An Optimization-oriented Simulation-based Job Shop Scheduling Method with Four Parameters Using Pattern Search

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Aiming at the elimination of tardy jobs in a job shop production schedule, an optimization-oriented simulation-based scheduling (OSBS) method incorporating capacity adjustment function is proposed. In order to determine the pertinent additional capacities and to control job allocations simultaneously the proposed method incorporates the parameter-space search improvement (PSSI) method into the scheduling procedure. In previous papers, we have introduced four parameters; two of them are used to control the upper limit to the additional capacity and the balance of the capacity distribution among machines, while the others are used to control the job allocation procedure. We found that a 'direct' optimization procedure which uses the enumeration method produces a best solution with practical significance, but it takes too much computation time for practical use. In this paper, we propose a new method which adopts a pattern search method in the schedule generation procedure to obtain an approximate optimal solution. It is found that the computation time becomes short enough for a practical use. Moreover, the extension of the parameter domain yields an approximate optimal solution which is better than the best solution obtained by the 'direct' optimization.

Key Words: Scheduling, Simulation, Production Planning, Job Shop, Parameter-space Search Improvement Method, Optimization, Due-date, Tardy Job

1. Introduction

In make-to-order production, the most important requirement for production scheduling is to fulfill the job due-date promised to individual customer. Make-to-order production is usually realized on a job shop production environment. It is not easy task to fulfill all the customer due-dates since all jobs, which naturally differ in due-dates from each other, have different routings with different process times and therefore the required capacity in each work center constantly changes time by time.

Production planning activity generally consists of upper-level production planning activity and production scheduling activity that makes the detailed manu-

facturing schedule on the basis of the production plan⁽⁵⁾. It is a general principle that the master production schedule and the capacity plan which are determined by the upper-level planning department are not to be changed in the scheduling department. However, in practice, they are often changed, by making full use of intuition and accumulated experiences in the scheduling department. Distinct improvements in scheduling may be gained by shifting capacity from one time-bucket in a work center to another or supplementing capacity.

When the tardy jobs occur in a scheduling process, the scheduling personnel tries to resolve the tardiness problem by taking the following measures step by step.

(a) To try to improve the scheduling result by changing operational order of jobs, lot splitting and so on in order to prevent the tardy job within the capacity given to each work center.

(b) To try to reduce the tardy jobs, if they are unavoidable, by supplementing capacity of the factory

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to some extent within the limit that the upper-level planning department grants. To be more concrete, overtime work, change of off days to work days and manpower movement between work centers are some examples of capacity supplement measures.

(c) To request that the upper-level department, in case measure (b) is not efficient enough, makes a change in the due-date of the jobs and the given capacity of the related work centers.

Measure (a) is considered to correspond to a minimization problem of tardy jobs under the given production planning condition, and measure (b) to a capacity supplement determination problem aiming at the elimination of tardy jobs under the condition that the scheduling generation method is fixed.

Many researches on a minimization problem of tardy jobs under the given production planning condition in a practical production environment have been carried out^{(2),(3)}. As for the research on capacity adjustment, various types of planning by infinite/finite capacity requirement planning method in production planning level have been actively discussed for several decades. In recent years, the unification method between production planning and production scheduling has been proposed. The aim of the method is to enhance production management quality by introducing higher capability of capacity adjustment and operational order setting.

As for research on the above mentioned item (b) of detailed scheduling, capacity adjustment methods in considering job operation sequences are limited^{(4),(5)}. In job shop production scheduling, it is generally difficult to decide when, to which machine and in what amount to add capacities so as to minimize of the number of tardy jobs after suppressing the added amount of the capacity.

In order to determine the pertinent additional capacities and to control job allocations simultaneously we incorporate the parameter-space-search-improvement method (named the PSSI method)⁽⁶⁾ into the scheduling procedure. The PSSI method is a framework for finding the best solution in a simulation method. It introduces a very few number of parameters which can systematically manipulate the relevant variables and the best solution is sought for on the parameter space spanned by the introduced parameters. In previous papers, we have introduced four parameters; two of them a and b are used to control the upper limit to the additional capacity and the balance of the capacity distribution among machines⁽⁷⁾. The best capacity addition plan is sought for on the parameter space $a \times b$, while the others c_1 and c_2 are used to control the job allocation procedure⁽⁸⁾ and the best schedule with respect to due-date

related criterion is sought for on the parameter space $c_1 \times c_2$. A 'direct' optimization procedures on the four-dimensional direct-product space which we named the OSBS/4 method is developed by merging the procedures separately treated on the two dimensional spaces⁽⁹⁾. By using scheduling data obtained from a practical large-scale system, the performance of the procedure was investigated. It is found that the number of tardy jobs and the added amount of the capacity are significantly reduced, but the computation time to obtain the best solution by enumeration is too long for practical application.

In order to shorten the computation time we propose in this paper a new method which adopts a local search method in the schedule generation procedure, and examine the performance of the propose method.

2. An Optimization-Oriented Simulation-Based Scheduling Method

As for scheduling aiming at the reduction of job tardiness, a certain type of the Backward/Forward Hybrid Simulation method (named BFHS/type C) is known to show excellent performance over the conventional forward simulation method with sophisticated dispatching rules⁽¹⁰⁾. In the BFHS/type C method, the due-date related information obtained from the first step backward simulation is utilized in the second step forward simulation, i.e., the order of the starting time for each job at each work center estimated in the backward simulation is used in the forward simulation to create a job priority order for the concerned jobs at each work center. We adopt this BFHS/type C method as a schedule generation method in the scheduling procedures in this section.

2.1 Parameters to control job allocations

Since the set of the parameters (c_1, c_2) shows the best performance with respect to tardiness related criteria such as the average tardiness, the maximum tardiness and the number of tardy jobs⁽⁸⁾, we adopt this parameter set to control the job allocation process in an optimization procedure.

This parameter set was introduced with the intention to control the job allocation process by utilizing the characteristics of the initially generated scheduling result. Focusing on the due-date lateness (=the job completion time—the due-date) and the accumulated waiting time of each job in the initial schedule, we intended to adjust the slack time for each job at every occasion when it is estimated for calculating the job priorities. In order to reduce the number of the degrees of freedom, we introduce the two parameters (c_1, c_2) and decide the adjustment quantity Δ_i for the slack time for a job i by the following equation

$$A_i = c_1 L_i + c_2 W_i, \quad (1)$$

where L_i and W_i stand for the due-date lateness and the waiting time, respectively, and the parameters c_1 and c_2 are used for all jobs in common. By systematically changing the parameter values, we can control the job allocation process.

2.2 Parameters to control capacity adjustments

When we limit the capacity adjustment to the supplement of capacities, the items to be determined in the capacity adjustment control procedure are the capacities when, to which machine and what amount, to add. In order to make a pertinent decision on the capacity supplements, we introduced two parameters a and b which can systematically manipulate the relevant decision variables⁽⁷⁾. The parameter a which takes the range $0 \leq a \leq 1$ controls the upper limit to the additional capacity and the parameter b which takes the range $-1 \leq b \leq 1$ the balance of the capacity distribution among machines.

With these two parameters, we determine the initially added capacity A_k for a machine k per day by the following equation:

$$A_k = a A_k^{\max} f(x_k, b), \quad (2)$$

where $A_k^{\max}$ denotes "the maximum additional capacity" for the machine k and x_k is a real number in the range $[0, 1]$ related to the machine k . In this paper, we decide the value x_k referring to the machine utilization under the given (initial) planning condition. According to the descendent order in the machine utilization, we set the values between 1.0 and 0.0 with equal interval by the following equation:

$$x_k = \frac{N - o_k}{N - 1}, \quad (3)$$

where N stands for the total number of machines involved in the planning condition and o_k denotes the ordinal number reflecting the utilization order of the machine k . By this equation, the machine with the highest utilization takes the value 1.0 and that with the lowest utilization takes the value 0.0.

By introducing the parameter a in Eq.(2), we can control the amount of capacity per day to add, and we may avoid the situation where jobs are allocated too earlier than required due to unnecessary addition of the capacity. The range of the parameter a is taken to be $[0, 1]$ in the following calculation.

On the other hand, we set the parameter b so as to change the balance of capacities to distribute among machines. We realize such function by changing the functional shape of $f(x, b)$ in terms of the parameter b . In this paper we set the concrete form of $f(x, b)$ as

$$f(x, b) = 1 - {}^{1-b}\sqrt{1-x^{1-b}} \quad \text{for } 0 \leq b \leq 1, \quad (4)$$

$$f(x, b) = {}^{1+b}\sqrt{1-(1-x)^{1+b}} \quad \text{for } -0 \leq b < 1. \quad (5)$$

The shape of $f(x, b)$ changes according to the param-

Fig. 1 The Functional Shape of $f(x, b)$ Plotted for Different Values of b

eter value b as illustrated in Fig. 1. When the parameter value b is set close to 1.0, the additional capacities are put equally distributed to all machines. On the contrary, when the parameter value b is set negative, the additional capacities tend to concentrate to machines with higher utilizations (i.e., machines with x_k values closer to 1.0).

By specifying the set of values for (a, b) , we can determine the initially added capacities for all machines.

2.3 Schedule generation procedure

A 'direct' optimization procedure to seek for an approximate optimal solution on the direct product space of the four parameters a, b, c_1 and c_2 was developed by Arakawa et al.⁽⁹⁾, where the enumeration method is adopted to select a best solution. In order to reduce the computation time, we adopt one of a direct search method termed the pattern search⁽¹¹⁾. Since the search method is adopted, the solution to be found can be an approximate optimal solution. However, the extension of the original parameter domain by the adoption of finer step sizes (grid spacing) and by the removal of the parameter boundaries yields an approximate optimal solution which can be better than the best solution defined on the original parameter domain.

The proposed scheduling procedure which we shall name the optimization-oriented simulation-based scheduling method with four parameters using a pattern search method (*the OSBS/4/P method* in abbreviation) starts with the initial schedule generation and calculation of necessary quantities followed by a pattern search procedure with modifications.

The pattern search procedure is described in Appendix B by Hooke and Jeeves⁽¹¹⁾ in a general form.

The procedure is applied to the current four-dimensional space problem: a space point is specified by the coordinates a, b, c_1, c_2 ; the 'function' is the procedure at a given space point to calculate the number of tardy jobs in a schedule and to decide the total additional capacity; and the 'functional value' corresponds to the number of tardy jobs as well as the total additional capacity.

We introduce the following modifications into the original pattern search procedure:

(1) In the original procedure, not all of the possible exploratory points are visited, and the new coordinate after a success move is retained and used at later steps. We modify this procedure to check all possible values obtained in exploratory moves, since 'functional value' is found to change discontinuously on the parameter space.

(2) When the reduced step size becomes smaller than the "minimum" step size δ , reset the base point by a move to a randomly chosen point and set the step size the initial value, and restart the pattern search as long as the accumulated number of the base point random jump is smaller than the preset limit. This modification is expected to allow a search not to be trapped in a local minimum on the parameter space.

During the search procedure we refer to the solution and the 'functional value' so far attained, which we shall call the reference solution and the reference value, respectively.

The OSBS/4/P method is described as follows:

Main procedure to search for an approximate optimal solution

Step 1. Search condition setup

Set the following values related to the search condition: the initial step size Δ_0 , the 'minimum' step size δ , the initial position $\mathbf{p}_0$ of the base point, the maximum number M of the base point random jump along with the stochastic function which gives a size of the random jump.

Step 2. Initial schedule generation and calculation of necessary quantities

Generate an 'initial schedule' subject to the initial planning condition by the BFHS/type C method, and calculate the quantities which will become necessary in the subprocedure: the machine utilization, the due-date lateness L_i and the accumulated waiting time W_i for every job i .

Step 3. Initial value assignments

Assign the initial value $\mathbf{p}_0$ to the base point $\mathbf{p}$, the initial value Δ_0 to the step size Δ , the initial schedule and the initial planning condition to the reference solution, and the number of tardy jobs in the initial schedule and the value zero of the total additional capacity to the reference value.

Step 4. Exploratory points arrangement and functional values estimation

Step 4a. Exploratory points arrangement

Arrange two exploratory points for each coordinate which locate both side of the base point $\mathbf{p}$ with the distance Δ . Since we have four coordinates, eight exploratory points are configured around the base point. When the coordinate values for a or b exceed their own boundary, put the exploratory point at each boundary point.

Step 4b. Solution generation and reference solution update

Repeat the following (1) and (2) for all eight exploratory points.

(1) At each exploratory point $\mathbf{x}$, execute the subprocedure $S(\mathbf{x})$ to obtain an exploratory solution and the functional value.

(2) Compare the functional value with the reference value. If the functional value is superior to the reference value, update the reference solution and the reference value by the exploratory solution and the functional value, respectively.

Step 5. Base point and step size update

According to whether or not the reference solution is updated at Step 4b, take the following actions:

(1) If the reference solution was updated, by using the exploratory point $\mathbf{r}$ where this reference solution is obtained, move the current base point $\mathbf{p}$ to the new base point $2\mathbf{r} - \mathbf{p}$, reset the step size Δ to the initial value Δ_0 , and go to Step 4.

(2) Otherwise, move the current base point $\mathbf{p}$ to the point $\mathbf{x}$ where the reference solution is obtained and replace the value of the step size Δ by one-half of it. If new step size is less than the minimum step size δ , go to Step 6, or else go to Step 4.

Step 6. Search termination or a base point jump

If the accumulated number of the base point jump is equal to the maximum number M , set the current reference solution to the approximate optimal solution, and terminate the search procedure.

Otherwise, reset the step size Δ to the initial value Δ_0 , move the current base point $\mathbf{p}$ to the point $\mathbf{p} + \text{rand}$ where rand means the value given by the random number with the normal distribution, and go to Step 4.

When the number of tardy jobs of the functional value is equal to that of the reference value at Step 4b (2), we compare the total additional capacities, and if the total additional capacity of the functional value is smaller than or equal to that of the reference value, we regard that the functional value is 'superior' to the reference value.

Subprocedure $S(\mathbf{x})$

Step s1. Capacity allocation

Determine the amount of additional capacities and the dates to add for each machine in terms of the coordinate values on the exploratory point x and distribute them to each machine. We shall call these capacities distributed at this step “initially added capacity”.

Step s2. Schedule generation by the BFHS/type C method with slack time modifications

Allocate the jobs to the machines by the BFHS/type C method subject to the planning condition that extends the initial planning condition to include the initially added capacity. In the backward simulation process of the BFHS/type C method, the slack time modifications specified by the parameters c_1 and c_2 are taken into account.

Step s3. Elimination of redundant capacity

On the basis of the schedule generated at *Step s2*, eliminate the “unused time period” from the initially set over time period (i.e., the added capacity on that day). Here we call the period that no job is allocated until the end of that day the “unused time period”. Moreover, if it is found that the added capacity on each machine on a particular day is not efficiently used, cancel the capacity addition on that machine on that day. We regard in this paper the capacity addition inefficient if the time interval between the end of the normal available time period and the onset time of a job allocated in the added capacity time period is larger than 30 minutes.

Step s4. Schedule regeneration by the BFHS/type C method with slack time modifications

Reallocate the jobs to the machines by BFHS/type C method taking account of the capacity alteration at *Step s3* and of the slack time modification.

Step s5. Elimination of “unused” capacity

On the basis of the schedule generated at *Step s4*, if an “unused time period” is again found, eliminate it from the modified over time period. At this step, the total additional capacity to all machines is fixed for a particular point on the parameter space.

Step s6. End of the subprocedure

Return the schedule generated at *Step s4* and the amount of the total additional capacity calculated at *Step s5*.

Figure 2 illustrates the relationship between the initially added capacity and the added capacity after the subtraction of the redundant capacity at *Step s5*. In the figure, “the maximum additional capacity” means the time period obtained by subtracting the normal/planned available time period and the period for the maintenance from 24 hours. It corresponds to the maximum capacity being able to add to each machine. In the above procedure, job allocation is

(a) Initial capacity allocation at Step s1

(b) Added capacity and “unused capacity” at Step s3

Fig. 2 The addition of capacities. This shows conceptually a case for a machine on a particular day.

performed two times. This is because that some overtime period to which a job is once allocated at *Step s2* may be taken off due to its inefficiency, and the reallocation is required to allocate the jobs to the available time period.

Because of the modification (1), there is possibility to select the space point once visited, we store on the computer memory every calculated result as the attribute of a space point, and reuse it to speed up the computation.

In the above procedure, the upper limit to the total amount of the additional capacity is not treated as a restriction. It can be considered by checking the total additional capacity of the exploratory solution at *Step 4b(2)* whether it exceeds the upper limit or not. This modification will narrow the search space, and the computation time can be reduced. However, the best solution and computation time to be compared with those of the proposed method are obtained without the restriction, we will not impose it in the following evaluation.

3. Evaluation of The Proposed Method

We examine the performance of the proposed method focusing on the time required to obtain the approximate optimal solution which has significance in a practical sense.

Since the OSBS/4 method which adopts the enumeration method shows excellent results on the scheduling data sets obtained from a large-scale system, we use the same data sets and compare the performance of the proposed method with that of the OSBS/4 method. The 48 data sets are extracted from the actual data in the currently working scheduling system obtained in the past 48 months. The characteristics of the actual data is given in Arakawa et al.⁽⁸⁾

The elapsed time required to obtain the approximate optimal solution by the proposed method is determined by the termination condition at *Step 6*, i.e.,

by the maximum number M of the base point random jump. The elapsed time T is directly governed by M , but also indirectly by the 'minimum' step size δ , because the number of the exploratory moves depend on δ at Step 5.

The values for M and δ should be chosen so that the practically significant solution can be derived by the proposed procedure. As an example for the practically significant solution, we cite the best solutions obtained by the OSBS/4 method which give the average values for the number of tardy jobs 8.7, and that for the total additional capacity 822.0 (hour). The value of δ is to be smaller than the values 0.1 and 0.2 adopted in the OSBS/4 method. We shall choose the value of $\delta=0.0125$.

The initial values for the initial step size Δ_0 and the initial position p_0 of the base point as well as the distribution of the stochastic function which gives a size of the random jump also affect the quality of the approximate optimal solution. The initial base point position is considered to be less important, because its effect may be compensated by the increase of M . On the other hand, the effects caused by the change of the initial step size Δ_0 and the distribution of the stochastic function are considered to be mutually related by themselves. We first investigate the dependence of the solution quality on the size of Δ_0 keeping the distribution constant. Figure 3 shows its dependence, where the results obtained by using the original pattern search procedure of Hooke and Jeeves⁽¹⁰⁾ is also included. We shall call the method which use the original pattern search procedure the OSBS/4/P [HJ] method. Both results shows similar trend as Δ_0 increases. As for the OSBS/4/P method, the number of tardy jobs decreases by about 2 jobs as Δ_0 increases from 0.2 to 0.8, and beyond $\Delta_0=0.8$ it slightly increases. In the following, we set the initial step size Δ_0 to 0.8.

Fig. 3 Dependence of the solution quality on the size of the initial step size

Next, let us investigate how the elapsed time T depends on M . Figure 4 shows the elapsed time measured on the PC (CPU : Pentium III, 1 GHz) when the M -th base point random jump occurred. The increments of the elapsed time decrease as M increases. This is due to the device to save the computation time stated in the previous section. The number of tardy jobs and the total additional capacity shown on the same figure indicate that after the first base point jump the significant solution can be almost attained and a choice $M=3$ can be sufficient in practice. The elapsed time at $M=3$ is about 10 minutes which is by far smaller than 13 hours of the time required to obtain the best solution in the OSBS/4 method, and the proposed method can be used for an actual problem.

4. Conclusion

We have proposed in a previous paper an optimization-oriented simulation-based scheduling method incorporating capacity adjustment function. The proposed method showed an excellent performance with respect to the elimination of tardy jobs in a production schedule, but it took too much computation time to obtain a best schedule for a practical large-scale problem. In order to shorten the computation time, we proposed in this paper a modified pattern search method and incorporated it in the scheduling procedure instead of the enumeration method.

The computation time required in the proposed method dramatically reduced to the level applicable to practical use while keeping the number of tardy jobs as well as the total additional capacity as the same or even slightly better than those of the previously proposed OSBS/4 method. Moreover, the extension of the parameter domain yields an approximate optimal solution which is better than the best solution obtained by the OSBS/4 method.

Fig. 4 Dependence on the number of the base point random jump

A comparison of performances of the proposed search method with other search methods such as the simulated annealing method and the genetic algorithm method is one of the subjects to be further studied.

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