Bending Load Capacity Enhancement Using an Asymmetric Tooth Profile*

(1st Report, Influences of Pressure Angle on Tooth Root Stress and Bending Stiffness)

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This research is to develop a new method to enhance the bending load capacity of gear tooth using asymmetric tooth profiles. For many power transmission devices operate in one direction, only one side of the tooth surfaces (the front of tooth) is in mesh. So the tooth is not necessary to be a symmetric form. The dimensions of the front and back profiles of the tooth could be designed individually. The back profile of the tooth that does not contact in running could be decided to decrease the maximum tensile stress at the root of tooth. Different standard pressure angles are used in the front and back profiles of an asymmetric tooth. Tooth root stress and bending stiffness of the asymmetrical tooth for several combinations of standard pressure angles were calculated by the use of finite element method. Based on the results of calculations, it is clarified that a larger standard pressure angle in the back profile of tooth makes the tooth root stress decrease remarkably and bending stiffness of tooth increase, but does not change the load sharing ratio.

Key Words: Gear, Tooth Profile, Pressure Angle, True Stress, Load Capacity, Bending Stiffness, Load-sharing Ratio

1. Introduction

The use of fine module gears is considered to be an effective method for low noise and vibration as well as the compact size of a power transmission device⁽¹⁾. However, a smaller module will lead to the insufficiency in bending load capacity, especially for the gears of high contact fatigue strength such as carburized or surface quality improvement gears. On

the other side, the problems of low bending strength in such as plastic gears and sintered gears are very obvious. The main methods to enhance the bending load capacity of tooth can be divided into two ways, one is to improve the fatigue strength of tooth through heat treatments or surface quality improvement such as carburization and shot peening⁽²⁾, another is to reduce the maximum tensile stress at the root of tooth using a gear cutter with a large radius of tip corner⁽³⁾ or changing the tooth profile of tooth root. Terauchi presented a tooth profile consisting of involute and cycloid curves to increase the contact and bending strengths of gear^{(4),(5)}. But, because of the problems in accuracy and manufacturing cost, the involute-cycloid profile gears are not in practical use.

In the meantime, design of the form of tooth considering the operating situation of the gear is also an effective method to increase the bending load capacity. In a normal involute cylindrical gear, tooth is in a symmetric form, so that the same performance

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can be obtained at the forward and backward rotation. However, in the practically used gear units for power transmission, both of the forward and backward rotation are not always required simultaneously, and even at the condition that the forward and backward rotation are necessary, the load and running conditions are quite different for different direction of rotation. For example, the load of the gears in an automobile transmission is undoubtedly different in running situation and engine brake situation. Then, the purpose of this research is to enhance the bending load capacity of tooth by the use of an asymmetric tooth form considering the practical situations of the gear power transmission device.

2. Design of Asymmetric Teeth

As above mentioned, it is practicable to decrease the maximum tensile stress at tooth root for the enhancement of bending load capacity of tooth by changing the left-right symmetric tooth form to an asymmetric one in such gears that only one side (the front profile) of tooth is in mesh. The back profile, which doesn't mesh in normal running condition, can be designed for a large critical section. In this research, for the compatibility with the involute cylindrical gears in use and the convenience in manufacture, the asymmetric tooth is realized by the use of different standard pressure angles in the front and back tooth profile under the same standard pitch with the normal gears. In the following discussions, the standard pressure angle of the front profile is expressed by α_F and called front standard pressure angle, the standard pressure angle of the back profile is expressed by α_B and called back standard pressure angle. The front standard pressure angle in this research is determined as a constant of 20 degrees in all following asymmetric teeth.

The tooth profile of a gear cutter is shown in Fig. 1, and the tooth form of a gear cut by this asymmetric cutter is shown in Fig. 2. There are two base circles because that the front profile and the back profile have different standard pressure angles but a common pitch circle. Four types of asymmetric teeth in different combinations of the front and back standard pressure angles are shown in Fig. 3. The front standard pressure angles of the teeth are 20 degrees, but the back standard pressure angle is 20, 25, 30 and 35 degrees respectively. From Fig. 3, it can be clearly understood that a larger back standard pressure angle will lead to a larger critical section. Therefore, the stress at the root of asymmetric tooth in a larger back standard pressure angle will be smaller than that of the standard tooth in the standard pressure angle of 20 degrees. The asymmetric cutters and gears shown in

Fig. 1 Profile of gear cutter

Fig. 2 Asymmetric tooth profile

Fig. 3 Tooth forms in different standard pressure angle combinations

Fig. 4 were produced to confirm the manufacturing processes. Compared with the involute-cycloid gear presented by Terauchi⁽⁴⁾, for that the asymmetric gear of this research can be made with the method and equipment used up to now, the possibility and convenience of the asymmetric gear in practical use are obvious. In the case of involute gears for power transmission, a small standard pressure angle is generally desirable for less vibration and lower noise⁽⁶⁾, but the problems of the decrease of critical section and bending load capacity shortage make difficulties to

Fig. 4 Samples of hob cutters and gears

the application of small standard pressure angle. In the presented asymmetric tooth, a small front standard pressure angle can be used and the insufficiency in bending load capacity can be solved by the use of a large back standard pressure angle.

3. Effects of Back Standard Pressure Angle on the Stress at Tooth Root

3.1 Finite element analysis for the stress at tooth root

The stresses of asymmetric were calculated using finite element method. The module of test gears was 2.0, number of teeth was 34, the tip diameter was 72.0 mm, addendum modification coefficient was 0, and the radius of the tip corner of hob cutter was 0.6 mm. The mesh of tooth and boundary conditions are shown Fig. 5. There are 1 344 elements and 720 nodes in the mesh. The nodes on the centers of the neighboring teeth are constrained. A normal load of 9.8 N was applied at the tip of tooth. A finite element analyzing software MARC & Mentat was used in the calculations.

3.2 Stress distribution for different back standard pressure angles

The maximum stress (principal stress σ_I) distributions at the root of tooth for different combinations of the front and back standard pressure angles are shown by gray scale figure in Fig. 6. The intensity of whiteness expresses the magnitude of the maximum stress. It is clear that the maximum stress decreases with the increase of the back standard pressure angle α_B . The stresses in Y (dedendum) direction σ_Y are shown by contour lines in Fig. 7, where the stress interval of the contours is 2.0 MPa. It is found from

Fig. 5 Tooth mesh and boundary conditions

Fig. 6 View of maximum stress distribution at tooth root

Fig. 7 that one contour line (number 8) disappears when the back standard pressure angle is increased from 20 degrees to 30 degrees. In addition, the result was also obtained that the maximum tensile stress at the root of tooth was decreased about 17% by changing the back standard pressure angle α_B from 20 degrees to 35 degrees.

Fig. 7 Contour lines of the stress σ_y for the back standard pressure angle of 20 deg and 30 deg

4. Calculation of the Maximum Tensile Stress

4.1 Distribution of the tensile stresses on tooth surface

The stress that takes a very important part in tooth root crack origination is considered to be the maximum tensile stress σ_t at the root of tooth, which is the stress component at the tangential direction of fillet. The tensile stress σ_t at the root of tooth was calculated using finite element method. The effect of the back standard pressure angle on the distribution of tensile stress is shown in Fig. 8. The horizontal axis of Fig. 8 is the length along tooth profile from standard pitch point. From Fig. 8, it is obvious that the use of a large back standard pressure angle is an effective method to decrease the tensile stress at the root of tooth, and the position of the maximum tensile stress approaches to tooth root for a larger back standard pressure angle.

4.2 Calculation of the maximum tensile stress at the root of tooth

Referring to Tobe's empirical formula for the maximum tensile stress for normal involute gears of standard pressure angle of 20 degrees⁽⁷⁾, the formula for the maximum tensile stress σ_{tmax} (MPa) due to a normal load P (N) for the asymmetric gears is presented as,

$$\sigma_{tmax} = \frac{P}{b \cdot m} K_z \cdot K_\lambda \cdot K_\alpha \quad (1)$$

where b is face width (mm), m is module, K_z is tooth number influence factor, K_λ is load position influence factor, and K_α is pressure angle influence factor.

The tooth number influence factor K_z was decided as the following expression by the least squares method based on the finite element analysis results

Fig. 8 True stress distribution

Fig. 9 Calculation of tooth number factor

under the conditions of the load at tooth tip, the back standard pressure angle α_B of 20 degrees and the number of teeth z within the range from 20 to 50.

$$K_z = 3.56 + 4.59\left(\frac{1}{z}\right) + 1740\left(\frac{1}{z}\right)^3 \quad (2)$$

The tooth number influence factor K_z vs. the number of teeth is shown in Fig. 9. A large number of teeth will lead to a small tooth number influence factor.

For the front standard pressure angle is equal to 20 degrees, the same with that of Tobe's formula, and the effect of the back standard pressure angle on the maximum tensile stress will be considered by the pressure angle influence factor, the load position influence factor K_λ was quote from Tobe's following formula.

$$K_\lambda = \exp\left[\left\{2.50\left(\frac{1}{z}\right) - 0.50\right\}\frac{\lambda}{m}\right] \quad (3)$$

Where z is the number of teeth, λ is the distance (mm) from loading point to the tip of tooth parallel to the centerline of the loaded tooth.

Pressure angle influence factor K_α was obtained

based on the finite element analysis results for the combinations of the front standard pressure angle of 20 degrees with the back standard pressure angles of 20, 25, 30 and 35 degrees. The pressure angle influence factor K_α vs. the back standard pressure angle is shown in Fig. 10. The formula for pressure angle influence factor K_α , was determined by the least squares method as,

$$K_\alpha = 1.32 - 1.82 \times 10^{-2} \alpha + 1.17 \times 10^{-4} \alpha^2 \quad (4)$$

Where α is substituted with the back standard pressure angle α_B .

Figure 11 shows the comparison between simulated and FEM calculated maximum tensile stress at the root of tooth, where normal load $P=9.8\text{ N}$, module $m=2.0$, the front standard pressure angle $\alpha_F=20\text{ deg}$. A good accuracy of the simulation is confirmed from Fig. 11, ignoring a little bit of large differences

Fig. 10 Calculation of standard pressure angle factor

Fig. 11 Correlation between simulated and FEM calculated maximum true stress of tooth

between simulated and FEM calculated values in the range of small stress where the loading positions λ are over 2.0 mm, namely the loading positions are lower than the standard pitch point of tooth. Considering that the worst loading position, the boundary of two pair of teeth mesh is near the tip of tooth, the empirical formulae of Eqs. (1)–(4) are applicable to practical use.

5. Bending Stiffness of Tooth and Load Sharing Ratio

The gearing and lubrication conditions of an asymmetric gear are the same with the normal gear whose standard pressure angle is equal to the front standard pressure angle of the normal gear. However, the bending stiffness, very important factor influencing the load sharing and vibration, will change for different back standard pressure angle. The bending stiffness of tooth and load sharing ratio of the asymmetric gears were investigated.

5.1 Definition of the bending stiffness of tooth

Ishikawa⁽⁸⁾ calculated the deflection of tooth with the model substituting the tooth fillet with a rectangle and a trapezoid. Since the front side profiles and backside profiles are different in an asymmetric tooth, the method of Ishikawa is not applicable to the case of asymmetric tooth. In the following discussion, the deflection of the tooth under a normal load is indicated in Fig. 12 ignoring the contact deflection. The bending stiffness of tooth is defined as the value of load P divided by deflection δ at the reference point in the direction of applied normal load.

5.2 Bending stiffness and load sharing ratio in different back standard pressure angle

The bending stiffness of the tooth whose dimensions are the same with that of the tooth mentioned in

Fig. 12 Deflection of tooth under normal load

Fig. 13 Changes of tooth bending stiffness in different standard pressure angle combinations

Fig. 14 Model for load sharing ratio calculation

stress analysis sections is plotted versus the radius of loading point to different back standard pressure angles in Fig. 13. A larger back standard pressure angle increases the bending stiffness of tooth uniformly.

Load sharing situation in the two pairs of teeth mesh region for the above mentioned gears in a gear ratio of 1.0 was analyzed with a parallel spring system shown in Fig. 14. Spring factors are substituted with the bending stiffness of teeth at the contact point. Ignoring the error in tooth profile, the load sharing ratio is shown in Fig. 15, the horizontal axis in the figure is the contact position on the line of action. As seen from the figure, almost no remarkable changes in load sharing ratio are recognized because of the

Fig. 15 Load sharing ratio in different standard pressure angle combinations

uniform changes in the bending stiffness of tooth for different back standard pressure angles as shown in Fig. 13. Therefore, it can be considered that the vibration characteristics of the asymmetric gears in forward running direction would not become worse comparing with that of the normal gears.

6. Conclusions

In order to enhance the bending load capacity of tooth, the asymmetric gears, which have different standard pressure angles in the front side profile and backside profile of tooth, were presented considering the practical situation of power transmission gears. These asymmetry gears were produced experimentally, and the tensile stress distributions at the root of tooth, stiffness and load sharing ratio under different back standard pressure angles were calculated using the finite element method. The results obtained are as follows:

(1) The critical section of tooth is increased, and the maximum tensile stress at the root of tooth is also drastically decreased by the use of a large back standard pressure angle.

(2) Based on the results of finite element analysis, the empirical formulae for the maximum tensile stress at the root of tooth of the asymmetry gear were obtained, and the validity of the formulae was confirmed.

(3) Bending stiffness of the asymmetric tooth increases uniformly with the increase in the back standard pressure angle, but no remarkable changes in load sharing ration are recognized for different back standard pressure angles.

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