Vibration and Sound-Radiation Analysis for Designing a Low-Noise Gearbox with a Multi-Stage Helical Gear System*

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A method for predicting gear noise from the vibration step to the sound-generation step of a complete gear system, including a gearbox, was developed. This method consists of three separate steps: gear-vibration analysis by an in-house program, gearbox-vibration analysis by an FEA program, and sound radiation analysis by a commercial software. By using this method, it can obtain the vibration behavior of the gear and gearbox, and the distribution of the sound-pressure around the gearbox and identify the areas from which noise radiates intensely. To validate the method, we measured the displacement of a gear shaft, the acceleration of points on the surface of the gearbox, and the sound-pressure levels around the gearbox. The measured results agreed well with the corresponding calculations. According to the calculated noise, a low-noise gearbox was designed by adding ribs near the antinodes of the gearbox's vibration mode.

Key Words: Gear, Modeling, Design, Multi-Stage Gear, Gearbox, Vibration Analysis, Sound-Radiation Analysis

1. Introduction

The increasing demand for quieter gear systems has created the need for precise analysis of gear-drive vibration. Furthermore, noise needs to be predicted more precisely during product design. Most structure-borne sound in gear systems is generated by gear vibration. Torque ripple in motors, errors in tooth profiles, and fluctuations in tooth stiffness can cause excitation during gear meshing. This mesh excitation propagates from the gear shafts to the bearings (where it is converted into a bearing force), excites the gearbox, and generates gear-vibration noise which is radiated from the surface of the gearbox. To design gearboxes with low noise, a method for precisely analyzing the vibration and noise of the gears and gearbox is therefore needed. There has been a lot of research on three-dimensional gear-vibration analysis using different models, such as a single gear pair\(^{(1,2)}\), a single gear pair with a gearbox\(^{(3)}\), multi-stage gears\(^{(4)-(6)}\), and multi-stage gears with a gearbox\(^{(7)}\). Houjoh and Wang included tilting vibration during tooth meshing in their gear-vibration model\(^{(8),(9)}\). Rigal’s model considers the anisotropic stiffness of a bearing\(^{(7)}\). The results of these gear-vibration analyses were used by several groups to create a numerical expression for calculating the gear-vibration noise by a simplified method\(^{(10),(11),(12)}\). The calculated sound-power level and sound-pressure level were used to optimize the shape of gears and their gearboxes to reduce noise\(^{(13),(14)}\). If the gearbox has a simple shape, these expressions can be used to roughly obtain the sound-pressure level of gear noise. However, the simplified method is not able to take into account of the effect of noise diffraction and reflection. Industrial gearboxes have complex shapes, therefore, to predict the gear noise precisely, it is important to take into account of the effects of the noise diffraction and reflection. Kato calculated sound power by using boundary element method (BEM)\(^{(15)}\),

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and this calculated result agrees well with the corresponding measured data. Moreover, for an effective low-noise design, the distribution of the sound pressure is required in order to identify where the noise is radiated from. Several experimental studies measured the noise radiating from a gearbox and identified the effects of differences between gear type (e.g., spur, helical, and double helical), helix angle, gear error, and gearbox rigidity \(^{(14)-(16)}\). However, they don't obtain the gear-noise distribution. Only Sabot calculated the sound-pressure distribution by using the simplified method \(^{(10)}\), though his model neither considered the effects of noise diffraction and reflection nor verified experimentally.

We have developed a method for predicting gear noise generated (i.e., from the vibration step to sound-generation step) in a complete gear system, which includes multi-stage gears, shafts, bearings, and a complex-shaped gearbox \(^{(17)-(21)}\). Furthermore, according to the rapid development of computer and software in recent years, it has become possible to produce a three-dimensional contour map that indicates the distribution of the sound pressure around the gearbox and considers the effects of noise diffraction and reflection by using BEM. Finally, using the calculated noise, a low-noise gearbox was designed by adding ribs near the antinodes of the gearbox's vibration mode \(^{(21)}\).

2. Sound-Radiation Analysis

As illustrated in Fig. 1, the developed method consists of the following steps: three-dimensional gear vibration analysis by an in-house program \(^{(17)-(18)}\), gearbox vibration analysis by a finite element analysis (FEA) program \(^{(22)}\), and boundary element analysis (BEA) of the sound field. To analyze the sound radiating from a gear system, a commercial software, LMS SYSNOISE\(^{(23)}\), is used. This software can estimate the sound field around the structure. The input data are the gearbox shape and material, the output shaft load, the gear-shaft dimensions, and the gear specifications. The output data are the frequency responses of the distribution of gear noise radiating from the gearbox.

In the first step, to predict the multi-stage helical-gear vibration, we construct a gear vibration model in which each gear shaft has six-degrees of freedom (DOF) (as illustrated in Fig. 2). This model has two features: to consider tilting vibration, tooth meshing is modeled as two parallel springs; to consider multi-stage gears, the bearing and tooth stiffness matrices are separated and overlaid on the total gear and gear shaft stiffness matrix (Ref. \(^{(17)}\)). By using this vibration model of multi-stage gears, we can analyze the frequency response of three-dimensional gear vibration. In this analysis, it is assumed that an angle-transmission error causes mesh excitation, and tooth profile error is inputted as the transmission error at both two parallel springs alternately, which takes into account of phase difference (Ref. \(^{(18)}\)).

In the second step, the FEA model of the gear system was divided into several sub-models, each constructed of component elements. To reduce the node number, the mesh of the gearbox was divided
into shell and solid elements. The main part of the gearbox model was made of shell elements, while the thick part (e.g., bearing housing) was made of solid elements (as shown in Fig 3). The calculation error of natural frequencies of the gearbox was less than 5%. The gear shafts were modeled using beam elements, and the gear meshing, bearings, and legs of the gearbox were modeled using spring elements (as shown in Fig. 4). We then combined these FEA submodels into a total model (Refs. (19), (20)).

By inputting each bearing force calculated from the results of the gear vibration analysis into each bearing in the FEA model, we can estimate the frequency responses of the gearbox. A gear vibration analysis program outputs the three-DOF vibration displacement ($\Delta D$) at the center of each bearing. The three-DOF vibration bearing force is given by

$$\Delta F_b = T \times k_b \times \Delta D,$$

where $T$ is the transmissibility from the gear shaft to the bearing housing and $k_b$ is the bearing stiffness. In the FEA analysis, $\Delta F_b$ is inputted to the FEA model as the exciting force at the center of each bearing model, and the program outputs the gearbox frequency response in terms of acceleration, velocity, and displacement.

The final step is to construct a BEA model, which is defined as the surface of the gearbox. First, the vibration velocities of the surface of the gearbox are calculated using an FEA program. Then, the frequency responses corresponding to these velocities are inputted into the BEA model. Finally, the three-dimensional distribution of sound pressure levels is calculated by the BEA software, SYSNOISE®. Using the results of the distribution of sound pressure and the sound-power level of gear noise calculated from the distribution, we can determine where the noise is radiated from; thus the gearbox's shape can be redesigned to reduce the noise, as shown by the broken line in Fig. 1.

3. Experimental Validation

To validate our sound radiation analysis method, we applied it to an experimental apparatus consisting of a second-stage helical reduction gears (Fig. 5). Before the calculation, we measured the sound pressure levels at points 1 m from the gearbox while it was being driven. The output torque was set at three values: 980.7, 490.4, and 98.1 N·m.

An example spectrum of the sound pressure is shown in Fig. 6. The first-stage–gear mesh-frequency components are most prominent. We therefore focused on the first-order mesh-frequency component of the first-stage gears and calculated the frequency response of the first-order sound.

While the speed of the gears increases, the frequency responses of the vibration displacements at the end of the second-axis of the gear shafts and the accelerations on the surface of the gearbox were measured. Simultaneously, sound-pressure levels at two separate points A and B were also measured.
Table 1 Inputted data for gears and bearings

<table>
<thead>
<tr>
<th></th>
<th>First axis</th>
<th>Second axis</th>
<th>Third axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_s ) kg</td>
<td>6.85</td>
<td>22.5</td>
<td>71.6</td>
</tr>
<tr>
<td>( J_t ) kgm^2</td>
<td>5.62 \times 10^{-4}</td>
<td>0.19</td>
<td>0.80</td>
</tr>
<tr>
<td>( J_{di} ) kgm^2</td>
<td>0.06</td>
<td>0.13</td>
<td>2.20</td>
</tr>
<tr>
<td>( L_{il} ) mm</td>
<td>-7.80</td>
<td>106.9</td>
<td>159.1</td>
</tr>
<tr>
<td>( L_{ir} ) mm</td>
<td>110.8</td>
<td>60.6</td>
<td>26.9</td>
</tr>
<tr>
<td>( k_{cil} ) N/m</td>
<td>1.49 \times 10^8</td>
<td>1.71 \times 10^8</td>
<td>3.34 \times 10^8</td>
</tr>
<tr>
<td>( k_{cir} ) N/m</td>
<td>1.53 \times 10^8</td>
<td>1.79 \times 10^8</td>
<td>8.02 \times 10^8</td>
</tr>
<tr>
<td>( k_{gil} ) N/m</td>
<td>1.76 \times 10^7</td>
<td>6.44 \times 10^7</td>
<td>3.89 \times 10^7</td>
</tr>
<tr>
<td>( k_{gir} ) N/m</td>
<td>6.77 \times 10^6</td>
<td>5.81 \times 10^7</td>
<td>5.75 \times 10^7</td>
</tr>
<tr>
<td>( k_{cil} ) N/m</td>
<td>1.60 \times 10^8</td>
<td>2.20 \times 10^8</td>
<td>3.33 \times 10^8</td>
</tr>
<tr>
<td>( k_{cir} ) N/m</td>
<td>1.18 \times 10^8</td>
<td>2.12 \times 10^8</td>
<td>7.77 \times 10^8</td>
</tr>
</tbody>
</table>

Table 2 Inputted data for gear meshing

<table>
<thead>
<tr>
<th></th>
<th>Horizontal</th>
<th>Axial</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>First stage N/m</td>
<td>1.91 \times 10^6</td>
<td>1.68 \times 10^6</td>
<td>6.02 \times 10^6</td>
</tr>
<tr>
<td>Second stage N/m</td>
<td>2.24 \times 10^6</td>
<td>2.00 \times 10^6</td>
<td>7.16 \times 10^6</td>
</tr>
</tbody>
</table>

Table 3 Modal damping ratio for each part

<table>
<thead>
<tr>
<th>Part</th>
<th>Modal damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear meshing</td>
<td>0.007</td>
</tr>
<tr>
<td>Bearing</td>
<td>0.05</td>
</tr>
<tr>
<td>Torsional vibration at shaft</td>
<td>0.005</td>
</tr>
<tr>
<td>Torsional vibration at coupling</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Point A is vertically 1 m from the gearbox and Point B is horizontally 1 m from it.

Figure 7 shows the measured and calculated frequency responses of the first-stage-gear mesh-frequency components. In the calculation, the anisotropy of bearing stiffness was taken into account. Inputted data for gears, bearings, and gear meshing are listed in Tables 1 and 2. Damping factors for each part are calculated by using the critical damping ratio derived from the mass and spring data in Tables 1 and 2 and by using the modal damping ratio assumed for each part (Table 3). The calculations agree well with the corresponding measurements. Regarding the gear-vibration analysis, the calculation error in the natural frequency was less than 5% ; however, the amplitude error at the resonance frequency was more than 20%. The reason for this difference is that the model does not take the anisotropy of bearing stiffness into account ; i.e., the inputted coefficients of radial bearing stiffness, \( k_x \) and \( k_z \), were the same values, because of that it wasn't considered in most previous studies.

The measurement apparatus has three shafts in the same horizontal plane. The middle shaft is affected by the transmission loads of both the first and the second-stage gears. The vertical load on the middle shaft is therefore more than three times larger than the horizontal load in the radial direction. Consequently, we took the anisotropy of the bearing stiffness into account in the calculation ; that is, the amplitude error was decreased less than 8% as shown in Fig.7. To predict the three-dimensional gear-vibration behavior accurately, it is therefore important to take into account the anisotropy of the bearing stiffness.

An example of the acceleration of a point on the gearbox surface obtained experimentally and by calculation is shown in Fig.8, which compares the responses of the first-stage-gear mesh-frequency component. Inputted data for FEA models are listed in Table 4. Measured modal damping ratio was 0.045. The measured and calculated resonance points agree well. The calculation error was less than 5%. However, according to the calculated result when the input transmissibility was constant, the amplitudes in the range from 250 to 350 Hz were more than twice the measured ones. The reason for this discrepancy is that the input transmissibility is the measured mean value of peaks, 0.22. According to the calculated
Table 4 Inputted data for FEA models

<table>
<thead>
<tr>
<th></th>
<th>Gearbox</th>
<th>Gear shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus (N/m²)</td>
<td>$1.27 \times 10^{11}$</td>
<td>$2.06 \times 10^{11}$</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>$7.20 \times 10^3$</td>
<td>$7.80 \times 10^3$</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>2989</td>
<td></td>
</tr>
</tbody>
</table>

Accelaration of point on gearbox

- Measured
- Calculated ($\tau$: constant, 0.22)
- Calculated ($\tau$: variable wrt freq.)

Fig. 8 Comparison between measured and calculated first-stage-gear mesh-frequency components of acceleration on the surface of a gearbox

result when the input transmissibility was variable with respect to the measured frequency, the amplitude error decreased. Therefore, the transmissibility should be variable with respect to the frequency for accurate analysis.

We used the calculated vibration on the surface of the gearbox as an input parameter of BEA to analyze the distribution of the sound-pressure levels around the gearbox. We calculated the frequency response of three-meter square equi-pressure contour maps in the horizontal and vertical directions. An example map calculated at 490 Hz is shown in Fig. 9. To take the effect of reflections from the surface of the floor into account, we modeled the floor as a baffle wall. Four models were used to calculate the frequency response of the distribution of the sound-pressure levels. E.g., model (1): the input transmissibility is constant; (2): the input transmissibility is constant and the floor model is added; (3): the input transmissibility is variable with respect to the frequency; (4): the input transmissibility is variable with respect to the frequency and the floor model is added.

To verify the effectiveness of our gear-noise-prediction method, we compared the measurements with the calculated the frequency responses of sound pressure. As shown in Fig. 10, the results of the model (4) were the closest to the measured data. We can thus sufficiently assess the noise radiating from the surface of a gearbox by inputting into the transmissibility variable with respect to frequency and taking into account the effect of reflections from the floor.

Fig. 9 Calculated contour map of sound-pressure level corresponding to the first-stage-gear mesh-frequency component at 490 Hz (taking into account effect of reflections from floor)

(a) Measured point A

(b) Measured point B

Fig. 10 Comparison between measured and calculated sound-pressure levels corresponding to the first-stage-gear mesh-frequency component at points A and B in Fig. 9

4. Designing a Low-Noise Gearbox

The validation of the new method means that it can be used to design low-noise gear systems. As an example of designing a low-noise gearbox, we used
the gear apparatus described in Section 2 and illustrated in Fig. 5. We modeled three different reduced-noise gearboxes and compared the calculated sound-power levels for the gear system in each one. In this apparatus, the mesh-frequency component of first stage gears is the most prominent (as mentioned above). When the apparatus is driven at maximum speed, the first-order mesh-frequency component of first stage is at 500 Hz. Therefore, to reduce the noise (the overall value of sound pressure), we tried to reduce the first-order mesh-frequency component of the first-stage (whose frequency range is less than 500 Hz).

We calculated the sound-power level according to JIS Z 8732 (ISO/DIS 3745) as follows:

\[
L_w = L_{pr} + 10 \log \left( \frac{S_1}{S_0} \right) + C,
\]

where

\[
L_{pr} = 10 \log (1/N) \left[ \sum_{i=1}^{N} 10^{k_i/10} \right],
\]

\[
S_1 = 4 \pi r^2 \text{ (m}^2\text{)},
\]

\[
S_0 = 1 \text{ (m}^2\text{)},
\]

\[
C = -25 \log \left( \frac{427/400}{273/(273 + \theta)} \right) \left( B/B_0 \right),
\]

\[
B_0 = 1.013 \times 10^5 \text{ (Pa)}
\]

\[r = 2 \times (h_o/2)^2 + (h_o/2)^2 + h_o^2 \text{ (m)},\]

where \(L_{pr}\) (dB) is the mean sound-pressure level on the surface of a calculated sphere, whose radius \(r\) (m) is twice the dimension of the gearbox which is \(h_o\) (m) in length, \(h_l\) (m) in width, and \(h_h\) (m) in height, referring to ISO3744: 1994; \(S_1\) (m\(^2\)) is the surface area of the calculated sphere; \(C\) is a term needed to compensate for temperature \(\theta\) (°C) and pressure \(B\) (Pa) at a measured or calculated point, and \(B_0\) (Pa) is atmospheric pressure.

The sound-power level of the apparatus in the range of mesh frequencies from 250 to 490 Hz was calculated. The maximum level was between 470 and 490 Hz as shown in Fig. 11, therefore, reducing the noise in this frequency range was focused. It is close to the fourth natural frequency of the gearbox, 484 Hz, whose vibration mode is that the side faces of the gearbox vibrate in opposite directions, as shown in Fig. 12. As shown in Fig. 13, the noise radiated most intensely from the antinodes (where the amplitude of vibration is large) of the gearbox’s vibration.

According to the calculated result in Fig. 13, we modified the gearbox by adding ribs near the antinodes, as shown in Fig. 14. In gearbox types 1 to 3, to increase the fourth natural frequency, ribs were added at the antinodes of the fourth vibration mode. Figure 15 shows calculated variations of lowering sound-power level corresponding to first-order mesh-frequency component of first-stage in modified gearboxes. As shown in Fig. 15, these three types thus reduce the sound-power levels by four or five decibels.
A method for predicting the noise generated (from vibration to sound-radiation) in a multi-stage helical gear system was developed. It consists of three separate analysis steps: gear-vibration analysis by an in-house program, gearbox-vibration analysis by an FEA program, and sound-radiation analysis by BEA software (SYSNOISE®). By using this method, it can obtain the vibration behavior of the gear and gearbox, and the distribution of the sound-pressure around the gearbox and identify the areas from which noise radiates intensely. To validate the method, we measured the displacement of a gear shaft, the acceleration of points on the surface of the gearbox, and the sound-pressure levels around the gearbox. The measured results agreed well with the corresponding calculations. For precise analysis, it is important to take into account of the anisotropy of bearing stiffness, the transmissibility with respect to frequency, and reflection from the floor. This method can accurately predict vibration and noise and according to the calculated noise, a low-noise gearbox was designed by adding ribs near the antinodes of the gearbox’s vibration mode.

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