Uncertainty in Cross Orthogonality Checks*

Yuichi MATSUMURA**

Finite element structural dynamic models are typically validated using data obtained from a modal testing. Although it is known that test variability may affect the verification, the test variability is typically ignored in the verification process. This paper describes the sensitivities of cross orthogonality check to the test variability. The cross orthogonality check is one of correlation techniques which is gaining acceptance in the structural dynamics community, because of its improved accuracy over the standard modal assurance criterion. Then an uncertainty index of the cross orthogonality check is proposed based on the fact that the sensitivities are much dependent on sensor placement. The results of some artificially generated test cases are presented to demonstrate the applicability of the proposed approach.

Key Words: Experimental Modal Analysis, Finite Element Method, Modeling, Cross Orthogonality Check, Uncertainty, Oblique Projection, Sensor Placement

1. Introduction

Orthogonality between analysis and test modes has been an important criterion for the determination of modeling accuracy. However some problems of orthogonality checks is known3-9): inexact reduction of analytical mass matrix, non orthogonality arising from repeated roots and large off-diagonal elements caused by small differences, and so on. This paper discusses the robustness of test-analysis model (TAM), which refers to the ability of TAM to provide dependable cross orthogonality checks by using mode shapes corrupted by noise.

Our recent research3-6 has shown that the computation of cross orthogonality check is a sort of modal decomposition techniques. Each term of cross orthogonality matrix is a modal weighting coefficient in the modal decomposition of Experimental Modal Analysis (EMA) modal vectors onto the basis spanned by Finite Element Analysis (FEA) modal vectors. It naturally follows that cross orthogonality matrix can also be computed with the general definition of projectors (oblique projectors). This technique is termed cross orthogonality checks via general definition of projectors (XORviaGDOP), and can be computed without the need for reduction of FEA mass matrix with no degradation in obtained results. The XORviaGDOP has clear physical meanings from the viewpoint of the projection, and is used for a system with non-proportional damping.

The formulation of XORviaGDOP shows a possibility that the small errors in measured mode shapes dominate a cross orthogonality matrix. The amplification rate of the measured mode shape error in cross orthogonality computation is dominated by the projector which is utilized for calculating the modal weighting coefficients in modal decomposition. Because the projector is calculated by FEA modal vectors of TAM which usually includes a few percent of the DOF contained in FE-model, the robustness of the cross orthogonality checks much depends on sensor placement—selected active set of DOFs. To circumvent this difficulty, orthogonality must be assessed based on the confidence region which is calculated from probability distribution of measurement error. Then this paper shows that an index derived from the projector can be employed as an uncertainty index of the cross orthogonality checks, in order to reduce the influence of measurement error and to select optimal sensor placement, by using the statistical method.

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Nomenclature

\[ [{\cdot}^T] \text{: transpose of a matrix} \]
\[ [{\cdot}^{-1}] \text{: inverse of a matrix} \]
\[ [{\cdot}^{-1}] \text{*: generalized inverse of a matrix} \]
\[ \lvert{\cdot}\rvert \text{: absolute value} \]
\[ I_k \text{: } k\text{-th order identity matrix} \]
\[ K_k \text{: FEA stiffness matrix} \]
\[ M_k \text{: FEA mass matrix} \]
\[ N \text{: the number of DOF} \]
\[ \text{POC} \text{: pseudo orthogonality check matrix} \]
\[ T \text{: transformation matrix for reduction method} \]
\[ \text{XOR} \text{: cross orthogonality check matrix} \]
\[ \text{XORviaGDOP} \text{: cross orthogonality check via GDOP matrix} \]
\[ a \text{: subscript denoting the active set of DOFs} \]
\[ d \text{: subscript denoting the deleted set of DOFs} \]
\[ x \text{: physical displacement vector} \]
\[ \phi_k \text{: } k\text{-th analytical modal vector} \]
\[ \phi_k \text{: } k\text{-th experimental modal vector} \]
\[ \phi_k \text{: modal vectors except for } k\text{-th modal vector} \]

2. Background Theory

2.1 Cross orthogonality checks

One simple way for assessing correlation between FEA modal vectors and EMA modal vectors has been termed a “Cross Orthogonality check (XOR)”:

\[ \text{XOR} = \phi_i M_k \phi_x, \]

where \( \phi_i \) and \( \phi_x \) are scaled to unity modal mass. Each element of this matrix represents the degree of correlation. This based on the well-known mass orthogonality relation:

\[ \phi_i \phi_k M = I. \]

When the EMA modal vectors are orthogonal with respect to the FEA mass matrix, diagonal terms must all be equal to one while off-diagonal terms remain at zero:

\[ \text{XOR} = I. \]

2.2 Reduction of mode shapes

Main problem in the computation of XOR was the incompatibility of DOF between analytical and experimental models. Therefore reduction or expansion procedures are employed to simplify the incompatibility. Because of the general representation of error propagation in orthogonality checks, this paper only deals with reduction based orthogonality checks.

With all reduction techniques, there is a relationship between the active DOF and deleted DOF which can be written in a general form as:

\[ x = \begin{bmatrix} x_a \\ x_d \end{bmatrix} = T x_a, \]

where \( T \) is a transformation matrix which can be computed with reduction techniques such as Guyan reduction\(^9\). As an example of the transformation matrix, the Guyan transformation matrix can be written as:

\[ T = \begin{bmatrix} I \\ -K_{ad} K_{da} \end{bmatrix}. \]

This relation can be derived from the assumption that the forces or loading on the deleted DOFs are zero:

\[ \begin{bmatrix} K_{aa} & K_{ad} \\ K_{da} & K_{dd} \end{bmatrix} \begin{bmatrix} x_a \\ x_d \end{bmatrix} = \begin{bmatrix} F_a \\ 0 \end{bmatrix}. \]

The reduced analytical mass matrix can then be written as:

\[ M_{aa} = T \text{M}_a T^T. \]

Then this matrix can be used in orthogonality checks:

\[ \text{POC} = \phi_a^T M_{aa} \phi_x. \]

2.3 Cross orthogonality checks as modal decomposition

Based on the Pseudo Orthogonality Checks (POC) theory\(^9\), the XOR was proven to be obtained without the need for expansion of experimental modal vectors or reduction of analytical mass matrix, by using System Equivalent Reduction Expansion Process (SEREP)\(^8\) as an adjuster of the incompatibility\(^9\).

\[ T = \phi_i \phi_d \]

The XOR for full model can be obtained using an active set of DOFs, without using an analytical mass matrix.

\[ \text{POC} = \phi_i \phi_x \]

Other formulation of the cross orthogonality criteria which do not require expansion or reduction procedures was derived and termed XORviaGDOP\(^4\). The background theory of the XORviaGDOP implies that the cross orthogonality computation can be regarded as the modal decomposition of EMA modal vectors onto the basis spanned by FEA modal vectors. Brief summary of the XORviaGDOP will be given here, based on the modal decomposition theory, in order to derive the amplifying rate of measurement error in cross orthogonality computation.

Let us assume that a \( l\)-th EMA modal vector \( \phi_l \) has following relation

\[ \phi_l = \alpha_1 \phi_{a1} + \alpha_2 \phi_{a2} + \cdots + \alpha_l \phi_{al} + \epsilon, \]

where \( d \) is the number of modes which are active in the interested frequency range. To compute the modal weighting coefficients \( \alpha_i \) is equal to compute each term of the matrix XOR in Eq.(1). Therefore the cross orthogonality computation can be generalized as the modal decomposition of EMA modal vectors onto the basis spanned by FEA modal vectors, and consequently the physical meanings of the each term of the XOR can be generalized as follows:

1. if \( |\alpha_i| = 1 \), then the \( l\)-th EMA modal vector is exactly the same as the \( k\)-th FEA modal vector
2. if \( |\alpha_i| = 0 \), then the subspace spanned by the \( l\)-th
EMA modal vector and the subspace spanned by the $k$-th FEA modal vector are disjoint

3. If $0 < |a_k| < 1$, then the $l$-th EMA modal vector $\phi_{xl}$ is a linear combination of the FEA modal vectors

The XORviaGDOP is a straightforward formulation of cross orthogonality checks. Each term of the XORviaGDOP can be computed as modal weighting coefficients $a_k$ based on the General Definition of Projectors (GDOP)\textsuperscript{10}:

$$[a_k] = [\phi_{l,k}^T P_{l,k}^{\text{FFA}}] \phi_{xl}^T,$$

where $P_{l,k}^{\text{FFA}}$ which denotes the projector onto $S(\phi_{l,k})$ along $S(\Phi_{\text{FFA}})$ defined as follows\textsuperscript{10}:

$$P_{l,k}^{\text{FFA}} = [\phi_{l,k} \ldots \phi_{l,k,l}] [\phi_{l,k}^T \ldots \phi_{l,k,l}^T]^T,$$

where $\Phi_{\text{FFA}}$ is the matrix which contains every FEA modal vector except for the $k$-th:

$$\Phi_{\text{FFA}} = [\Phi_{l,k} \ldots \phi_{l,k,l}] = [\phi_{l,k} \ldots \phi_{l,k,l}] = [\phi_{l,k} \ldots \phi_{l,k,l}] = [\phi_{l,k} \ldots \phi_{l,k,l}] = [\phi_{l,k} \ldots \phi_{l,k,l}]$$

Therefore the XORviaGDOP matrix was defined as follows:

$$\text{XORviaGDOP} = \begin{bmatrix} a_1 & a_2 & \cdots & a_d \\ a_1 & a_2 & \cdots & a_d \\ \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & \cdots & a_d \end{bmatrix}$$

where each column of $\Phi_k$ and $\Phi_x$ are normalized with its vector norm. Equation (13) can also be abbreviated as follows.

$$[a_k] = [\phi_{l,k}^T P_{l,k}^{\text{FFA}} \phi_{xl}]$$

The explicit expression of the GDOP\textsuperscript{10} was first defined in terms of a generalized inverse\textsuperscript{10}. The essential computation of Eq.(15) is, for a system with proportional damping, the same as Eq.(10), and the XORviaGDOP can be, in a sense, considered to be the detailed version of the POC. However, the XORviaGDOP has clear physical meanings for analyst from the viewpoint of projection, and it can be computed between EMA complex modal vectors because the XORviaGDOP is not based on the mass orthogonality which assumes proportional damping.

3. Effect of Mode Shape Error on Cross Orthogonality Checks

3.1 Propagating uncertainty in cross orthogonality checks

The difference between the standard XOR and the XORviaGDOP is only in a scaling scheme: the standard XOR uses mass scaled vector, but the XORviaGDOP uses unity scaled vectors. Both computation is equal in the sense of computation of modal weighting coefficients of Eq.(11). The $(k, l)$ element of Eq.(1) and Eq.(15) is described as:

$$\text{XOR}(k, l) = \phi_{kl}^T M_k \phi_{xl},$$

$$\text{XORviaGDOP}(k, l) = \phi_{kl}^T \phi_{l,k}^T [\phi_{l,k} \cdots \phi_{l,k,l}]$$

We call $w_{kl}$ for the XOR,

$$w_{kl} = \phi_{kl}^T M_k \phi_{xl},$$

and for the XORviaGDOP,

$$w_{kl} = \phi_{kl}^T \phi_{l,k}^T [\phi_{l,k} \cdots \phi_{l,k,l}]$$

For illustration purposes of error propagation, the $(k, l)$ element of the XOR will be written for a 2DOF system:

$$\text{XOR}(k, l) = \begin{bmatrix} w_{kl}(1) & w_{kl}(2) \end{bmatrix} \begin{bmatrix} \phi_{xl}(1) \\ \phi_{xl}(2) \end{bmatrix}$$

Considering the situation that the second DOF of $\phi_x$ contains an error $\epsilon$:

$$\phi_{xl} = \begin{bmatrix} \phi_{xl}(1) \\ \phi_{xl}(2) + \epsilon \end{bmatrix}$$

and $\phi_{kl}$ is orthogonal to $\phi_{xl}$:

$$\text{XOR}(k, l) = 0.$$

Substituting the vector $\phi_x$ into Eq.(21) instead of $\phi_x$, $\text{XOR}(k, l) = w_{kl}(2)$. This indicates that the amplification rate of an EMA mode shape error on the $n$-th DOF is decided by the $n$-th element of $w_{kl}$, and that each row of XOR has different amplification rate.

3.2 Error propagation

Let us assume that each element of an EMA modal vector contains random variables with mean values $\mu_1, \mu_2, \ldots, \mu_n$, and the covariance matrix of the random variables are $C_n$.

Based on the error propagation theory, the amplified error in $k$-th row of XOR to an error $e(n)$ on the $n$-th element of the EMA modal vector can be written as

$$e(n) w_{kl}(n).$$

Then the error of resulting orthogonality matrix $\Delta\text{XOR}(k, l)$ can be written as:

$$\Delta\text{XOR}(k, l) = w_{kl}(1) e(1) + w_{kl}(2) e(2) + \cdots + w_{kl}(n) e(n)$$

$$= \sum_{n=1}^{N} w_{kl}(n) e(n).$$

From the Eq.(19) or (20), this equation also expressed as:

$$\Delta\text{XOR}(k, l) = \sum_{n=1}^{N} \frac{\partial\text{XOR}(k, l)}{\partial \phi_{xl}(n)} e(n).$$

From this relation, the maximum error of XOR$(k, l)$ is determined by:

$$\Delta\text{XOR}(k, l)_{\text{max}} = \sum_{n=1}^{N} |w_{kl}(n)| e(n).$$

The mean value and the standard deviation of $\Delta\text{XOR}(k, l)$ are respectively determined by:

$$\sum_{n=1}^{N} |w_{kl}(n)|$$

$$\sum_{n=1}^{N} w_{kl}(n) w_{kl}(n) C(l, n).$$

Equation (31) gives the sensitivity of the cross ortho-
ognality to mode shape errors in EMA when we know the error covariance matrix $C$. However the $C$ is usually an unknown parameter, because the most of modal identification method does not give the information about $C$.

3.3 Optimal sensor placement

Mode shape variations have been variously characterized: normal distribution with the addition of a small proportion of outliers,\textsuperscript{12,13} transducer cross sensitivity and miss alignment error models,\textsuperscript{14} and so on. Because the objective of this paper is not to characterize the test variation, this paper deals with a popular case that the error of modal vectors is an accidental error with same standard deviation for all sensor points. This assumption is based on the suggestion by Cai et al.\textsuperscript{13}: "Mode shape error is normally distributed on a DOF basis and qualitatively has relatively equal variances". Then mode shape variations are assumed to have normal distribution such that,

$$e_a(n) \sim N(0, \sigma^2).$$

In this case, each element of the $k$-th row of $\text{XOR}$ contains error with normal distribution:

$$N(0, \sigma^2 \sum_{n=1}^{\infty} (w_{kn}(n))^2),$$

because the covariance matrix of error is obtained as follows when the mode shape errors has no correlation.

$$C=\text{diag}(\sigma^2, \sigma^2, \ldots, \sigma^2)$$

This equation shows that the accuracy of the $k$-th row of $\text{XOR}$ is depend on $\sum_{n=1}^{\infty} (w_{kn}(n))^2$.

It is generally important to make the accuracy of each measured value even in order to minimize the error in indirect measurement. For cross orthogonality checks, in the same way, it is important to make the error amplification coefficients $(w_{kn}(n))^2$ even for each sensor points : $n=1, 2, \ldots, N$. Therefore optimal sensor placement for cross orthogonality check can be estimated based on this law if the $\sigma$ is an unknown parameter.

3.4 Uncertainty index

Error distribution in $\text{XOR}$ can be statistically quantified with the confidence region. The error distribution in diagonal and off-diagonal terms of $\text{XOR}$ has, for instance, the 95% confidence interval between $-1.96\sigma \sqrt{\sum_{n=1}^{\infty} (w_{kn}(n))^2}$ and $1.96\sigma \sqrt{\sum_{n=1}^{\infty} (w_{kn}(n))^2}$.

Note: Because the off-diagonal terms of $\text{XOR}$ are absolute values, the 95% confidence interval is $0 < \text{XOR}(k, l) < 1.96\sigma \sqrt{\sum_{n=1}^{\infty} (w_{kn}(n))^2}$ (as shown in Fig. 1).

Because the standard deviation $\sigma$ is usually an unknown parameter, this confidence interval cannot be estimated. However, the coefficients $1.96 \sqrt{\sum_{n=1}^{\infty} (w_{kn}(n))^2}$ can be calculated with only FEA mode shapes and used as an uncertainty index of $\text{XOR}(k, l)$ to show the degree of error in each row of $\text{XOR}$. One possible way to utilize this uncertainty index is to note the value $1.96 \sqrt{\sum_{n=1}^{\infty} (w_{kn}(n))^2}$ by the side of the $k$-th row of $\text{XOR}$ for ease of the interpretation of $\text{XOR}$. This index also gives us easy and direct estimation of the error distribution of the $\text{XOR}$ by using the known standard deviation or roughly estimated standard deviation of the mode shape error.

We will show how to use this index using an example shown in Table 1. For the off-diagonal terms of the first row, the 95% prediction interval is $0 < \text{XOR}(k, l) < 0.064$ when our estimation of standard deviations of the error in the EMA modal vectors is 0.03. The off-diagonal terms of the first row are all in the interval, and consequently we cannot explicitly say that there are differences between FEA and EMA modal vectors.

4. Case Studies

4.1 A Simple beam structure

Consider the cantilever beam shown in Fig. 2, which is modeled with 8 grid points and 7 elements. The great differences of physical DOFs in the FE-model from those in the EM-model, in most cases,
Table 2 Uncertainty index for active set of DOFs

<table>
<thead>
<tr>
<th>Set</th>
<th>Grid Points</th>
<th>Uncertainty Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>All</td>
<td>2.12</td>
</tr>
<tr>
<td>2</td>
<td>3, 4, 5, 6, 7, 8</td>
<td>2.12, 2.14, 2.16, 2.17, 2.18</td>
</tr>
<tr>
<td>3</td>
<td>4, 5, 6, 7, 8</td>
<td>2.86, 9.43, 13.7, 14.4, 8.18</td>
</tr>
</tbody>
</table>

Fig. 2 Bending modes of the beam

![Bending modes of the beam](image)

Fig. 3 Error distribution of the k-th row of XORviaGDOP for active set 3 (1000 evaluations)

![Error distribution](image)

 Demand a particular choice of active set of DOFs—sensor placement. Suppose following three active sets:

1. We place 7 accelerometers to all grid points except for fixed first point.
2. We place 6 accelerometers to point number: 3, 4, 5, 6, 7, 8.
3. We place 5 accelerometers to point number: 4, 5, 6, 7, 8.

Table 2 shows the uncertainty index of XORviaGDOP to the each active set. These results indicate that the error amplification rate in XORviaGDOP extremely varies depending on sensor placement.

Let us assume that FE-model is exactly the same as EM-model: \( \Phi_a = \Phi_x \), but measured EMA modal vectors \( \Phi_x \) contain the normal distribution error:

\[
\varepsilon(n) \sim N(0, 0.05^2), \quad n = 2, 3, \ldots, 8.
\]

For this case, XORviaGDOP computation between \( \Phi_a \) and \( \Phi_x \) was done with 1000 evaluations to obtain the distributions of amplified error. Because the error amplification rate is depend on the row of XORviaGDOP, only the first column of XORviaGDOP is shown. The result for active set 3 is shown in Fig. 3. This figure is the histogram, and dotted line shows the product of the uncertainty index by the standard deviation (95% confidence interval). The resulting distribution shows that the most of the error is in the predicted interval, and the selection of the active set remarkably influences the error amplification. Therefore, the proposed uncertainty index is useful for predicting the modeling error when the standard deviation of the error in the EMA modal vectors is known. When the standard deviation of mode shape error is unknown, the uncertainty index also indicates which row of XORviaGDOP must be improved, and is useful for selecting optimal active set of DOF for the XORviaGDOP. This discussion is also effective for general XOR, because there is no special assumption in the computation which distinguish XORviaGDOP from XOR.

4.2 A frame structure

A frame structure as shown in Fig. 4(a) was used to show the pre-test planning for the cross orthogonality checks. The frame was constructed of steel round bar (150 mm length x 4 mm radius). A finite element model using grounded Rahmen structure was developed with 34 nodes and 42 elements. Experimental modal data was simulated by strengthening the lower part of analytical model (Fig. 4(b)). The first five modes is used in orthogonality checks, and the frequencies of those are shown in Table 3.

One hundred patterns of FEA mode shapes were made with random choice of active set of DOFs. The number of sensor point is ten, which is taken on locations in \( xy \)-direction. Figure 5 shows the standard deviation of the elements of \( w_{ab} \) for each active set. Figure 5(a) shows the results for XORviaGDOP, and Fig. 5(b) shows the results for XOR using Guyan reduction. The standard deviation in Fig. 5 shows whether the error amplification coefficients \( (w_{ab}(n))^2 \) is even for each sensor points or not. This gives the criterion of sensor placement under the general law of error propagation of indirect measurement, because of the reason described in section 3.3. The XORviaGDOP is often sensitive as compared to the Guyan reduction based XOR. Figure 6 shows relation between the standard deviation in Fig. 5(a) and the


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condition number of FEA mode shape matrix for each active set of DOFs: \( \Phi_{ab} \). Because the cross orthogonality check is a modal decomposition, the quality of projector is much dependent on the condition number of \( \Phi_{ab} \). The standard deviation is relatively proportional to the condition number in Fig. 6. Therefore the large standard deviation of \( \left( v_{ab}(n) \right)^2 \) is caused by ill-conditioned \( \Phi_{ab} \). A brief criterion of the robustness of TAM is to compute the condition number of FEA mode shape matrix.

Table 3 Selected target modes

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural Frequency [Hz]</th>
<th>Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.78</td>
<td>1st bending in x-direction</td>
</tr>
<tr>
<td>2</td>
<td>18.21</td>
<td>1st bending in y-direction</td>
</tr>
<tr>
<td>3</td>
<td>22.82</td>
<td>Torsion in x-direction</td>
</tr>
<tr>
<td>4</td>
<td>51.50</td>
<td>2nd bending in x-direction</td>
</tr>
<tr>
<td>5</td>
<td>55.68</td>
<td>2nd bending in y-direction</td>
</tr>
</tbody>
</table>

![Analytical](image1)
![Simulated Experimental](image2)

Fig. 4 Model of the test article

![XORviaGDP](image3)

![XOR (Guyan reduction)](image4)

Fig. 5 Standard deviation of \( w_{ab} \)

Table 4 Sensor configurations for best and worst set

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Best Set</th>
<th>Worst Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-x, 10-x, 15-y, 12-y, 18-y, 22-x, 25-y, 27-y, 28-y</td>
<td>8-x, 10-x, 11-y, 12-y, 14-y, 22-x, 25-y, 27-y, 28-y</td>
<td></td>
</tr>
</tbody>
</table>

![EMA](image5)

Table 5 XORviaGDP and Uncertainty Index for best set

<table>
<thead>
<tr>
<th>EMA</th>
<th>Uncertainty Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.866</td>
</tr>
<tr>
<td>2</td>
<td>0.001</td>
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<tr>
<td>3</td>
<td>0.016</td>
</tr>
<tr>
<td>4</td>
<td>0.138</td>
</tr>
<tr>
<td>5</td>
<td>0.005</td>
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</tbody>
</table>

![EMA](image6)

Table 6 XORviaGDP and Uncertainty Index for worst set

<table>
<thead>
<tr>
<th>EMA</th>
<th>Uncertainty Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.222</td>
</tr>
<tr>
<td>2</td>
<td>0.029</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
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<tr>
<td>4</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.024</td>
</tr>
</tbody>
</table>

![Condition Number of Reduced Mode Shape Matrix](image7)

Fig. 6 Effect of condition number to the distribution of \( w_{ab} \)

Table 4 shows the best and worst active set of DOFs in Fig. 5 (a). The XORviaGDP was computed with these two sets, and the results are shown in Tables 5 and 6. In these tables, uncertainty index is also noted by the side of resulting XORviaGDP. In Table 5, XORviaGDP(2, 5) and XORviaGDP(5, 2) show relatively large value. However, the uncertainty indexes are even. Therefore resulting XORviaGDP indicates the correlation between EMA and FEA modal vectors in second and fifth mode shapes. In Table 6, second and fifth row of XORviaGDP have large values. However, the uncertainty indexes for these rows are larger than those for other rows. Therefore, there are possibilities that these large values are not modeling errors.

5. Conclusions

The work presented in this paper concentrated on
a method to assess the robustness of TAM, based on the cross orthogonality matrix resulting from experimental mode shapes corrupted by noise. An error amplification rate in cross orthogonality computation was cleared to show how mode shape error contributes to the cross orthogonality matrix. Then, an uncertainty index to check a resulting cross orthogonality matrix was developed assuming the error of test mode shapes has zero-mean normal distribution. The results of case studies show that the distribution of amplified error can be predicted using the uncertainty index when the standard deviation of the error is known. The uncertainty index is also useful for predicting the modeling error, and for selecting optimal sensor placement in the cross orthogonality computation when the standard deviation of the error is unknown. In addition, the condition number of FEA mode shape matrix of TAM can be employed as a brief criterion of the robustness of TAM.

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References