Thermal Responses in Control Loop in Indirect Control of Indoor Environment of Non-Air Conditioned Space with Quasi-Steady-State Model*

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This work presents the results from the controlling of the thermal environment of an enclosed space using Euler-Lagrange equations in a variable control problem, and compares it with three semi-empirical controller settings. Thermodynamic characteristics of the space of the administrative building equipped with convectors, disseminating heat energy supplied by steam, provided its mathematical model, which is fully commandable and observable. The model of the enclosed space—single-zone room—was reduced to the desired order dimension applied on the model's transfer function. Set in feedback control-loop with a PS controller, the indoor air temperature, heat fluxes as well as heat consumption over screened period were simulated under certain weather conditions and internal thermal loads, thus the thermodynamic responses of the space were obtained. Comparing the outputs with measurements and the results obtained for each simulation case showed energy savings and the optimal variable control method itself as an effective alternative, if Lagrange multipliers in Euler-Lagrange equations could be solved separately apart from model state-form variables. Further, maintaining Hamilton function as time-independent within defined boundaries, proves along with optimal control solution the readiness for its application in control algorithms of current heating systems.

Key Words: Simulation Model, System Order Reduction, (Convective) Heat Transfer Coefficient, (Reference Room) Thermodynamic Characteristics, Hamilton Function, Transfer Function

1. Introduction

The qualitative new approaches to dynamic systems over the past twenty years brought significant changes in traditional system modeling and led to new insights and conceptually new methods over the whole area of thermal energy modeling in buildings^{(1),(3),(7)}. Utilizing computational techniques in the areas such as weather data simulations, hour-by-hour simulation of heat energy transfer, and implementation of complex, sophisticated control algorithms in order to maintain the desired thermal environment, often tends to introduce a robust measurement device in praxis, analytical and control techniques, or at least suggests a comprehensive simulation and control tool, which is

capable of accomplishing all these objectives.

Fleming (1981) pointed out that the current situation in building energy use modeling was state of the art and full satisfying. Based on his study of measured energy use data from a string of hotels, he concluded that the computer predictions about energy use matched very well on a month-by-month basis accurately. However, similar opinions are not seen frequently and are confined to specific problems and concrete cases. This dichotomy between reasonable energy model analysis and an appropriate model for its monitoring and eventual control continues right now at the very beginning of the new century.

Obviously, the most reliable results come from comparing two or more solutions, where at least basic energy analysis and energy audit could be made. However, that course is more expensive and the whole comparison of the suggested alternatives would go only to some-costly still reasonable-extent in order to

* Received 4th March, 2002 (No. 02-4054)

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demonstrate the right choice^{(3),(4)}. The focus is on the energy transfer modeling through the building or its parts, and its eventual control through choosing the right control technique (control algorithm). Considering how these concepts affect energy models as well as simulation models, we point to some hints that could save a bulk of work if they are kept in mind in the model-building-up stage.

They include the concepts of :

- decomposition (familiar from classical system theory)⁽⁵⁾,
- system order reduction, with respect to the required accuracy, both the model itself and coupled with a controller⁽⁶⁾,
- choice of simulation method, model and suitable (software) tool^{(7),(8)}.

Appropriate choice of any from number of quasi-steady-state methods is beyond the scope of this discussion, and besides there is also no general rule of proposing it. The three steps above-succeeding in many energy models-were completed by utilizing the Hamilton function without forcing subsequent conditions guaranteeing its full optimality through its trajectory. As they must follow necessary conditions respecting the limitations of input variables (mainly controllable ones), the solution could not be found in frame of a classical variable method at all. The Hamilton function we suggested would include neither time nor sampling time variable and still leave controllable variable(s) free of similar restriction. To accommodate this, the approach was realized in the schema with an internal adaptive loop, which maintains tracking of an observed variable within the vicinity of the working point and its limits.

These important aspects of the classical system theory have been translated and solved within the behavioral setting, thus leading to powerful generalizations of well-known theorems obtained for the input/output or state-space models. One of the most interesting is undoubtedly the well-known decomposition theorem, stating that every complete behavior can be expressed as the direct sum of a (uniquely determined) controllable behavior and of an autonomous one, which can be chosen with a certain degree of freedom. Such decomposition represents the natural generalization of the free and forced evolution process of decomposition, a well-known result for classic state-space models.

The controllable part of the transient function matrix represents a typical case of its sub-behavior. Aiming at more general results as far as behavior decomposition is concerned, one can investigate the expected decompositions in which the sub-behavior of one of the two components is given. In particular, one

can search for conditions insuring that within such set of these decompositions there exists a direct one : this means that the two components are devoid of common trajectories.

In this contribution, we reduced the system order on separate parts of the original matrix of dynamics **A** while necessary and sufficient conditions for the existence of such direct decomposition are maintained and would not affect its stability. On the other hand, when these conditions are not satisfied, it is always possible to obtain a decomposition in which the two components exhibit an autonomous (and hence finite-dimensional) intersection. The reduced model in its order, as well as its approximations of non-linear terms within matrix of dynamics served in control closed loop with PS (proportional-sum) controller aiming at reducing the heat supply into the reference room. The comparison showed certain energy savings for the case using Lagrange multipliers in the minimization criterion.

Nomenclature

- A** : surface area (m^2)
- A, B, C, D** : state-space model matrices
- c* : specific heat ($\text{J}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}$), constant
- e* : (control) deviation (—) or (K)
- G, H** : transfer function matrix
- h* : heat transfer coefficient ($\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$)
- k* : thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$), constant
- J* : criterion formula
- K* : system gain
- L* : spatial dimension (m)
- M** : minor terms of matrix of dynamics
- n* : system order
- p* : Laplace variable
- q* : heat flux per unit surface area ($\text{W}\cdot\text{m}^{-2}$)
- Q* : heat rate (W)
- r₀* : controller gain
- R** : symmetric, positive-definite matrix
- s* : (wall) thickness (m)
- t, T* : temperature, thermodynamic temperature ($^{\circ}\text{C}$, $^{\circ}\text{K}$)
- u** : input vector
- x, y* : distance (m), vector component
- x** : state-space variable (vector)
- y** : output vector
- Bi, Gr* : Biot, Grashof number
- Nu, Ra* : Nusselt, Rayleigh number
- MIMO** : multi-input and multi-output (system)
- MOS** : model of reference room, Ref. (2)
- Greek letters
- β : volume coefficient of expansion (K^{-1})
- γ : gain-to-loss ratio (—)
- μ : dynamic viscosity ($\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$)

- ε : emissivity (—)
 ν : ventilation rate (—), (overall) dumping coefficient, section 4.2
 Π_H : design temperature difference (K)
 ρ : density ($\text{kg}\cdot\text{m}^{-3}$)
 σ : Stefan-Boltzmann constant
 τ : time (s), transmissivity (—)
 $\Delta\tau$: sampling period (s)

Indexes

- a, b : air, layer
 c : ceiling
 cv : convective
 cd : conductive
 f : floor
 g : globe, global
 h : heater
 i : indoor, internal
 inf : infiltration
 o : outdoor
 r : radiant
 s : system, solar
 st : stream
 w, wo : wall, outside wall
 wf : window

2. Decomposition of Transfer Function Matrix

2.1 System reducibility with respect to energy storage interaction

Our goal as described in the previous chapter is reducing the system order dimension of model of one single-zone room in an administrative building supplied by a (steam) central heating system. Here we kindly refer the reader to Ref.(2) with a description of the tested room, its thermal conditions and governing equation system of the space represented by Eqs. (3.a)–(3.g) in Ref.(2). It consists of several qualities expressed in vectors $\mathbf{u}(\tau)$, $\mathbf{x}(\tau)$ and $\mathbf{y}(\tau)$ that serve in other sub-calculations such as parameters of convection heat fluxes and overall heat losses estimations. With respect to the accuracy of required outputs and, on the other hand, to simplify the model there is a practical rule of not pursuing higher system order unless qualitative measurement apparatus or even method with regard to desired accuracy takes place. Because of the very purpose of the proposed model—to provide controller with reliable data conforming well with actual space thermal characteristics—the next step focused on the model's reliability when reducing its order⁽⁵⁾. Beside that process the stability of matrix of dynamics was verified in order to exclude an unexpected behavior caused either by the model itself, or (as precaution) by false controller's outputs.

The order of the system itself is initially defined by the level of heat generating sources, resp. number

of significant heat energy storage capacities, thermodynamic characteristics of individual construction parts including spaces with their boundaries capable of heat energy storage. Such sorting is made usually in an early stage during thermal energy analysis and the level-in-detail may be shown eventually from the signal schema (Fig. 3 in Ref.(2)). The type of the system with respect to interactive and non-interactive relations, multiple connection and other features (linearity, autonomy) shall be determined also during the analysis. In the control theory such classification helps to arrange the input-state-output qualities in order to reduce unnecessary interactions in equation build-up-process and exclude redundancies.

Monitoring the energy flow directions cannot be used for system interactivity judgement^{(8),(9)} and system border setting has to be selected carefully, otherwise additional relations may cause an increase of the system order. From such point of view the system may qualify for the treatment described here:

- system decomposition (and possibly approximation to subsystems of the 1st-order),
- reconsider secondary interactions in order to eliminate irrelevant variables and redundant relations⁽⁶⁾,
- classify heat energy storage capacities with respect to the distributed-parameter system (as opposed to the continuous one).

The controlled space volume dictates spatial limits of storage parts: the larger parts are contained within the 1st-order subsystem, the greater the extent of the system order reduction one may expect. On the other hand, more detailed division means that characteristics of the controlled space volume could help in sensors' location, particularly with respect to asymmetrical space disposition and/or heat sources location.

The 12th-order MOS system⁽²⁾ forms set of coupled the first-order differential equations, which coefficients by state vector $\mathbf{x}(\tau)$ shape the matrix of dynamics $\mathbf{A}$. In the model structure Eq.(11) of Ref.(2) among disturbances of input vector $\mathbf{u}(\tau)$ term $u_1(\tau)$ serves as controllable variable from a controller. Its minimal, respective maximal value determines range of $x_{11}(\tau)$ and that of $x_{12}(\tau)$. From the change of input $u_1(\tau)$ the MOS transient responses of our interest have to be calculated and be grouped into the output vector $\mathbf{y}(\tau)$. Through transfer function matrix $\mathbf{F}(p)$ in Laplace transform continuous space:

$$\mathbf{F}(p) = \mathbf{y}(p)/\mathbf{u}(p) \quad (1)$$

with initial conditions $\mathbf{x}(0) = \mathbf{x}_0$ and performing Laplace transform on equation system (3) in Ref.(2) yields:

$$\begin{aligned} p\mathbf{x}(p) - \mathbf{x}(0) &= \mathbf{A}\mathbf{x}(p) + \mathbf{B}\mathbf{u}(p) \\ \mathbf{y}(p) &= \mathbf{C}\mathbf{x}(p) + \mathbf{D}\mathbf{u}(p) \end{aligned} \quad (2)$$

where its transfer function matrix is:

$$\mathbf{F}(p) = \mathbf{C}(p \cdot \mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D} \quad (3)$$

The difference of unit Laplace matrix and matrix of dynamics $(p \cdot \mathbf{I} - \mathbf{A})$, from Eq. (4) in Ref. (2), $\mathbf{I}$ is the unit matrix:

$$(p \cdot \mathbf{I} - \mathbf{A}) = \begin{bmatrix} (p - a_{1,1}) & 0 & \cdots & a_{1,11} & a_{1,12} \\ a_{2,1} & (p - a_{2,2}) & \cdots & a_{2,11} & a_{2,12} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{12,1} & a_{12,2} & \cdots & a_{12,11} & (p - a_{12,12}) \end{bmatrix}, \quad (4)$$

has its inversion for $|(p \cdot \mathbf{I} - \mathbf{A})| \neq 0$:

$$(p \cdot \mathbf{I} - \mathbf{A})^{-1} = \frac{\mathbf{M}}{|(p \cdot \mathbf{I} - \mathbf{A})|} = \frac{1}{|(p \cdot \mathbf{I} - \mathbf{A})|} \times \begin{bmatrix} M_{1,1} & \cdots & M_{1,12} \\ M_{2,1} & \cdots & M_{2,12} \\ \cdots & \cdots & \cdots \\ M_{12,1} & \cdots & M_{12,12} \end{bmatrix} \quad (5)$$

with minor terms M_{ij} , $i, j = 1, 2, \dots, 12$. These terms follow from transposed matrix

$$(p \cdot \mathbf{I} - \mathbf{A})^T \quad (6)$$

created through reciprocal sub-determinants of transposed matrix

$$(p \cdot \mathbf{I} - \mathbf{A})^T = \begin{bmatrix} (p - a_{1,1}) & -a_{2,1} & \cdots & -a_{12,1} \\ 0 & (p - a_{2,2}) & \cdots & -a_{12,2} \\ \cdots & \cdots & \cdots & \cdots \\ -a_{1,12} & -a_{2,12} & \cdots & (p - a_{12,12}) \end{bmatrix} \quad (7)$$

A part of input matrix $\mathbf{B}$ containing outdoor climate disturbances as well as thermal influence of internal heat sources will be subtracted from now. This allows division of one multi-dimensional system into two more transparent subsystems, Fig. 1.

From the decomposed upper part of transfer function matrix $\mathbf{G}(p)$, Fig. 1, comes the analytical solution $\mathbf{y}^1(p) = \mathbf{G}(p)\mathbf{u}(p)$ with output equations containing transition matrix $e^{A\tau}$, resp. $e^{A(\tau - \tau_{ad})}$ in (continuous time):

$$\mathbf{X}(\tau) = e^{A\tau} \mathbf{X}(\tau_0) + \int_{\tau_0}^{\tau} e^{A(\tau - \tau_{ad})} \mathbf{B} \mathbf{u}(\tau) d\tau \quad (8)$$

These responses have physical meaning and could serve directly for extraction of time constant(s) of the space and their approximation may complete previous system-order reduction^{(5),(6)}. Along with this the envelope of investigated space has to be subjected to reasonable division into parts with respect to the amount of energy storing ability and variability of properties throughout their bodies. The conventional thermal balance method⁽¹⁰⁾ was used to incorporate the effects of thermal mass of interior's envelope and clarify time differences among heat fluxes' responses as the main judgment factor in partition of interior's surrounding walls. The outside wall is classified as a medium mass construction wall (with relatively high overall U -value $3.1 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$) apart from internal

Fig. 1 System decomposition of transfer function matrix with respect to input variables (MIMO blocks, multi-input-multi-output).

Fig. 2 Reference room allocation on climate-surface relief for total design temperature difference Π_H [K] at heat losses 45 W/m^2 . $T_1(h)$ —time constant of object; $\gamma(-)$ —gain-to-loss ratio.

walls being adjacent to neighboring spaces. Important value of gain-to-loss ratio $\gamma = 0.12$ was not among state variables and was calculated separately a priori, Fig. 2 (program Klimaflaeche by ETH Zurich, Physics of Buildings, Swiss 1998).

This enables us to estimate the probable value of the design temperature Π_H of steam converter and provides a simple tool for simultaneous comparison of heat fluxes for various weather conditions. Knowing the value means a direct assessment of reference space gain K with respect to the heater (convector) output. Figure 3 shows the program line for input steam temperature as a function of the outside air temperature enabling us to determine the necessary heat amount for covering the overall heat losses in

Fig. 3 Program curve-input steam temperature as inverse to outdoor air temperature $t_{st,i} = f(t_a)$. Near thermal-steady state (quasi-equilibrium) is $(t_{st,i} - t_{st,p}) \geq 0.6$ K. $t_{st,i}$ —steam input temperature, $t_{st,p}$ —effective (temperature), approximately $\frac{T_{st,i} + T_{st,o}}{2}$. Working point P_{st} of effective temperature of heater is always positioned lower than temperature $t_{st,i}$.

steady or nearly quasi-steady condition as the curve could be seen as a string of thermal equilibrium. The working point is in our model with vector term u_1 shifted to the lower temperature at its end as a result of the heat amount being removed from the convector's body surface continuously into the room.

The classification of input variables (disturbances) to the subsystems was respected in the separation process, Fig. 1. The simplification is reasonable, as one of the goals of the method of system design and commissioning processes is to linearize the relationship between the control signal and controlled variable, e.g., by canceling non-linearity connected with valve functioning and simply putting the input after this item. Although the model simplifications reduce seemingly the potential accuracy and performance of the scheme, a major advantage is that the parameter values may be obtained from typically available information from basic temperature measurement⁽¹¹⁾, rather than requiring calibration data and additional tuning efforts on the model side.

2.2 Input and output vectors classification

Selected inputs and outputs variables respect major outdoor climate disturbances as well as thermal influence of the internal heat sources, Eqs. (5), (6) and Fig. 3 in Ref. (2):

1. $u(\tau)$ —input vector:

$$u(\tau) = [u_1(\tau), u_2(\tau), u_3(\tau), u_4(\tau), u_5(\tau), u_6(\tau), u_7(\tau), u_8(\tau), u_9(\tau)]^T$$

$u_1(\tau), u_6(\tau), u_7(\tau)$ and $u_8(\tau)$ —steam temperature, transmitted solar radiation and internal heat sources^{*1}

$u_2(\tau), u_3(\tau), u_4(\tau), u_5(\tau), u_9(\tau)$ —adjoining rooms environment characteristics, infiltration ratio, outdoor temperatures and wind speed.

2. $y(\tau)$ —output vector:

$$y(\tau) = [y_1(\tau), y_2(\tau), y_3(\tau), y_4(\tau)]^T$$

with controlled $y_3(\tau) = T_g(\tau)$

Transfer function matrices $G(p)$ of dimension 5×12 with the $H(p)$ of dimension 4×12 form the eventual vector composed back in transient responses from our model:

$$y(p) = G(p) \cdot u(p) + H(p) \cdot z(p) \quad (9)$$

where $z(p) = [u_2(\tau), u_3(\tau), u_4(\tau), u_5(\tau), u_9(\tau)]^T$ —input vector of outdoor climate disturbances.

The basic thermal responses could be obtained from Eq. (9) for output vector $y(p)$ with loads excluding internal heat sources, sun radiation and wind, i.e.

$$u_i(\tau) = 0 \text{ kJ, for } i = 6, 7, 8 \text{ and } 9. \quad (10)$$

This will reduce input matrix B , Eq. (11) to dimension (12×5) , with heat transfer coefficients $h_r, (\tau), h_{cv}(\tau)$ kept updated with sampling period of a measurement (10 minutes):

$$B = \begin{bmatrix} 0, 0, 0, 0, 0, \frac{f_{wo}}{k_{wo,i}} \frac{A_{wo}}{A_{tot}}, \frac{f_{wf}}{k_{wo,i}} \frac{A_{wo}}{A_{tot}} \\ 0, 0, 0, 0, 0, \frac{f_{wj}}{k_{wj}} \frac{A_{wj}}{A_{tot}}, \frac{f_{wj}}{k_{wj}} \frac{A_{wj}}{A_{tot}} \\ 0, 0, 0, 0, 0, \frac{f_{wf}}{k_{wf}} \frac{A_{wf}}{A_{tot}}, 1 - \alpha_{wf} \\ 0, 0, 0, 0, 0, \frac{f_h}{k_h} \frac{A_h}{A_{tot}}, \frac{f_h}{k_h} \frac{A_h}{A_{tot}} \\ 0, 0, 0, -\frac{h_{wo,e}}{k_{wo,e}}, 0, 0, \frac{\alpha_{wo,e}}{k_{wo,e}} \\ 0, 0, 0, -\frac{h_{wf,e}}{k_{wf,e}}, 0, 0, \frac{\alpha_{wf,e}}{k_{wf,e}} \\ 0, -\frac{h_{w5,e}}{k_{w5,e}}, -\frac{h_{w5,e,r}}{k_{w5,e,r}}, 0, 0, 0, 0 \\ 0, 0, 0, 0, \frac{Ca}{3600(w_{rc} + m_a C_a)}, 0, 0 \\ \frac{\dot{m}_{st} C_{st}}{m_h C_h}, 0, 0, 0, 0, 0 \end{bmatrix} \quad (11)$$

Substituting Eqs. (5), (7) with Eq. (11) into Eq. (2) yields the desired transfer function matrix, ($D=0$):

$$G(p) = [y_1, y_2, y_3, y_4]^T \frac{1}{[(p \cdot I - A)]} \times \begin{bmatrix} M_{1,1} & \cdots & M_{1,12} \\ M_{2,1} & \cdots & M_{2,12} \\ \cdots & & \\ M_{12,1} & \cdots & M_{12,12} \end{bmatrix} \cdot B(p) + D \quad (12)$$

Equation (12) serves as a basic expression for transient responses—one of the data characteristics of the reference space. These data, along with time constants and overall gain values of the space provide worthwhile information to a PS-controller we used in indirect controlling of the reference space in order to maintain the desired indoor air temperature inside the room. State variables within the system (A, B and C) shall also be treated in similar manner. Separation of matrices $G(p)$ and ($H(p)$) help to eventually express transient responses of Eq. (2) in a way suit-

*1 Including one occupant during work hours.

able for some approximation with a polynomial form.

3. Heat Energy Consumption

There are many well developed models handling qualities related to heat fluxes of each separable item within the object such as its heat capacity, absorptivity, mean radiant (surface) temperature, etc. (Refs. (7), (12), (13)). Most models allow assessment of heat energy demands of the investigated space fairly well, if they can handle:

- at least hour-by-hour weather data,
- both infiltration and forced air exchange rate n sub-hourly,
- selected (dominant) internal heat sources,
- heat storage capacity of surrounding walls and
- convective heat transfer coefficient of outside wall(s) as variable.

The discussed model shall also satisfy the following simplifying assumptions:

1. Surface temperatures are united for each part as the average values; this value changes also at time again as average of the changes. Even if the material of all area is not isotropic, it is possible to find it as one representative value.
2. Heat transfer through walls is considered as one-dimensional, i.e. in perpendicular direction to the surface.
3. Heat accumulation capability of glassing is also omitted (compared to volumetric heat capacity of other construction parts).
4. Air is assumed perfectly stirred, otherwise must be at least selected representative values adjacent to the respective construction parts.

While both convective and radiative heat exchanges occur from and around surfaces, they affect heat exchanges as heat transfer coefficients indicate. To predict heat loss (gain) precisely means to attempt to calculate actual h_{cv} and h_r permanently (non-stop), or at least in reasonable time checks, as is predicted by models with the most popular group of quasi-steady state models (this is 'a must' feature by full dynamic mode). Heat transfer coefficients are calculated simultaneously with sampling time and are based on relations described through several dimensionless numbers (Nu , Pr , Re) in literature.

What follows is common practice in determining the convective heat transfer coefficients through groups of these numbers, where the dimensional analysis was employed to estimate heat transfer from vertical surfaces on the surface area $x_h \times b_y$. Assuming a steady motion without viscous dissipation, based on π -theorem, yields for 2-dimensional motion (L_x, H_y) with these relations^{(14),(15)}:

$$Nu \cdot Re^{-1/2} = f\left(Pr, \frac{Gr}{Re^{5/2}}, \frac{b_y}{x_h} \cdot Gr^{*1/5}\right),$$

$$Nu \cdot Gr^{*-1/5} = f\left(Pr, \frac{Re}{Gr^{*2/5}}, \frac{b_y}{x_h} \cdot Gr^{*1/5}\right) \quad (13)$$

The equations are valid within a laminar mixed convection motion over a vertical plate, and basically are outgoing formulas for several other expressions, valid in a more or less wide region of either Nusselt or Raileigh numbers, not including the turbulent case ($Gr \cdot Pr > \sim 10^9$)⁽¹⁶⁾. On the surfaces are unknown heat fluxes $q_w(j)$ and the velocity profiles (u , v -velocity components in horizontal, resp. vertical direction) in the area close to the walls, both resulting from complex of quantities forming Grashof, Nusselt and Prandtl numbers and can be grouped as^{(17),(18)}

$$h_{cv} = h_{cv}(x_h, b_y, \beta, \rho, \mu, k, c_p, \tau)$$

Particularly for the calculation of free convection the second equation in Eq.(13) yields:

$$Nu \cdot Gr^{*-1/5} = f\left(Pr, \frac{b_y}{x_h} \cdot Gr^{*1/5}\right) \quad (14)$$

Free convection motion by window (assumed $t_a < t_i$) moves downwards along the coordinate b_y and in the opposite direction, i.e. with forced convection the exponents of the above functional relations change, because of the flow changes behind the separation cross point as well. On the vertical wall surface, such flow is described through a system of Eq. (15)^{(19),(20)}:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \beta g + \nu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial \vartheta}{\partial x} + v \frac{\partial \vartheta}{\partial y} = a \frac{\partial^2 \vartheta}{\partial y^2} \quad (15)$$

with the boundary conditions $u=0$, $v=0$ and $q_w(j) = -\lambda \left(\frac{\partial \vartheta}{\partial y} \right)_{x_h=0}$ at $b_y=0$ and

$$u=0; \vartheta=0 \text{ at } b_y \rightarrow \infty; \vartheta = t - t_{\infty}. \quad (16)$$

Thus, convective heat transfer coefficient h_{cv} calculated for measured temperatures and obtained velocities represents, barely averaged value around unknown fluctuating ones, valid at time of measurement⁽²¹⁾. Obtained values belong to certain thermal states over the surface area of room-faced outer wall, and serve as a basis for extracting the averaged value with respect to the heat flux through the wall, (Fig. 4).

In Fig. 4 was approached form $h=f(T_a - T_w, H)$ over the net temperature area $T_g \times T_{wo} \subseteq [(284; 294) \times (290; 298, 5)K]$, on the right side ($Pr \approx 0.71$). In this figure are also seen heat transfer coefficients values for the same initial and boundary conditions recommended by Jacob and Hawkins (on the left)^{(10),(20)}. It was revealed that the area in the wall bottom would include bigger errors (in the coefficient value) and corresponding heat flux, respectively, than in the

Fig. 4 Values of convective heat transfer coefficient of room-faced outside wall over projected comfort area $h_{cv}=f(H, T_a - T_{wo})$: (a) according Jacob, Hawkins, (b) this study.

Fig. 5 Reference room arrangement with occupied space (dashed line) and internal heat sources location (* temperature sensor's checkpoint height according to ISO 7726).

middle and upper parts. Similar procedures were used for the other surrounding walls, and their respective values substitute for older ones in the matrix of dynamics A in each time step simultaneously with the sampling period $\Delta\tau$.

The investigated interior space showed in Fig. 5 was equipped with an air-conditioner, not used in the winter heating season, to manage the humidity of the indoor environment. The conventional steam convector along with a secondary infrared gas heater together with some other heat sources are listed in Fig. 6, on the left. Actual room dimensions with its occupied zone—dashed line—are also seen in Fig. 5.

The aim of the control task was to maintain the desired indoor temperature 293 K, resp. 294 K (20 or 21°C) within the occupied zone⁽²²⁾, while the infiltration was assumed to be 0.9 air exchange per hour, still above a limit for no-smoking space as is stated in most national hygienic standards and norms^{(10),(23)}.

Obtained heat losses 1.4 kW of the reference

Table Overview of internal heat sources

No	Type	Max. consumption (W)
1	Occupant(s)	96*
2	Appliances (printer, monitor, PC)	330
3	Lighting	36 x 9 tubes = 324
4	Infra-red heater	$3.83 \cdot 10^3$
5	(Steam) convector	$5 \cdot 10^3$
	Sum	9.6 kW

* reduced from male adult value to the female one.

Fig. 6 Internal heat sources in reference room. Overview table (left) and percentage share (right).

Fig. 7 Profiles of mean radiant temperature (a) and indoor air temperature (b) in reference room (workday 7 a.m.-24 p.m.) after 48-hour-period-off without heating-control feedback.

room for considered thermal state (outdoor temperature 268 K (-5°C)^{(15),(24)} are adequate to about 42% glassing of outside wall (one-plate glass) and barely 200 mm thick concrete wall without any significant thermally insulated layer. Profiles of indoor temperatures (indoor air temperature $t_{a,i}$ and global temperature t_g) in the day following weekend-off-heating period shows Fig. 7. Within 24-hour period without automatic control a full temperature swing occurs as is being altered by both, heat released from the steam convector and outdoor temperature profile (along with internal heat sources and infiltration influence). The profiles were represented in our model with central values for each hour in rough simulation case and then of 10-minute intervals for detailed cases.

Separately were set transfer functions for all four outputs in order to accommodate needs for intended use of a PS-controller (proportional-sum) adjustment⁽²⁵⁾. They were estimated through formula, where at least three parameters of the 2nd-order differential equation must be calculated:

$$a_2 p^2 y(p) + a_1 p y(p) + a_0 y(p) + 1 = u(p) \quad (17)$$

From classical control theory, the gradual integration method could be helpful, if the intended differential equation is up to the 2nd-order at most:

$$a_0 = \frac{u(\tau_n) - u(\tau_0)}{y(\tau_n) - y(\tau_0)} \quad (18)$$

with value a_0 relatively accurate, however, with more overburden coefficients a_1 , resp. a_2 :

$$\begin{aligned}
 a_1 &= \frac{1}{y(\tau_n) - y(\tau_0)} \left[a_0 \int_0^{\tau_n} \bar{y}(\tau) d\tau - \int_0^{\tau_n} \bar{u}(\tau) d\tau \right] \\
 a_2 &= \frac{1}{y(\tau_n) - y(\tau_0)} \left[\int_0^{\tau_n} \int_{\tau}^{\tau_n} \bar{u} d\tau^2 - a_0 \int_0^{\tau_n} \int_{\tau}^{\tau_n} \bar{y}(\tau) d\tau^2 \right. \\
 &\quad \left. + a_1 \int_0^{\tau_n} \bar{y}(\tau) d\tau \right] \quad (19)
 \end{aligned}$$

An alternative—rough approximation of the 2nd order—gives two time-constants greatly exceeding reliable values. The approximation of transfer function of the 3rd-order with different time constants was used instead⁽²⁵⁾:

$$F_3(p) = \frac{K}{(1+p\tau_1)(1+p\tau_2)(1+p\tau_3)} e^{-p\tau_{td}} \quad (20)$$

where K —system gain, $K=0.073$ and τ_1, τ_2, τ_3 —time constants of reference room, Eq.(15) in Ref.(2). Similarly, the 4th-order equation seeks expression of the same model with the 4-time multiple time constant $\tau_3^{(25)}$. For $D=0$ and zero-inputs except $u_1(\tau)$, i.e. for matrix B including only input steam temperature and with dead time τ_{td} :

$$F_4(p) = \frac{K}{(1+p\tau_s)^4} e^{-p\tau_{td}} \quad (21)$$

While both relations include a term with dead time, from their comparison

$$F_3(p) \cong F_4(p) \quad (22)$$

were obtained its time constants 1650 s, about half an hour.

It is obvious that either full-dynamic or quasi-steady-state models have to match any possible thermal states, including all steady state ones. Usually, there is always on hand heat demand (or heat costs) for any desired period (a day, a week or month). Beside that, advanced models offer the most energy saving option, screening heat fluxes in and out of the object and pinpoint parts and components where heat rate appears in excess, provided there is measured variable(s) enabling such comparison. Other groups of models may offer more time-efficient numerical calculation of qualities, which are used in control systems aimed at energy savings on these objects. As criterion they use various approaches, however each of them ultimately employs a control criteria seeking its minimum. In this study we suggest a form $J = \int_{\tau_0}^{\tau_1} f(x, u, \tau) d\tau$, where $f(x, u, \tau)$ —new functional of state-form variables, which have to be minimized.

The model calculates the heat consumption Q subhourly, in Fig. 8 is depicted an one-day-period heat consumption Q_{24hr} with chosen tempering regime t_{temp} between two consecutive heating periods. In the figure are compared two profiles of the daily heat consumption energy reached under semi-empirical settings of a PS-controller (Takahashi method), case B, and settings which utilize Hamilton function, case D^{(15),(25)}. It reveals energy saving at some extent of

Fig. 8 Reference room heat consumption with controller settings case B (Takahashi method) and optimal control of heat supply (case D) for profiles B, D in Fig. 14

Fig. 9 Heat energy consumption in two consecutive days. At nighttime zone is energy demand increased in tempering regime of about 291.6 K (18.5°C) (angle β) compared with temperature 290 K (16.8°C) (angle α).

more than 8% compared to the Takahashi method, quite favorable result for the 10.5 hours-long period. The saving ratio is also influenced by nighttime regime: in Fig. 9 are depicted slopes for two different tempering temperatures 16.8°C and 18.5°C (290 K and 291.6 K), following two consecutive workdays. Just increasing the nighttime setback up about 1.5 K caused energy consumption increase 18%(!). A typical one-week period (including 2-days-heating-off) comprised about 75 kWh or more than 320 kWh monthly.

4. Control of Heat Supply Delivery

4.1 Control practice at present—open loop approach

Open loop approach means controlling the input temperature of steam $u_1(\tau)$ into the heating system according an algorithm inverse to the outdoor temperature $u_4(\tau)$ or $u_5(\tau)$ (Adelman 1988)^{(2),(15)}. The refer-

ence signal to the controller is calculated from the outdoor temperature by using a calibration curve, which relates the outdoor temperature and the set point for the supply steam temperature: this control principle is called feed forward compensation. Such practice has no feedback from the controlled space's thermal state ($y_s(\tau) = T_s(\tau)$) and is insufficient, if not completed by any secondary temperature modulation.

The new calibration curve of input steam temperature (Fig. 3) can be approximated by the curve of the second (or higher) order expressed by an appropriate formula (for inst. Newton interpolation formula) in operating temperature range of $+267 \dots +287$ K, and representing quasi-steady states near thermal equilibrium with the polynomial expression of the new curve together with known coefficients a_0 , a_1 and a_2 , instead of the whole set of measurement data.

$$N(t_d) = a_0 + a_1(t_e + 7) + a_2(t_e + 14)(t_e - 1) \quad (23)$$

In the case where was no temperature sensor installed (as is still the case in many large administrative buildings with central heating systems) or has been damaged, Eq.(23) provides basic reliable information to the controller for an open-loop approach. With a priority of signals from sensors, both of air temperature t_a as well as global temperature t_g , the model matrix of dynamics is regularly updated and the de facto transfer function offers an effective path for indirect-control approach: the whole decision process regarding the main outcome—input steam temperature $t_{s,i}$ —emanates from the model itself and all measurement data is basically forwarded to Eq. (23) and input vector $u(\tau)$.

4.1.1 Simulation tools & environment

While each case requires conducting a preliminary probe with the purpose of selecting the most fitting control strategy, it is not easy at all and there is no one set of rules leading solely to one control option. The main point is that we have to be sure of the parameters of the object itself, i.e. if there are some changes of the object dynamics, then such demand has to be adjusted at the sampling time either by experience or on the basis the simulation case. Load disturbances obviously change with outdoor climate conditions and with the purpose the space within (internal heat sources' regime, technology etc.). The variations and complexity of all disturbances make it very difficult to compare two different methods under the same (thermal) conditions, thus there is clear advantage to simulation methods. The program used here takes advantage of the powerful toolbox Simulink, developed for an almost universal MATLAB^{(27),(28)}, and gives a convenient option for setting controller parameters employing several reli-

able tuning methods, such as semi-empirical (Ziegler-Nichols, Cohen-Coon), or any other designated to that purpose^{(15),(25)}. In our study, we sought to adjust those controller parameters, which could pursue the controlled value with respect to the minimum transient time^{(10),(28)}, together with treatment of the first peak (guarded limit). Both are frequent requirements in many similar control tasks, and the later is due to securing maximal temperature limit of secondary heat medium. An effective simulation tool, such as Simulink, already contains a library with components that can be linked together into a complete structure, where the controller's and object's parameters are the only ones inserted by the user. This compatibility among modules of such added values became standard over the past few years among well-renowned simulation tools TRNSYS (Andrews, 1979)⁽⁸⁾, DOE-2 (Sullivan et al., 1985)⁽¹³⁾ and recently Energy-Plus, among others.

4.2 PS controller. Control function requirements

The control function aims to maintain system output $y(\tau)$ according a program set to control selected system outputs within acceptable range. With the above depicted model, selected outputs $y(\tau) = [y_1(\tau), y_2(\tau)]^T$ are set for two heating regimes depending on the day of the week, respectively. Such combinations represents common practice at most large residential buildings with central heating system. The controller gets information at the sampling time period about both qualities simultaneously, and thus for integral of the square of the error (ISE):

$$J(\tau)_{ISE} = \int_{\tau_0}^{\tau_1} [e(\tau)^T S \cdot e(\tau)] d\tau \quad (24.a)$$

S : symmetrical positive-definitive matrix $n \times n$,
 e : deviation vector, $e(\tau) = W(\tau) - y(\tau)$,

$\Delta\tau$: sampling time period, $\Delta\tau = \tau_1 - \tau_0$,

or, sufficiently for integral of the absolute value of error (IAE):

$$J(\tau)_{IAE} = \int_{\tau_0}^{\tau_1} |e(\tau)| d\tau \quad (24.b)$$

Under a PI or PS control regime, the set point error is supposed to reach zero in a steady-state condition (considering class of controllers with no derivative function), and Eqs.(24.a), (24.b) allow as well to compare responses that use different sets of controller parameters. A response that gives minimum overshoot with respect to maximal allowed output steam temperature from the heat exchanger, and 30% decay ratio is considered as a satisfactory response. Heuristically tuning the PS parameters is particularly due above given reason excluded. The parameters—dead time τ_{tz} and sampling time $\Delta\tau$ —are

both well below the time scale of any time constant of the object being controlled (difference by the second, respectively the first order), so for such relatively slow responding systems it means full readiness on the side of the controller and measurement (temperature sensor dynamics⁽²⁵⁾). For such applications, also in the transient period between two thermal states is a vast time for an inexpensive controller's processor chip, idling even when cycled in order to keep track in the nearest vicinity of the working point p_{st} on program curve, Fig. 3. The overview in Table 1 shows the values for three methods applied while controlling the indoor thermal environment, selection of other parameters depends on the order of the model being used, and relates to the accuracy of the model and the degree of accuracy.

Table 1 Reference room thermodynamic characteristics with PS-controller semi-empirical settings (times in seconds).

Quality	Symbol	REFERENCE ROOM	CONTROLLER		
			T	Z-N	C-C
Dead time	τ_d	<1 min			
System/Controller gain	K/K_c	0,073	29	36.1	26.9
Time constant of enclosed space (1st-order tr. f.)	τ_s	> 5500	-	-	-
Time constant of enclosed space (4th-order tr. f.)	τ_s'	> 1700	-	-	-
Integral time Constant	τ_i	-	9050	8040	4030
System Transient Time	τ_n	7000	-	-	-
Time coordinate of point of inflexion	τ_i	4500	-	-	-

Abbreviations: T- Takahashi, Z-N – Ziegler-Nichols, C-C – Cohen-Coon

4.3 Controller parameters adjustment (Takahashi method)

In this chapter Takahashi control formulas⁽²⁵⁾ will be mentioned with respect to proportional-sum processing (PS) of controlled deviation e and suitable software environment for simulating, visualizing, and evaluating multivariable controller performance for our quasi-linear system. Different simulation and animation models may be opted for by the user. Users should be able to alter model and controller parameters, thus allowing them to quickly examine different scenarios. Simulink toolbox and MATLAB's three-dimensional graphics are used for signal-processing^{(17),(28)}. Graphical indicators and real-time plotting are also employed to help users. The environment also accommodates data exchange⁽²⁶⁾ in a format common to other programmable languages (Visual Basic, Pascal) and spreadsheets (MS Excel). Other interactive software tools enabling direct control of

the model (full dynamic programming, DDC) appeared highly-valuable in advanced simulation environments nowadays, and created environment have proved to be a valuable tool for enhancing both research and design. Examples are presented in validation charts to demonstrate its versatility.

Next, we will consider an $n+1$ ($n=4$) class linear system containing time-varying parameters whose behavior is not exactly known. The order increasing of one grade with feed-back control loop causes only the integral part of PS-controller. We assume that the parameters vary within known intervals and there are known bounds on their rates of variation. We give a computationally verifiable condition for stability of the system for all possible parameter variations. In particular, we first show that the information on the rate bounds can be exploited by considering an augmented system described by the model also containing the PS-controller equation. Through the analysis of the implicit system using some numerical solution methods, we can obtain a stability conditions for the original system. Also, the computational aspects of the proposed stability conditions are addressed in terms of a linear matrix, which can be solved efficiently with a numerical method.

For the control algorithm is utilized time-discretisation form with proportional and sum component (PS), i.e. its continuous formula, which follows from:

$$u(\tau) = r_0 \left[e(\tau) + \frac{1}{T_i} \int_0^\tau e(\tau) d\tau \right] \quad (25)$$

where integration is substituted through the sum of controlled deviations e :

$$u(k) = r_0 \left[e(k) + \frac{\Delta\tau}{T_i} \sum_{i=0}^k e(i) \right] \quad (26)$$

with $\Delta\tau$ -sampling period (s). Recommended values of both parameters of PS controller—proportional constant and time constant⁽²⁵⁾:

$$r_0 K = \frac{0.9 \cdot \tau_n}{\tau_u + \frac{\Delta\tau}{2}} \cdot \frac{0.135 \cdot \tau_n \cdot \Delta\tau}{\left(\tau_u + \frac{\Delta\tau}{2} \right)^2}$$

$$\frac{\Delta\tau}{T_i} = \frac{0.27 \cdot \tau_n \cdot \Delta\tau}{r_0 \cdot K \left(\tau_u + \frac{\Delta\tau}{2} \right)^2} \quad (27)$$

T_i : integral constant (s), τ_n : transient time period (s), τ_u : timelag (s), $\Delta\tau$: sampling period (s), r_0 : proportional constant (gain) of controller (—), K : system gain (—).

Substituting $R = K/\tau_n$, $K_p = r_0$, and $K_i = r_0 \cdot \Delta\tau/T_i$ follows:

$$K_p = \frac{0.9}{R \left(\tau_u + \frac{\Delta\tau}{2} \right)} - \frac{1}{2} K_i \quad K_i = \frac{0.27 \cdot \Delta\tau}{R \left(\tau_u + \frac{\Delta\tau}{2} \right)^2} \quad (28)$$

These settings on the PS-controller were used while simulating a heating-up period, Fig. 10 next page.

Fig. 10 Case A,B,C. Comparison of global air temperature profiles of reference room without overheating. *E*-measurements, A, B, C-simulation case. PI-controller with Ziegler-Nichols, Takahashi and Cohen-Coon settings, $|0.1| \cong 0.5$ K; $T_{ref} = 293$ K.

Fig. 11 4th-order simulation module of single-zone space with steam heater's transfer function (transfer blocks Fcn1, Fcn5 and transport block Delay 1). Simulink/MATLAB environment.

Also, the other two temperature profiles with semi-empirical settings of their controllers are drawn in this figure: Ziegler-Nichols (Case A) and Cohen-Coon (Case C). Their profiles are drawn from bottom upward in order A, B and C. Although, there are some remarkable differences among them, the first swing avoided a boost in time following a nighttime setback ($\tau < 0$ s), resulting in an acceptable magnitude of about $|0.3|$ K (case B). The Takahashi course seems here to be quit balanced as its output of global temperature t_g remains nearly centered (averaged values) between cases A and C almost all of the simulation time, 5×10^5 s.

Transient responses of outside wall with its 4th-order model, Fig. 11, for outdoor harmonic (sinus) temperature changes, shows crossing through a common point of tangents of inflexion point *P*, Fig. 12. The shifting appears at first on the outdoor-faced outside wall and effectively 'neutralizes' the amount of heat being stored in the wall in the way, that reversing the heat flow from that point causes a 'cut' of heat thermal capacity, being accounted as available to the room. In reality, summoned *P*-points create a virtual plane moving further to the inside of wall and

Fig. 12 Outside wall temperature distribution with convective boundary conditions $h_{wo} = 5.7$ W·m⁻²K⁻¹. 6 nodes, $x = 40$ mm: time step 180 s, $\tau = 6400$ s, $T_i = 293.15$ K; $T_e = 276.85 \pm 6.75$ K ($-3.05 \dots +10.45$ °C), harmonic (sinus) temperature change. Runge-Kutta 3. order, $Bi \cong 1.2$.

determine a new location of the plane equation $\frac{dt(\tau)}{dx} \Big|_{x=x_p} = 0$ (K·m⁻¹)⁽²⁹⁾. Taking into account the Biot number as if it was the surface, beginning in the current location of the moving *P*-plane, the time lag will become effectively shorter ($h_p \cdot (x_p + \Delta x) < k$).

The simulation test, including thermal periodic (sinus) disturbance (Fig. 12) caused by exposing the outside wall to the external heat load, allowed us to find out its thermodynamic characteristics—crucial to the investigated space⁽²⁰⁾. Providing the air in the space is perfectly stirred (as is usually assumed in models of that kind), the wall-effect 'shifts' together with the dead time period into the room and enables one to estimate the system time constant:

$$\tau_s = \frac{\tau_{inf} - \tau_{tz}}{n - 1} \quad (29)$$

τ_s : system time constant (Table 1), τ_{tz} : dead time (Table 1) and n : system order. Temperature amplitudes' magnitude on outer and inner walls (Fig. 12), while outdoor climate-driven disturbances occur, are useful for evaluating gains if thermal equilibrium would not be sufficiently reached. For outside wall, the overall damping coefficient was:

$$\nu_{wo} = \frac{\Delta t_{wo,i}}{\Delta t_{wo,e}} = 0.19 \quad (-) \quad (30)$$

$\Delta t_{wo,e}$: two, amplitudes of temperature wave on out-faced outside wall (K), and $\Delta t_{wo,i}$: two amplitudes of temperature wave at room-faced inner surface of the outside wall (K).

4.4 Optimal control utilizing criterion with Lagrange multipliers (Hamilton canonical function)

The basic specifications for an optimal control of the system Eq.(2), with initial conditions $x_0 = x(\tau_0)$

and the objective vector—eventual state vector $\mathbf{x}_N = \mathbf{x}(\tau_N)$ are completed with limitations on input vector $\mathbf{u}(\tau)$:

$$|u_i(\tau)| \leq U_i, \quad i=1, \dots, r. \quad (31)$$

We have formed the problem of finding optimal trajectory $\mathbf{u}(\tau) = \mathbf{u}^*(\tau)$ as the variable task with quality control criterion^{(25),(30)}

$$J = J_1 + \sum_{i=1}^N p_i(\tau) \phi_i(\mathbf{x}, \mathbf{u}, \mathbf{p}, \tau) d\tau \quad (32)$$

where $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in \mathbf{R}^r$ of the state-form system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \tau) \text{— equation system (2),}$$

$$-\phi_i(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{u}, \tau) = 0, \quad i=1, 2, \dots, n \text{—boundary conditions of a newly introduced functional}$$

$$\phi_i(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{u}, \tau) = \phi_i(\mathbf{x}, \mathbf{u}, \tau) - \dot{\mathbf{x}}_i,$$

$$-\mathbf{p}(\tau) = [p_1(\tau), p_2(\tau), \dots, p_n(\tau)]^T \text{—new vector of Lagrange multipliers,}$$

$$\text{—common optimal criterion } J_1 = \int_{\tau_0}^{\tau_1} L(\mathbf{x}, \mathbf{u}, \tau) d\tau.$$

With applied energy criterion will be further sought an optimal control as optimal time-dependent controller output $y_i(\tau)$. The corresponding criterion, ($c_i = \text{const}$, $n \approx 1$):

$$J_2 = \int_{\tau_0}^{\tau_1} \sum_{i=1}^r c_i u_i^n(\tau) d\tau \quad (33)$$

is in common matrix form

$$J_2 = \int_{\tau_0}^{\tau_1} [\mathbf{u}^T \mathbf{R} \mathbf{u}] |_{\mathbf{u} \in \mathbf{R} \times \mathbf{R}} d\tau \quad (34)$$

where $\mathbf{R}$ is symmetric, positive-definite matrix ($r \times r$). Further was employed the new scalar—

Notice: Hamiltonian-Like Model

In the classical dynamics of Hamiltonian models, there exists a wide variety of nonlinear circuit models and mechanical models where energy storage elements determine the structure of the state-form description. They represent system states like the standard spatial coordinate system, which are determined through a vector of positions $\mathbf{s}$ together with the associated group of vector of momenta $\mathbf{m}$. The differentiable system of Hamiltonian function $\mathbf{H}(\mathbf{s}, \mathbf{p})$ in matrix form

$$\begin{bmatrix} \frac{\partial \mathbf{s}}{\partial \tau} \\ \frac{\partial \mathbf{p}}{\partial \tau} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \nabla_{\mathbf{s}, \mathbf{p}} \mathbf{H}(\mathbf{s}, \mathbf{p}) \quad (\text{N.1})$$

can sustain nondissipativity of the system by the derivatives of $\mathbf{H}$ to be zero along its trajectories:

$$\begin{aligned} \frac{d\mathbf{H}[\mathbf{s}(\tau), \mathbf{p}(\tau)]}{d\tau} &= [\nabla \mathbf{H}(\mathbf{s}, \mathbf{p})]^T \\ &\times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} [\nabla \mathbf{H}(\mathbf{s}, \mathbf{p})] = 0 \end{aligned} \quad (\text{N.2})$$

The reader is encouraged here to learn more about the systems-candidates eligible to be Hamiltonian-like system model in Ref. (30), chapter 4.

Hamilton function— $H = H(\mathbf{x}, \mathbf{u}, \mathbf{p}, \tau)$

$$\begin{aligned} H(\mathbf{x}, \mathbf{u}, \mathbf{p}, \tau) &= L(\mathbf{x}, \mathbf{u}, \tau) + \sum_{i=1}^n p_i(\tau) f_i(\mathbf{x}, \mathbf{u}, \tau) \\ &= L + \mathbf{p}^T \cdot \mathbf{f} \end{aligned} \quad (35)$$

This function applied on Eq.(32) yields a completed 'new' minimization criterion:

$$J^* = \int [H(\mathbf{x}, \mathbf{u}, \mathbf{p}, \tau) - \mathbf{p}^T \cdot \dot{\mathbf{x}}] d\tau \quad (36)$$

with familiar (the first two) canonical Hamilton conditional equations:

$$\dot{\mathbf{x}} = \frac{\partial H}{\partial \mathbf{p}}, \quad \dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{x}} \quad \text{and} \quad \frac{\partial H}{\partial \mathbf{u}} = 0 \quad (37)$$

Applied on Eq.(2) with input variables $\mathbf{u}(\tau) = [u_1(\tau), u_4(\tau), u_5(\tau)]^T$:

$$\dot{\mathbf{x}}_1(\mathbf{p}) = \mathbf{x}_2(\mathbf{p})$$

$$\dot{\mathbf{x}}_2(\mathbf{p}) = -\frac{\bar{\tau}}{\tau^*} \mathbf{x}_2(\mathbf{p}) - \frac{1}{\tau^*} \mathbf{x}_1(\mathbf{p}) + \frac{K^*}{\tau^*} \mathbf{u}(\mathbf{p}) \quad (38)$$

$$y_3(\mathbf{p}) = \mathbf{x}_1(\mathbf{p}):$$

where

$\bar{\tau}$: individual time constant (s)

τ^* : compound time constant (s), $\tau^* = f(\tau_i)$, $i=1, 2, \dots, n$

K^* : individual gain (—), $K^* = f(K_i)$, $i=1, 2, \dots, n$

Utilizing energy criterion Eq.(32) yields the Hamilton function:

$$\begin{aligned} H(\mathbf{x}, \mathbf{u}, \mathbf{p}, \tau) &= \frac{1}{2} \mathbf{u}^2(\tau) + \mathbf{p}_1(\tau) \mathbf{x}_2(\tau) \\ &+ \mathbf{p}_2(\tau) \left[-\frac{1}{\bar{\tau}} \mathbf{x}_1(\tau) \right] - \mathbf{p}_2(\tau) \frac{\tau^*}{\bar{\tau}} \mathbf{x}_2(\tau) \\ &+ \mathbf{p}_2(\tau) \frac{K^*}{\bar{\tau}} \mathbf{u}(\tau) \end{aligned} \quad (39)$$

Completed with canonical Hamilton conditional relations (37), Lagrange multipliers were finally obtained:

$$\dot{p}_1 = -\frac{\partial H}{\partial x_1} = \frac{1}{\bar{\tau}} p_2 \quad \wedge \quad \dot{p}_2 = -\frac{\partial H}{\partial x_2} = -p_1 + \frac{\tau^*}{\bar{\tau}} p_2 \quad (40)$$

They could be solved separately, apart from Eq. (38), thus guaranteeing way for a consistent optimal trajectory of searched vector $\mathbf{u}(\tau) = \mathbf{u}^*(\tau)$ within defined space $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in \mathbf{R}^r$. The Eq.(40) with two Lagrange multipliers has one solution expressed with initial conditions $\mathbf{x}_0 = \mathbf{x}(\tau_0)$ and a final state with eventual state vector $\mathbf{x}_N = \mathbf{x}(\tau_N)$: for the first term of input vector $\mathbf{u}^*(\tau)$ yields

$$\begin{aligned} \mathbf{u}^*(j\Delta\tau) &= \mathbf{u}^*(j\Delta\tau) \\ &= -\frac{\bar{\tau}}{K^*} \mathbf{x}_2[(j-1)\Delta\tau] j\Delta\tau - \mathbf{x}_1[(j-1)\Delta\tau] \end{aligned}$$

$$\Delta\tau: \text{ sampling period (s), } j=1, 2, \dots, n \quad (41)$$

with new initial conditions $\mathbf{x}_1(j-1)$, $\mathbf{x}_2(j-1)$ in each sampling time step $j\Delta\tau$, $j=1, 2, \dots, n$.

Comparison of the optimal control with Hamilton function using Lagrange multipliers (case D) with semi-empirical settings of controller are shown in Figs. 8 and 14.

Fig. 13 Comparison of global air temperature of reference room while overheating occurred in two consecutive 24-hour-long heating-on/off periods with steam heater and infrared gas heater used simultaneously. 1-measurement (no control feedback), 2-controlled (PS controller with Takahashi, Cohen-Coon and Ziegler-Nichols settings; $|0.1| \cong 2$ K; $T_{ref} = 294$ K).

Increase of heat energy consumption during the heat supply within working hour period (8 a.m. – 6 p.m.) revealed about 8% while using Takahashi controller settings (Fig. 8, case B). In average, for 22 workdays it means monthly savings for the reference room more than 12.3 kWh. As for the heat energy supplied in the room over the same period, similar outcomes are seen among semi-empirical methods (Takahashi, Ziegler-Nichols and Cohen-Coon)^{(10),(25)}. Compared with each other, less significant results are demonstrated: during two consecutive days with the night period supply off, considerable savings are seen, if controllers continued to work and maintained a nighttime tempering of about 17°C (Fig. 9).

The temperature profiles (global temperature T_g of the measured space within the occupied zone), referring to both control methods (optimal with Hamilton equation and Takahashi method) over the same 10-hour-working-day period, differ at most according to expectations in the heating-up transient period (about the first two hours after heating up), in cases B and D in Fig. 14. At the end of heat supply, the controller is switched off and the temperature is dropping toward a weekend heating-off-period level. The third curve in Fig. 14 (case E) represents measured global temperature T_g , with the simplest regime of central heating steam (CHS) supply: two-heating-up and two-heating-off periods. The two peaks in the graph correspond to the times when heat supply is likely to culminate—the first shortly before midday, the second at the end of working hours—with the lowered gap referring to the time heat supply is being suspended. The huge thermal energy gains in the middle of the day transmitted by direct solar radiation

Fig. 14 Case B, D, E. Optimal control of global air temperature of reference room with Hamilton function (case D) and semi-empirical method (case B). Case E (measurement) -no control feedback; $|0.1| \cong 2$ K; $T_{ref} = 294$ K.

basically aggravate the impact of the outdoor temperature peak shortly after midday and increase the temperature of already overheated space. On the other hand, the effect of a substantive reducing of radiative solar heat gain through the fenestration by using opaque roller shades together with a moderate heat supply creates particular indoor thermal conditions that are most acceptable. The highest value of indoor global temperature ($T_g = 294$ K (21°C) at $\tau = 2.93 \times 10^4$ s) with the lowest one ($T_g = 292$ K (19°C) at $\tau = 0$ s) was the best result achieved, Fig. 10, curve E.

With several empirical controller parameters adjustments—cases A, B and C—the desired temperature 293/294 K (20/21°C) was reached after about one hour, Figs. 10, 14, with costs results in Fig. 8 for respective controllers' settings in Table 1 below. The settings using Hamilton function will save well up 9% of heat costs Fig. 8, with properly adjusted tempering through the night period, Fig. 13. The difference among three semi-empirical settings was less than 0.5% in the transient area while the optimal setting approach managed to reach the desired temperature value (either 20 or 21°C) earlier in the same period. Further, it uses only the initial output values from previous time steps and the desired value from program curve, Fig. 3.

5. Conclusions

A mathematical model of the single-zone reference room was used to evaluate its thermal performance and provide a ready-to-work model in an on-line control loop with a PS-controller. Evaluated thermal characteristics aimed to facilitate both system-order reduction, the number of inputs, and provide basic parameters for controller settings.

1. As a thermal system, which is invariant, non-linear, and reciprocal, a system reduction, with a decomposed matrix of dynamics, led to the separation terms. For both parts of the matrix dynamics of the 4th and 5th-order, respectively, their transfer functions were estimated with gains ranging from value 0.073 for steam heater and almost 0.19 for outdoor temperature. Radiative and convective heat transfer are treated in a similar way as in the associated emitter model, however, since convective heat transfer may vary significantly, respective coefficients are subject to change simultaneously with sampling time. Thus, the model is capable of relatively accurate evaluation of thermal states in typically shorter periods of time than normal in similar models available nowadays.

2. Controller settings gave satisfactory results (Fig. 10) with values in Table 1. They were compared with optimal settings using the Hamilton function; differences shown in the transient area with the most closely matching semi-empirical method was less than 3% in controlled variable. After a series of simulation tests with different internal and external loads, it was possible to confirm the suggested parameters but also the wage errors of both output temperatures and decide between them for the specific case.

3. Considerable heat losses through a single-glazed window and overheating led to increased consumption of primary heat medium — steam — distributed to the whole building. The size of the night-time setback was decisive for energy saving as well as time of the heating-up period. However, it is not obvious how to select the appropriate values for it while exact heat losses for the time remaining to the start of heating period with or without boost are unknown.

4. The optimal control trajectory derived with Lagrange multipliers would save up to 9% of heat energy supply in comparison to any semi-empirical control method. There is a certain advantage to using the Hamilton function, as its outcomes do not need a time constant, nor system gain knowledge a priori, required by the other empirical control methods.

Appendix: Dynamic Optimization Problem Transformation to Static Optimization of One Variable

In the case where we would not be able to accommodate the condition concerning the time-invariant Hamilton function along an optimal trajectory ($\frac{\partial H}{\partial u} = 0$ cannot exist), we can find expansions $u_0, u_1, \dots, u_{N-1}$: ($\forall u_i \in U$) from discrete functional criteria

$$J = \sum_{k=0}^{N-1} L_i(x_k, u_k) \Delta \tau + \Phi_1[x(N \Delta \tau)] \\ = \sum_{k=0}^{N-1} L_i(x_k, u_k) + \Phi(x_N) \quad (\text{A.1})$$

and reload the problem to the task of simpler gradual minimization of one variable. Then the new variable v_i will include all $x \in R^n$, $u \in R^r$ as summarized into matrix V :

$$V = \begin{bmatrix} \tau_1 \cdots v_1 & x_1(1) \cdots v_{N+r \cdot Nr+1} \\ \tau_2 \cdots v_2 & x_2(1) \cdots v_{N+r \cdot Nr+2} \\ \vdots & \vdots \\ \tau_N \cdots v_N & x_N(1) \cdots v_{N+r \cdot N+n} \\ u_1(0) \cdots v_{N+1} & x_1(r) \cdots v_{N+r \cdot N+n+1} \\ \vdots & \vdots \\ u_r(0) \cdots v_{N+r} & x_r(r) \cdots v_{N+r \cdot N+n+r} \\ u_1(1) \cdots v_{N+r+1} & x_1(N) \cdots v_{N+r \cdot N+(N-1)n+1} \\ \vdots & \vdots \\ u_r(1) \cdots v_{N+r+r} & x_r(N) \cdots v_{N+r \cdot N+n \cdot N} \end{bmatrix} \quad (\text{A.2})$$

The newly obtained $N(n+r+1)$ variables can be substituted into the Eq.(A.3), which was rewritten from Eq.(A.1):

$$J_N = \sum_{k=0}^{N-1} L[x(k), u(k-1), k] \tau_k + \Phi[x(N)] \quad (\text{A.3})$$

The Eq.(A.3) now refers to the new task of static optimization with $J_N = f(v)$, with more variables, however, letting to formulate other conditions on state form variables x_i , which are uneasy in dynamic methods.

Acknowledgments

The study is part of a project carried out at Faculty of Engineering of Yamaguchi University where the measurements part was enabled and kindly supported by Prof. M. Okada. We would like to acknowledge also the full cooperation we received from colleagues Mr. Y. Isobe and Mr. Y. Yamashita as well as by heat exchange station staff who provided us with the data on heating system and assisted in the experiment.

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