Motion and Vibration Control of Flexible-Link Mechanism With Smart Structure*

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This study is concerned with the motion and vibration control strategy for a flexible-link mechanism composed of smart structures in order to achieve the high performance and stability. Reducing vibration and making positioning time faster are simultaneously required in this system. Technology of smart structures is introduced in the flexible-link system to achieve the vibration reduction. The smart flexible-link is composed of the flexible-link and the piezoelectric film which has the sensor/actuator functions for itself, and so its mechanism is extremely suitable for controlling the vibration. Modeling and control strategy are presented to enhance the motion and vibration control performance.

Key Words: Vibration Control, Motion Control, Flexible Structure, Smart Structure, Piezoelectric Sensor and Actuator

1. Introduction

In various multibody systems including flexible structural elements, it is required that the enhanced control performance for positioning and vibration reduction should be achieved. The miniaturization and lightening are simultaneously promoted in the recent systems. The modeling approach of multibody systems appropriate for the control system design and the control strategy have to be established in order to achieve these requirements. It is expected that the control performance can be enhanced if the smart mechanism is introduced in the system as well as the servomotor.

Recently, modeling of multibody system including flexible structure is actively studied. Ghiringhelli et al. analyzed a large deformation problem of multibody system composed of multiple beam structure using the finite volume concept⁽¹⁾. Arteaga presented a modeling approach of a flexible robot-manipulator by the Lagrange's equation of motions and the assumed modes method⁽²⁾. Edelstein and Rosen described the system as chains of flexible beam

elements⁽³⁾. Rai and Asada studied an integrated design methodology of structure and control system to achieve a faster positioning⁽⁴⁾. It has been expected that technology of smart structure contributes to the development of small and light-weight mechanism with the required performance. So far, research on smart structures has been carried out with electro-rheological fluids, shape memory alloys and piezoelectric materials. The piezoelectric materials are generally installed on the structural surface as actuator and sensor to control the static and dynamic responses⁽⁵⁾. To date, distributed parameter systems such as cantilever type structures have been mainly taken up as the controlled object. Bailey and Hubbard proposed simple control algorithm for transient vibration control by constant amplitude actuator and constant gain controller⁽⁶⁾. Crawly and de Luis analyzed the stiffness effect of piezoelectric actuators to the elastic property of host structures⁽⁷⁾. Tzou investigated the piezoelectric effect on the vibration control through a modal shape analysis⁽⁸⁾. Choi proposed a robust force tracking control of a flexible gripper driven by a piezoceramic actuator⁽⁹⁾ and Ge et al. investigated a dynamic modeling of a smart material robot⁽¹⁰⁾. A lot of researchers focused only on the dynamic analysis of the multibody systems but the modeling strategy appropriate for the control system design has not been studied in the past. Moreover, the smart structure was discussed with simple cantilever-like structure in almost research works but has not been introduced in the multibody system.

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In this study, modeling and control strategies are proposed for a smart flexible-link mechanism in order to achieve the positioning and vibration suppression requirements. The sensor and actuator as a base of the smart structure are composed of a thin piezoelectric film in this study⁽¹¹⁾. The piezoelectric film is a very frivolous high polymer material which is suitable to be installed on the structural surface. An accelerometer is also used as a sensor which does not require an external reference such like the piezoelectric film sensor. First, a modeling method of the flexible multibody systems is presented based on modal analysis taking the characteristics of the piezoelectric film into account. In this method, each flexible body modeled by the finite element method (FEM) is transformed into the modal space at the 1st stage of the model reduction and then the total system is constructed by synthesizing each modal model and reducing it into the modal space at the 2nd stage reduction. This model is described as a state space form and appropriate for the control system design. The control system is constructed with the designed dynamic compensator based on the mixed H_2/H_∞ control problem. The motion and vibration controllers are designed by the separated control problems and actuated by the DC servomotors and piezoelectric film actuators, respectively. The enhanced performance with respect to the motion and vibration control is achieved by the proposed smart flexible-link mechanism.

2. Controlled Object and Modeling

The controlled object with two flexible-links is shown in Fig. 1. In the modeling process, each flexible body modeled by FEM is transformed into the modal space at the 1st stage of the model reduction and then the total system is constructed by synthesizing each modal model with the servomotor and piezoelectric film characteristics and reducing it into the modal space at the 2nd stage reduction. The system can be described as a state space form which is appropriate for the control system design.

Fig. 1 Model of flexible-link mechanism

2.1 Modeling of structure

Consider the modeling of the structure 1 in Fig. 1. The spatial matrices are calculated by FEM with $\theta_1=0$ and the model with arbitral angle of θ_1 can be easily obtained by the rotational coordinate transformation. The equation of motion of the structure 1 is

$$\bar{\mathbf{M}}_1 \ddot{\mathbf{x}}_1 + \bar{\mathbf{K}}_1 \mathbf{x}_1 = \mathbf{B}_{s10} \boldsymbol{\eta}_0 + \mathbf{B}_{s11} \boldsymbol{\eta}_1 + \mathbf{B}_{s12} \mathbf{f}_1 + \mathbf{B}_{s13} \mathbf{w}_1 + \mathbf{B}_{s14} \mathbf{u}_{p1} \quad (1)$$

where

$$\bar{\mathbf{M}}_1 = \mathbf{T}_1^T \mathbf{M}_1 \mathbf{T}_1, \quad \bar{\mathbf{K}}_1 = \mathbf{T}_1^T \mathbf{K}_1 \mathbf{T}_1$$

$\mathbf{M}_1$ and $\mathbf{K}_1$ are the mass and stiffness matrices obtained by FEM with $\theta_1=0$ and $\mathbf{T}_1$ ($=\mathbf{T}_1(\sin(\theta_1), \cos(\theta_1))$) is the rotational coordinate transformation matrix with respect to θ_1 . $\mathbf{B}_{s10}$ and $\mathbf{B}_{s11}$ are the connection matrices at the points 0 and 1, respectively. $\mathbf{B}_{s12}$, $\mathbf{B}_{s13}$ and $\mathbf{B}_{s14}$ are the input matrices with respect to the servomotor, disturbance and control input, respectively. $\mathbf{x}_1$, $\boldsymbol{\eta}_0$, $\boldsymbol{\eta}_1$, $\mathbf{f}_1$, $\mathbf{w}_1$ and $\mathbf{u}_{p1}$ are the vectors of the displacement, states at the connecting points 0 and 1, torque by the servomotor, disturbance and control input by the piezoelectric film actuator, respectively. The degrees of freedom of Eq.(1) is usually so large that the model is not appropriate for constructing the total system and designing the control system. The modal coordinate transformation of Eq.(1) by $\mathbf{x}_1 = \mathbf{T}_1^T \boldsymbol{\Phi}_1 \boldsymbol{\xi}_1$ yields

$$\mathbf{I}_1 \ddot{\boldsymbol{\xi}}_1 + \mathbf{A}_1 \dot{\boldsymbol{\xi}}_1 = \boldsymbol{\Phi}_1^T \mathbf{T}_1 \mathbf{B}_{s10} \boldsymbol{\eta}_0 + \boldsymbol{\Phi}_1^T \mathbf{T}_1 \mathbf{B}_{s11} \boldsymbol{\eta}_1 + \boldsymbol{\Phi}_1^T \mathbf{T}_1 \mathbf{B}_{s12} \mathbf{f}_1 + \boldsymbol{\Phi}_1^T \mathbf{T}_1 \mathbf{B}_{s13} \mathbf{w}_1 + \boldsymbol{\Phi}_1^T \mathbf{T}_1 \mathbf{B}_{s14} \mathbf{u}_{p1} \quad (2)$$

where $\mathbf{I}_1$ is the identity matrix, $\mathbf{A}_1$ is the matrix composed of the eigenvalues at the diagonal elements, $\boldsymbol{\xi}_1$ is the modal displacement vector and $\boldsymbol{\Phi}_1$ is the modal matrix composed of the eigenvectors of the structure 1. The modal model of the structure 2 is obtained by the same manner of the structure 1 as

$$\mathbf{I}_2 \ddot{\boldsymbol{\xi}}_2 + \mathbf{A}_2 \dot{\boldsymbol{\xi}}_2 = \boldsymbol{\Phi}_2^T \mathbf{T}_2 \mathbf{B}_{s21} \boldsymbol{\eta}_2 + \boldsymbol{\Phi}_2^T \mathbf{T}_2 \mathbf{B}_{s22} \mathbf{f}_2 + \boldsymbol{\Phi}_2^T \mathbf{T}_2 \mathbf{B}_{s23} \mathbf{w}_2 + \boldsymbol{\Phi}_2^T \mathbf{T}_2 \mathbf{B}_{s24} \mathbf{u}_{p2} \quad (3)$$

where $\mathbf{A}_2$, $\boldsymbol{\xi}_2$ and $\boldsymbol{\Phi}_2$ are the eigenvalue matrix, the modal displacement vector and the modal matrix of the structure 2, respectively.

Next, modeling of the total system is carried out by synthesizing the above two structures. It is assumed that each substructure is connected by rigid spring except z -axis coordinate element at the connecting points, namely the points 0 and 1. From this assumption, the connection vectors are described as

$$\begin{cases} \boldsymbol{\eta}_0 = -\mathbf{F}_{kf} \mathbf{y}_{0F} = \mathbf{C}_{0F} \mathbf{x}_1 \\ \boldsymbol{\eta}_2 = -\mathbf{F}_k (\mathbf{y}_{2F} - \mathbf{y}_{1F}) = -\mathbf{F}_k (\mathbf{C}_{2F} \mathbf{x}_2 - \mathbf{C}_{1F} \mathbf{x}_1) = -\boldsymbol{\eta}_1 \end{cases} \quad (4)$$

where $\mathbf{F}_{kf}$ and $\mathbf{F}_k$ are the stiffness matrices which have large elements at the connecting points. $\mathbf{C}_{0F}$, $\mathbf{C}_{1F}$ and $\mathbf{C}_{2F}$ are the output matrices at the points 0 and 1 of the structure 1 and the point 1 of the structure 2, respectively. $\mathbf{y}_{0F}$, $\mathbf{y}_{1F}$ and $\mathbf{y}_{2F}$ represent the displacement corresponding to each output matrix. According

to Eq.(4), the equation of motion of the two-link system is described with the modal displacement vector $\xi = (\xi_1^T, \xi_2^T)^T$ as

$$I_t \ddot{\xi} + K_t \xi = B_t f + B_w w + B_u u_p \quad (5)$$

where

$$K_t = \begin{bmatrix} \Lambda_1 + \Phi_1^T T_1 B_{s10} F_{kf} C_{0F} T_1^T \Phi_1 + \Phi_1^T T_1 B_{s11} F_k C_{1F} T_1^T \Phi_1 \\ - \Phi_2^T T_2 B_{s21} F_k C_{1F} T_1^T \Phi_1 \\ - \Phi_1^T T_1 B_{s11} F_k C_{2F} T_2^T \Phi_2 \\ \Lambda_2 + \Phi_2^T T_2 B_{s21} F_k C_{2F} T_2^T \Phi_2 \end{bmatrix}$$

$$B_t = \begin{bmatrix} \Phi_1^T T_1 B_{s12} & 0 \\ 0 & \Phi_2^T T_2 B_{s22} \end{bmatrix},$$

$$B_w = \begin{bmatrix} \Phi_1^T T_1 B_{s13} & 0 \\ 0 & \Phi_2^T T_2 B_{s23} \end{bmatrix},$$

$$B_u = \begin{bmatrix} \Phi_1^T T_1 B_{s14} & 0 \\ 0 & \Phi_2^T T_2 B_{s24} \end{bmatrix} \quad (6)$$

f , w and u_p are described by $f = (f_1^T, f_2^T)^T$, $w = (w_1^T, w_2^T)^T$ and $u_p = (u_{p1}^T, u_{p2}^T)^T$. Furthermore, transforming Eq.(5) into the modal space with $\xi = \Phi_t v$ yields the reduced order model:

$$I_r \ddot{v} + \Lambda_t v = \Phi_t^T B_t f + \Phi_t^T B_w w + \Phi_t^T B_u u_p \quad (7)$$

where Λ_t and Φ_t are the eigenvalue and eigenmode matrices calculated by Eq.(5), respectively. In this study, DC servomotors are used as actuator for the motion control. The relation between the torque τ applied by the servomotor, the input voltage u_M and the angle velocity $\dot{\theta}$ is expressed as $\tau = R_a^{-1} K_t (u_M - K_e \dot{\theta})$. In this relation, R_a , K_t and K_e are the armature resistance, torque constant and induced voltage constant, respectively. The input f by the servomotor can be described as the form:

$$f = B_M u_M - F_M \dot{v} \quad (8)$$

The total system including the motor characteristics is written by

$$I_r \ddot{v} + \Phi_t^T B_t F_M \dot{v} + \Lambda_t v = \Phi_t^T B_w w + \Phi_t^T B_t B_M u_M + \Phi_t^T B_u u_p \quad (9)$$

Describing the state as $q_g = (v^T, \dot{v}^T)^T$ and the control input as $u = (u_M^T, u_p^T)^T$, the reduced order state equation is obtained by

$$\dot{q}_g = A_g(\bar{\theta}) q_g + B_{1g}(\bar{\theta}) w + B_{2g}(\bar{\theta}) u$$

$$y = C_g(\bar{\theta}) q_g \quad (10)$$

where

$$A_g(\bar{\theta}) = \begin{bmatrix} 0 & I \\ -\Lambda_t & -\Phi_t^T B_t F_M \end{bmatrix}$$

$$B_{1g}(\bar{\theta}) = \begin{bmatrix} 0 \\ \Phi_t^T B_w \end{bmatrix}, \quad B_{2g}(\bar{\theta}) = \begin{bmatrix} 0 & 0 \\ \Phi_t^T B_t B_M & \Phi_t^T B_u \end{bmatrix} \quad (11)$$

In Eqs.(10) and (11), $\bar{\theta}$ represents the angles at the shoulder and elbow according to the configuration for the modeling of the mechanism. y is the observed output for the feedback.

2.2 Modeling of piezoelectric film sensor/actuator

It is difficult for the servomotor used in this study

to control the minute vibration due to the friction and non-linear characteristics. Piezoelectric films made of PVDF are employed as actuator and sensor in the smart structure to control the minute vibration. Figure 2 shows the geometry of the flexible structure and the piezoelectric film. The thickness of the piezoelectric films is 40 μm and this film is bonded to the both sides of the beam surface by the very thin adhesion layer. Each side film is used as a sensor and an actuator, and the films of both sides can be used as actuator. This film does not change the structural characteristics and dynamics due to the very thin thickness and is appropriate to be installed in the smart structure with the vibration control function. According to the formulation by Crawley and de Luis⁽⁷⁾, the moment caused by the piezoelectric film under the assumption of the perfect adhesion is described as

$$M = -E_B t_B b \frac{1}{\psi + \alpha} \left\{ \left[\frac{\epsilon_B^{S+} + \epsilon_B^{S-}}{2} + \frac{\epsilon_B^{S+} - \epsilon_B^{S-}}{2} \bar{x} \right] + \frac{\alpha}{\psi} \Lambda \right\}$$

$$\psi = \frac{E_B t_B}{E_c t_c}, \quad \Lambda = \frac{d_{31} V_a}{t_c} \quad (12)$$

where V_a is the input voltage to the piezoelectric film actuator. E_B , t_B , and b are Young's modulus, the thickness and width of the beam, and E_c , t_c , and d_{31} are Young's modulus, the thickness and piezoelectric constant of the piezoelectric film, respectively. ϵ_B^{S+} , ϵ_B^{S-} are the strains at the boundaries of the piezoelectric film and $\bar{x}$ is the non-dimensional coordinate. The substructure equilibrium parameter α is 3 in case of one side actuation and 6 in case of both sides actuation for the bending deformation. On the other hand, the output voltage from the piezoelectric film sensor is described as

$$V_s = \frac{1}{2} \frac{t_c t_B}{L} \frac{d_{31} E_c}{\epsilon} \left(\frac{\partial u}{\partial x} \Big|_{x_1} - \frac{\partial u}{\partial x} \Big|_{x_2} \right) \quad (13)$$

where u is displacement with respect to y -axis and L is the length of the piezoelectric film sensor. Above formulations are introduced in the system model in Eq.(10). The relation of

Fig. 2 Geometry of flexible structure and piezoelectric

$$\varepsilon_B = \varepsilon_C = -\frac{t_B}{2} \frac{\partial^2 u}{\partial x^2} \quad (14)$$

is used for the derivation of Eq.(13) in case of the perfect adhesion. ε_c is the strain of the piezoelectric film.

3. Control System Design

The control system is designed based on the mixed H_2/H_∞ control problem considering the motion/vibration control performance and the control input limitation. Figure 3 shows the block diagram of the control system. In Fig. 3, $P(s)$ is the controlled object, $K(s)$ is the controller to be designed, z_2 is the controlled variable on the motion and vibration, z_∞ is the controlled variable on the control input limitation, W_1 and W_2 are the frequency weight functions with respect to z_2 and z_∞ , respectively, r is the command input for positioning, e is the error between r and y , and u is the control input. Figure 4 shows the actual control system adopted in this study in which the motion controller K_M and the vibration controller K_P are designed by the separated control problems and actuated by the DC servomotors and the piezoelectric film actuators, respectively.

3.1 Motion controller design

Frequency weight functions W_i ($i=1, 2$) are given by

$$W_i(s) = \begin{bmatrix} A_{wi} & B_{wi} \\ C_{wi} & D_{wi} \end{bmatrix} \quad (15)$$

Synthesizing Eqs.(10) and (15) yields the augmented state equation:

$$\begin{aligned} \dot{q} &= A(\bar{\theta})q + B_1 r + B_2(\bar{\theta})u \\ z_\infty &= C_{\infty} q + D_{\infty 1} r + D_{\infty 2} u \\ z_2 &= C_2(\bar{\theta})q + D_{21} r + D_{22} u \\ y &= C_y(\bar{\theta})q + D_{y1} r \end{aligned} \quad (16)$$

where

Fig. 3 Block diagram of control system

Fig. 4 Control system composed of motion and vibration controllers

$$\begin{aligned} A(\bar{\theta}) &= \begin{bmatrix} A_g(\bar{\theta}) & 0 & 0 \\ -B_{w1}C_g(\bar{\theta}) & A_{w1} & 0 \\ 0 & 0 & A_{w2} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ B_{w1} \\ 0 \end{bmatrix} \\ B_2(\bar{\theta}) &= \begin{bmatrix} B_{2g}(\bar{\theta}) \\ 0 \\ B_{w2} \end{bmatrix}, \quad C_\infty = [0 \quad 0 \quad C_{w2}] \\ C_2(\bar{\theta}) &= [-D_{w1}C_g(\bar{\theta}) \quad C_{w1} \quad 0], \\ C_y(\bar{\theta}) &= [C_g(\bar{\theta}) \quad 0 \quad 0] \\ D_{\infty 1} &= 0, \quad D_{\infty 2} = D_{w2}, \quad D_{21} = D_{w1}, \quad D_{22} = 0, \quad D_{y1} = 0 \end{aligned} \quad (17)$$

The control problem is that the H_2 norm $\|T_{z2r}(s)\|_2^2$ of the transfer function matrix $T_{z2r}(s)$ between r and z_2 is minimized under the constraint in which the H_∞ norm of the transfer function matrix $T_{z_\infty r}(s)$ with respect to z_∞ has to be smaller than a given γ , $\|T_{z_\infty r}(s)\|_\infty < \gamma$. The controller $K_M(s)$ at the configuration $\bar{\theta}$ is designed by LMI approach, resulting in the dynamic compensator as the state space form:

$$\begin{aligned} \dot{q}_K &= A_K q_K + B_K(r - y) \\ u_M &= C_K q_K + D_K(r - y) \end{aligned} \quad (18)$$

Synthesizing Eqs.(16) and (18), and describing the state vector as $x_{cl} = (q^T q_K^T)^T$ yields the closed-loop state equation as

$$\begin{aligned} \dot{x}_{cl} &= A_{cl} x_{cl} + B_{cl} r \\ z_\infty &= C_{cl1} x_{cl} + D_{cl1} r \\ z_2 &= C_{cl2} x_{cl} + D_{cl2} r \end{aligned} \quad (19)$$

where

$$\begin{aligned} A_{cl} &= \begin{bmatrix} A - B_2 D_K C_y & B_2 C_K \\ -B_K C_y & A_K \end{bmatrix} \\ B_{cl} &= \begin{bmatrix} B_1 + B_2 D_K (I - D_{y1}) \\ B_K (I - D_{y1}) \end{bmatrix} \\ C_{cl1} &= [C_\infty - D_{\infty 2} D_K C_y \quad D_{\infty 2} C_K] \\ D_{cl1} &= [D_{\infty 1} + D_{\infty 2} D_K (I - D_{y1})] \\ C_{cl2} &= [C_2 - D_{22} D_K C_y \quad D_{22} C_K] \\ D_{cl2} &= [D_{21} + D_{22} D_K (I - D_{y1})] \end{aligned} \quad (20)$$

3.2 Vibration controller design

The direct velocity feedback (DVFB) and the dynamic compensator obtained by the mixed H_2/H_∞ control problem are considered as the feedback law for the vibration control. The DVFB controller is adopted when each side film is used as a sensor and an actuator. The collocation is established in this case, and so the damping effect can be easily increased by the control for each mode. On the other hand, when the films of both sides are used as the actuator, the dynamic compensator is designed by the mixed H_2/H_∞ control problem with the accelerometer output. In this case, the control design process is a bit complicated, but a larger control input can be applied to the system compared with the one side actuation.

The observed output for the DVFB control is described as

$$y_{1P} = C_{1Pt} \dot{v} \quad (21)$$

where C_{1Pt} is the matrix for extracting the outputs. The control input u_P for the vibration control is

$$u_P = -K_P y_{1P} \quad (22)$$

where K_P is the constant feedback gain matrix. The detected y_{1P} of the actual system is the derivative value of the output voltage from the piezoelectric film sensor. On the other hand, the observed output for the dynamic compensator with the accelerometer is

$$y_{2P} = C_{y2Pt} \ddot{v} \quad (23)$$

where C_{y2Pt} is the matrix for extracting the detected acceleration. The controlled variable z_{2P} with respect to the vibration response and $z_{\infty P}$ with respect to the control input are defined by

$$z_{2P} = W_{1P} C_{z2Pt} v \quad (24)$$

$$z_{\infty P} = W_{2P} u_P \quad (25)$$

where C_{z2Pt} is the matrix for extracting the controlled response, W_{1P} and W_{2P} are the frequency weight functions with respect to the controlled response and control input, respectively. In this study, both W_{1P} and W_{2P} are defined as the constant matrix in order to prevent the increase of the controller order, and described as

$$W_{1P} = \text{diag}[w_{11P}, w_{12P}, \dots, w_{1n1P}] \quad (26)$$

$$W_{2P} = \text{diag}[w_{21P}, w_{22P}, \dots, w_{2n2P}]$$

The feedback law with the dynamic compensator $K_P(s)$ obtained by LMI approach is

$$u_P = -K_P(s) y_{2P} \quad (27)$$

4. Simulation and Experiment

The smart mechanism with two flexible-links and the parameters of the experimental set-up are shown in Fig. 5 and Table 1, respectively. The dielectric constant of the piezoelectric film is 13. The rated output, mass, inertia and reduction ratio of the servomotor are 1.7 (W), 83 (g), 16 ($\text{kg} \cdot \text{m}^2$) and 1:80, respectively. Both flexible-links are made of aluminum and their dimensions are 2 mm thickness, 35 mm width and 160 mm length for each link. Each link is divided into 10 elements with 11 nodal points for FEM calculation. The system is modeled by including the first two natural modes on the bending whose frequencies are 7.75 Hz and 32.3 Hz. The sampling frequency of the control system is 1 kHz. The piezoelectric film is installed only to the link 1, since the elastic oscillation is especially remarkable on the link. The piezoelectric films are bonded on both surface of the link 1 and an accelerometer is also installed at the tip of the same link. A gap sensor is placed at a target position to detect the vibration of the link 1 after the positioning. Two cases of the piezoelectric film functions are considered here, namely each side film is used as the sensor and the actuator in one case and the films of both sides are used as the actuator in another

Table 1 Parameters of experimental set-up

k_t	Nm/A	1.11
k_c	V/(rad/s)	1.15
r_a	Ω	8.6
E_B	GPa	5.1×10
t_B	m	2.0×10^{-3}
b	m	3.5×10^{-2}
d_{31}	m/V	1.24×10^{-9}
t_C	m	4.0×10^{-5}
E_C	GPa	3.1

Fig. 5 Composition of experimental set-up

case.

The following approximation is applied to Eq.(14) for describing the system.

$$\frac{\partial^2 u}{\partial x^2} \Big|_x = \frac{1}{\Delta x} \left(\frac{\partial u}{\partial x} \Big|_{x+\Delta x} - \frac{\partial u}{\partial x} \Big|_x \right) \quad (28)$$

The output y_{kP} from the piezoelectric film at a nodal point k of the structure 1 is described as

$$y_{kP} = C_{kP} T_1^T \Phi_1 \xi_1 = C_{kPt} v \quad (29)$$

where C_{kP} is the output matrix on the piezoelectric film at the nodal point k . From Eqs.(28) and (29), the strains, ϵ_B^{S+} and ϵ_B^{S-} , at the both ends of the piezoelectric film, $\bar{x} = \pm 1$, are described as

$$\epsilon_B^{S+} = -\frac{t_B}{2l} (y_{11P} - y_{10P}) = -\frac{t_B}{2l} (C_{11Pt} - C_{10Pt}) v \quad (30)$$

$$\epsilon_B^{S-} = -\frac{t_B}{2l} (y_{2P} - y_{1P}) = -\frac{t_B}{2l} (C_{2Pt} - C_{1Pt}) v \quad (31)$$

where l is the distance between the both ends of the piezoelectric film. The suffix of y and C shows the nodal point number of the both ends. In this case, the control input u_1 composed of the input moments, M^+ and M^- , applied to the structure 1 by the piezoelectric actuator at $\bar{x} = \pm 1$ becomes

$$u_1 = \begin{Bmatrix} M^- \\ M^+ \end{Bmatrix}$$

$$\begin{aligned}
&= E_B t_B^2 b \frac{\alpha d_{31}}{\Psi t_c (\Psi + \alpha)} \begin{bmatrix} 1 \\ -1 \end{bmatrix} u_P \\
&\quad - \frac{E_B t_B^3 b}{2l(\Psi + \alpha)} \begin{bmatrix} C_{2Pt} - C_{1Pt} \\ -(C_{11Pt} - C_{10Pt}) \end{bmatrix} v \\
&= B_P u_P - F_P v
\end{aligned} \quad (32)$$

Accordingly, the system matrices of Eq.(11) can be described as

$$\begin{aligned}
A_g(\bar{\theta}) &= \begin{bmatrix} 0 & I - \Phi_t^T B_u F_P \\ -\Lambda_t & -\Phi_t B_t F_M \end{bmatrix} \\
B_{1g}(\bar{\theta}) &= \begin{bmatrix} 0 \\ \Phi_t^T B_w \end{bmatrix}, \quad B_{2g}(\bar{\theta}) = \begin{bmatrix} 0 & 0 \\ \Phi_t^T B_t B_M & \Phi_t^T B_u B_P \end{bmatrix}
\end{aligned} \quad (33)$$

4.1 Effect of motion control

The motion control system is designed by the control strategy described in the section 3.1 without the piezoelectric film input. The controlled response is the error between the command and the output angle at each joint. The control input is the voltage fed to each joint servomotor. The frequency weight function $W_1(s)$ has an integral property so as to diminish the error in low frequency region. On the other hand, $W_2(s)$ has a high gain in high frequency region to satisfy the robust stability against the structural uncertainty with respect to the high order modes. The controller is designed by LMI approach based on the mixed H_2/H_∞ control problem and the obtained results are shown in Fig.6 with the time responses of the closed-loop system. In Fig.6, the command angle is 30 degrees for each joint and the positioning time is 0.6 second. The angle velocity command is given by the constant acceleration for 1st 0.2 seconds, the constant velocity for next 0.2 seconds and the constant deceleration for final 0.2 seconds. Figure 6(a) and (b) show the angles at the shoulder and elbow joints, respectively, and it is observed that the excellent positioning performance is achieved by the control with the servomotors. The vibration response at a part of the link 1 detected by the gap sensor is shown in Fig.6(c) in which the response for beginning 0.6 seconds is in motion and excluded from the evaluation. It is confirmed that the vibration control ability is not enough in this system due to the friction and the non-linear properties of the servomotor. It is also observed from Fig.6 that the responses between the simulation and the experiment agree well, and so the effectiveness and validity of the proposed modeling approach have been verified.

4.2 Control with smart link mechanism

The motion and vibration are controlled by the servomotor and piezoelectric film actuator. As mentioned in the section 3.2, consider the two cases of the control strategy concerning the piezoelectric film utilization; the DVFB control with the piezoelectric

(a) Angle of shoulder

(b) Angle of elbow

(c) Gap sensor output

Fig. 6 Responses by motion control

film sensor/actuator on each side of the link and the dynamic compensator based on the mixed H_2/H_∞ control problem with the piezoelectric film actuators on both sides of the link and the accelerometer. The feedback gain for the DVFB control is given as $K_P = 180$. The weight parameters for the dynamic compensator, W_{1P} and W_{2P} , are set as 1.0×10^{-4} and 1.0×10^{-6} , respectively. The observed vibration responses of the closed-loop system for the same command input with the previous case of the motion control are shown in Fig.7.

Figure 7(a), (b) and (c) show the responses without vibration control, with control by the DVFB control and with control by the dynamic compensator, respectively. The response of the beginning 0.6 seconds is excluded from the evaluation. The convergence time of the vibration in Fig.7(b) and (c) become 60% and 40% compared with the response without vibration control, respectively. The result of

(a) without control

(b) with DVFB control (one side actuator)

(c) with mixed H_2/H_∞ control (both sides actuator)

Fig. 7 Gap sensor outputs with smart link mechanism

the motion control concerning each joint angle is almost same with Fig. 6(a) and (b). It is verified from these results that the proposed smart link mechanism demonstrates a remarkable effect on both motion and vibration control, and both sides actuation achieves an enhanced vibration suppression as against one side actuation.

5. Conclusions

A modeling approach of multibody system with flexible structural elements is proposed based on FEM and modal analysis. The validity and effectiveness of this modeling method which derives the reduced order model as a state space form appropriate for the control system design are confirmed by the simulation and experiment. The smart flexible-link mechanism is also proposed with the control system design strategy in which the motion and vibration controllers are designed by the separated control problems. Concerning the piezoelectric film utilization, the characteristics by one side actuation with DVFB control and both

sides actuation with output feedback control are studied and compared with each other. It is verified from this study that the proposed smart link mechanism demonstrates a remarkable effect on both motion and vibration control, and both sides actuation with the mixed H_2/H_∞ control achieves an enhanced vibration suppression over one side actuation with the DVFB control.

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