become better and better by on-line trained in control process. Figure 10 (b) shows the pressure is varied from 0.1 MPa to 1 MPa i.e. \( \Delta P \approx 0.9 \text{ MPa} \) in control process. The time responses of pressure variation in both the P controller and the neural controller condition are similar.

6. Conclusion

In industrial application, the traditional PD controller has been used to control the VDAPP system. The proposed neural network model-following controller can improve the stability of VDAPP control system, and reduce the overshoot of pump output pressure. In addition, the neural controller enhances the performance of EHPV, as mentioned in experiment it can improve the angular response of swashplate from 0.05 seconds to 0.1 seconds of settling time.
Furthermore, the model-following characteristic becomes better in on-line learning, and become robustness for avoiding the disturbance due to load variation, command variation, parameter variation and surrounding changing.

Appendix A : The Transfer Functions of the Components

In this section, the transfer functions and the block diagram of the components are evaluated in the frequency domain. A Laplace transformed variable is indicated by capital letters, a linearized variable by its infinitesimal differential $\Delta$ about an operating point. Terms with small influence of the dynamic response are ignored. See also Lantto(31).

A1. Control valve of meter-out flow

The following equation forms the block diagram in Fig. 11 according to Lantto(31). If the control valve is noncompensated, the flow gain is $K_\delta = \partial Q_\delta / \partial X_\delta$ and the flow pressure coefficient is $K_\pi = \partial Q_\delta / \partial P_\pi$.

$$\Delta Q_\delta = K_\delta \Delta X_\delta + K_\pi \Delta P_\pi$$

(A1)

Fig. 11 Control valve of meter-out flow

Fig. 12 Control valve of meter-in flow

Fig. 13 The pump volume and its block diagram

A2. Control valve of meter-in flow

The following equation forms the block diagram in Fig. 12 according to Lantto(31). If the control valve is noncompensated, the flow gain is $K_\delta = \partial Q_\delta / \partial X_\delta$ and the flow pressure coefficient $K_\pi = \partial Q_\delta / \partial (P_\pi - P_\infty)$.

$$\Delta Q_\delta = -K_\delta \Delta X_\delta + K_\pi (\Delta P_\pi - \Delta P_\infty)$$

(A2)

If it has an infinite fast pressure compensator spool without spring forces, the flow pressure coefficient $K_\pi$ will approximate 0.

A3. Volume of pump

The continuity equation of the pump volume gives the block diagram in Fig. 13.

$$\Delta Q_\delta - \Delta Q_\pi = C_\delta \Delta P_\pi$$

(A3)

where $C_\delta = V_\delta / \beta_\delta$, $\Delta Q_\delta$=total flow in the pump volume, $\Delta Q_\pi$=total flow out from the pump volume.

$$\Delta Q_\delta = d_\delta V_\delta \Delta \theta_\delta$$

(A4)

$$\Delta Q_\pi = Q_\pi + C_\delta \Delta P_\pi + \Delta Q_\delta + Q_\infty$$

(A5)

Substituting Eqs.(A4) and (A5) into Eq.(A3) gives

$$\Delta P_\pi = \Delta Q_\pi$$

(A6)

Appendix B : Pressure variation of control actuator

B1. Control valve of meter-out flow condition

Flow continuity in the control volume is expressed by

$$\frac{dP_\delta}{dt} = \frac{2}{V_\delta} \left( -\frac{dV_\delta}{dt} + Q_{in} - Q_{out} \right)$$

(B1)

From Fig. 14 $Q_{in} = 0$, $Q_{out} = Q_\delta$, $\Delta Q_\delta = K_\delta \Delta X_\delta + K_\pi \Delta P_\pi$, and the volume variation of control actuator expressing by
then from Eqs. (B1) and (B2), the pressure variation of control actuator expressed in frequency domain by
\[
\Delta P_a = \frac{\beta_P}{V_i} (D_1 \Delta \alpha - \Delta Q_i)
\]  
(B3)
\[
\Delta P_i = \frac{1}{C_i S} (D_1 \Delta \alpha - \Delta Q_i)
\]  
(B4)
where 
\[
C_i = \frac{V_i}{\beta_e}
\]

**B2. Control valve of meter-in flow condition**

Flow continuity in the control volume is expressed by
\[
\frac{dV_i}{dt} = \frac{\beta_e}{V_i} \left( \frac{dV_i}{dt} + Q_{in} - Q_{out} \right)
\]  
(B5)
From Fig. 15, \(Q_{in} = Q_i, \ Q_{out} = 0, \ \Delta Q_i = -K_s \Delta X_r + K_r (\Delta P_i - \Delta P_s)\), and the volume variation of control actuator expressed by
\[
\frac{dV_i}{dt} = -D_1 \frac{d}{dt} \alpha_r
\]  
(B6)
then expressed in frequency domain by
\[
\Delta P_i = \frac{\beta_P}{V_i} (D_1 \Delta \alpha_r + \Delta Q_i)
\]  
(B7)
\[
\Delta P_i = \frac{1}{C_i S} (D_1 \Delta \alpha_r + \Delta Q_i)
\]  
(B8)
where 
\[
C_i = \frac{V_i}{\beta_e}
\]

**Appendix C: The Torque Exerted on Swashplate**

The output force and torque of control actuator are
\[
F_a = P_A A_1
\]  
(C1)
\[
T_a = P_A D_1
\]  
(C2)
The output force and torque of bias actuator are
\[
F_b = P_B A_2
\]  
(C3)
\[
T_b = P_B D_2
\]  
(C4)
The torque of pressure differential across the pump is
\[
T_r = K_P P_r
\]  
(C5)
The relationship between torque and pressure differential across the pump, swashplate angle, and angular velocity, defined by Akers, is
\[
T = K_r P_3 + J_s \omega + K_0 \Delta \omega + K_a \Delta \omega
\]  
(C6)
The torque exerted on the swashplate is balanced by the actuators, which have spring and pressure forces acting on them. And linearized about the null point, can be expressed as
\[
\Delta P_i D_1 - \Delta P_s D_1 = J s^2 \Delta \omega + K_0 \Delta \omega + K_a \Delta \omega + K_d \Delta \omega
\]  
(C7)

**References**

7. Psaltis, D., Sideris, A. and Yamamura, A.A.


