Compliance Cancellation for a Magnetic Guide Using a Smart Multitasking Active Vibration Control Algorithm*

Martin RUSKOWSKI**, Lars REICKE** and Karl POPP**

Although active magnetic guides (AMG) can obtain an infinite static stiffness and high damping using sophisticated multi-degree-of-freedom control, still a non-negligible dynamic compliance is inherent. This is especially true when high controller gains cannot be achieved due to spill over effects. Several approaches are known to overcome this problem, repetitive control and frequency estimation being two of them. Both have drawbacks concerning flexibility, robustness and compensation quality. This paper proposes a smart frequency identification and compensation scheduling algorithm along with tracking compensation oscillators to cancel arbitrary periodic disturbances. The proposed algorithm has been implemented and tested on an AMG. The introduced smartness provides a very robust behavior and leads to an infinite steady state stiffness.

Key Words: Active Vibration Control, (Active) Magnetic Guide, Signal Identification, Real-time Implementation, Measurements

1. Introduction

Active magnetic guides (AMG) qualify as an alternative guiding principle for many machine tool applications. Like active magnetic bearings (AMB) in rotational applications they avoid friction forces and wear. Possible applications include High Speed Cutting (HSC) machine axes or long-stroke, high speed machines as used e.g. in the wood processing industry. AMGs have extreme advantages in clean rooms as in harsh environments, since no lubrication and no sealing of the active parts is required.

In a joint research project at the University of Hannover a new generation of High Speed Cutting machines is under development aiming at an acceleration of 5 g in all axes. Our project part deals with the integration of an AMG as an axis for a HSC machine.

1.1 Experimental setup

For an examination of the achievable control dynamics and thereby the mechanical stiffness, a prototype guide (cp. Fig. 1) had been built\(^{(3)}\). It consists of an aluminum frame with a steel reinforcement and has a mass of 45 kg. The additional payload can be up to 100 kg. The frame is guided by six pairs of electromagnets in differential arrangement. Six eddy current displacement sensors are used to determine the position relative to the rails.

The control is realized on a 366 MHz PowerPC VME process computer using the hard real-time operating system RTOS-UH from Hannover University\(^{(3)}\). The system is capable to generate flexible sample rates of up to 20 kHz while being used as a general purpose computer. The programming is done in the high level real-time language PEARL allowing for very efficient real-time programming. RTOS-UH is scalable and available for microcontrollers as well as for high-end multiprocessor systems.

1.2 Control strategy

The control strategy used for the AMG is a

---

* Received 15th November, 2002 (No. 02-5191)
** Institute of Mechanics, University of Hannover, Appelstrasse 11, D-30167 Hannover, Germany.
E-mail: ruskowski@ifm.uni-hannover.de

Fig. 1 The active magnetic guide (AMG)
decoupled cascade control (DCC)\(^{(12)}\). It is designed to control the five degrees of freedom (DOF) excluding the feed direction \(x\) using an inverse model of the (constant) couplings in the plant (cp. Fig. 2). It is based on the known mass matrix \(M\) and the Jacobian \(J = \partial x / \partial \theta\), where the magnet displacements are given by the \((6 \times 1)\) vector \(x\) and the guide displacement is denoted in generalized coordinates by the \((5 \times 1)\) vector \(q = [y, z, \phi, \phi, \theta]^T\) (cp. Fig. 1) for a fixed reference point in the geometrical center of the guide.

The control itself is designed in a normalized domain for the generalized coordinates \(q\), calculating the desired generalized acceleration \(\ddot{q}\) from the displacement vector \(q\). For an optimal suppression of sensor noise, the control is realized as five independent discrete-time PID-equivalent controllers with KALMAN-filters for the observation of the velocities\(^{(9, 10)}\). An inverse magnet model makes their characteristics transparent to the control\(^{(8)}\). External disturbances in form of static or periodic forces can as well be transformed to disturbance accelerations. Thus, this paper focuses only on the normalized controller.

### 2. Motivation

To examine its suitability for a milling machine, the prototype guide has been deployed to a conventional milling machine (cp. Fig. 3). Milling tests have been performed and the deflection of the guide measured. Figure 4 shows a typical displacement spectrum while milling an aluminum block. Obviously, the guide oscillates with the frequency of the cutter rotation and some harmonics depending on the number of cutting edges.

The oscillation results from the compliance frequency response of the AMG inherent to the control, cp. Fig. 5. To reduce the amplitude of oscillation, a higher stiffness is required. This can be obtained using a higher controller gain or acceleration measurement\(^{(9)}\).

Both methods are not always applicable, thus this paper proposes a smart disturbance identification and active vibration control algorithm.

#### 2.1 Compensation using repetitive control

Hillerström\(^{(9)}\) has shown that remaining oscillations can be partially compensated for the steady state case using feedforward repetitive control based on a time delay and a memory. Thus, the measured deviation is applied to the plant input cyclically using a feedback gain. With high feedback gains the controller tends to get unstable while low gains lead to a long settling time. Especially high frequency oscillations tend to build up as no additional signal processing is provided. Several enhancements proposed still cannot resolve these basic problems resulting from the

---

**Fig. 2** Layout of the DCC loop

**Fig. 3** AMG deployed to a milling machine tool

**Fig. 4** Typical frequency spectrum of the lateral deflection \(y\) while milling on the AMG

**Fig. 5** Vertical compliance frequency response of the prototype AMG.
fact that the repetitive control calculation is performed at the same sample rate as the position feedback control.

2.2 Compensation using frequency estimation

As an enhancement, an identification of the external disturbance has been proposed\(^{(n-o)}\), compensating the disturbance using an oscillator which is tuned on line. The above implementations of this algorithm are based on the time-domain disturbance determination described later. The main drawback of these implementations is the fixed coupling of the frequency determination to the position control sample time, thus the performance is limited by computation time availability and noise sensitivity.

2.3 New multitasking concept

A more promising concept seems to decouple the computation of the compensation acceleration from the position controller. Thus, an improved and more robust disturbance recognition is possible. Periodic disturbances have to be identified by frequency, amplitude, and phase. With this information, an independent oscillator can be tuned which generates a signal to compensate the external disturbance.

For robustness reasons, the disturbance recognition is better not calculated at the sample rate of the position control but in an independent background task looking at sequences of the position signal. Thus, a real-time operating system with priority-scheduled task management is needed. RTOS-UH is one of the most reliable systems in this field and PEARL an advanced real-time language offering task scheduling and synchronization as built-in high-level commands.

3. Frequency Identification

To enable the compensation of a disturbance, first its frequency has to be identified. This can be performed either in time or in frequency domain. A robust and significant determination requires for high frequency resolution and noise insensitivity.

3.1 Identification in frequency domain

The DFT (Discrete Fourier Transform) \( X[k] \) of an arbitrary time signal \( x[k] \), \( k=1, \ldots, N \) divides the frequency range into \( N \) wave bands of bandwidth \( \Delta f=f_s/N \) with the sample frequency \( f_s \). The magnitude \( |X[k]| \) is a measure for the power content of the respective wave band \( k \), so the DFT spectrum is a reliable hint for present oscillations. As a drawback, there might be a mismatch between the real frequency \( f_s \) and the detected frequency \( k\Delta f \) due to the quantization of the DFT.

The DFT is insensitive to noise due to the high signal to noise ratio in each wave band and the possibility to detect multiple frequencies simultaneously. The discrete frequency resolution \( \Delta f \) can only be improved by increasing the number \( N \) of samples or decreasing \( f_s \). This leads to longer calculation or measurement times, respectively.

3.2 Identification in time domain

Alternatively to the frequency domain identification, several different approaches to determine the frequency of a signal \( x[k] \) in time domain are known\(^{(n-o)}\). This paper focuses on the frequency determination through a difference equation using three successive samples \( x[k-2], \ldots, x[k] \) of the signal. Thus, the frequency is estimated from the curvature of the time signal.

With \( \omega \) being the angular frequency of the time signal, \( \Delta T \) the sample time and \( Q=\omega \Delta T \) the normalized angular frequency,

\[
x[k]=2 \cos Q \cdot x[k-1]-x[k-2],
\]

(1)

can be derived using the \( z \)-transformation of a general harmonic oscillation

\[
x[k]=\alpha \cos Qk
\]

(2)

and \( \omega \) can be obtained analytically as

\[
\omega=Q \frac{1}{\Delta T} \cos^{-1} \left( \frac{x[k]+x[k-2]}{2x[k-1]} \right).
\]

(3)

For \( 0<Q<\pi \) Shannon's law is fulfilled and the result of Eq.(3) remains unique. In practice, Eq.(3) is successively applied to a time series of the signal and the average is taken.

As a result, the frequency identification by means of a difference equation leads to a non-iterative and simple algorithm with a continuous frequency resolution. However, it shows a high sensitivity to noise and therefore is not applicable with high sampling rates. Further, it is restricted to oscillations of a single frequency. Hence, a preceding signal processing is necessary.

3.3 Combined frequency identification

The above mentioned requirements, i.e. high frequency resolution combined with noise insensitivity, are neither fulfilled with a single time domain or frequency domain approach. However the disadvantages of the time domain identification can be compensated as follows by a frequency domain signal preprocessing.

After determining the wave bands of the occurring oscillations using the DFT, for each frequency a signal preprocessing is established including bandpass filtering and consecutive subsampling. The filter separates the corresponding oscillation from the mixture and thus reduces the noise power. Subsequently, the signal-to-noise ratio is further decreased using optimal subsampling with a sample frequency of about four times the signal frequency. The signal is then well conditioned for reliable continuous frequency determination in the time domain by use of the difference Eq.(3).
4. Disturbance Compensation

The compensation task itself is performed by tracking compensation oscillators (TCO). A TCO is characterized by two parameters: its angular frequency \( \omega_r \) and complex Fourier coefficient \( C_r \). To track the oscillator to a disturbance \( d(t) \), it is necessary to identify the disturbance by frequency \( \omega d \) and coefficient \( C_d \).

4.1 Disturbance identification

At the plant input, the unknown harmonic disturbance \( d(t) \) is supposed to be present. Further, the (known) compensation signal \( r(t) \) is applied. Then the plant input \( u(t) \) is given by

\[ u(t) = d(t) + r(t) = C_d e^{j \omega_d t} + C_r e^{j \omega_r t} \]  

(4)

With the complex plant transfer function \( G(j \omega) \), the plant output \( y(t) \) results as

\[ y(t) = G(j \omega_d) C_d e^{j \omega_d t} + G(j \omega_r) C_r e^{j \omega_r t} \]  

(5)

For small frequency mismatches \( \Delta \omega = \omega - \omega_r \), \( G(j \omega_d) \approx G(j \omega_r) \) can be assumed. For the determination of \( C_d \), the measured plant output

\[ \text{Re} \{ y(t) \} = \frac{1}{2} \left[ G(j \omega_d) (C_d e^{j \omega_d t} + C_r e^{j \omega_r t}) + G^*(j \omega_r) (C_d e^{-j \omega_d t} + C_r e^{-j \omega_r t}) \right] \]  

(6)

is demodulated with the normalized reference signal \( m(t) = e^{j \omega_d t} \), resulting in a complex signal

\[ A(t) = \frac{1}{2} \left[ G(j \omega_r) (C_d e^{j (\omega_d - \omega) t} + C_r e^{j (\omega_d + \omega) t}) + G^*(j \omega_r) (C_d e^{-j (\omega_d - \omega) t} + C_r e^{-j (\omega_d + \omega) t}) \right] \]  

(7)

with \((\cdot)^*\) denoting the complex conjugate. Filtering of the modulated signal with a low-pass filter of cut-off frequency \( \omega_c < \omega_r \) then results in the complex signal

\[ A(t) = \frac{1}{2} G^*(j \omega_r) (C_d e^{j \omega_c t} + C_r) \]  

(8)

At this point, \( \Delta \omega \) shall be assumed to be negligible. For a complete compensation \( A(t) \) has to be iteratively minimized. Thus, for \( r(t) \) to compensate \( d(t) \), the Fourier coefficient \( C_r \) has to match \(-C_d\). Resolving Eq. (8) for two successive iteration steps \( k \) and \( k+1 \) leads to

\[ C_r[k+1] = C_r[k] - V_c \frac{\hat{A}^*(t)}{G^*(j \omega_r)}, 0 \leq V_c \leq 1 \]  

(9)

where an additional feedback gain \( V_c \) has been introduced. A dead-beat design is given by \( V_c = 1 \), correcting the coefficient mismatch in a single step. For robustness reasons it is sometimes better to choose a slower tracking dynamic.

The compensation time step \( \Delta T_c \) needs not to be the sample time of the position control and needs not to be constant either. In a multitasking control system the tracking can be performed as a lower priority background task with the time step depending on the CPU load.

For the low-pass filtering of \( A(t) \) a linear phase is vital, so here a FIR implementation is first choice. Furthermore, the frequency response \( G(j \omega) \) of the plant is required, which can be easily measured using a stepped sine excitation. For the AMG it is of simple form (cp. Fig.5). Here, the inverse frequency response from actuators to sensors is sufficient.

4.2 Frequency tracking

As an extension to the Fourier coefficient tracking described above, a similar tracking of the disturbance frequency \( \omega_r \) is possible. When the frequency mismatch \( \Delta \omega \) does not vanish, \( \hat{A}^*(t) \) is not constant but of the form \( \hat{A}^*(t) = \hat{A} e^{j \Delta \omega t} \).

Thus, between two consecutively determined Fourier coefficients \( C_r[k] \) and \( C_r[k+1] \) at time steps \( T \) and \( T + \Delta T_c \), a phase shift \( \Delta \phi = \Delta \omega \Delta T_c \) occurs. To correct the frequency mismatch a tracking formulation similar to Eq. (9) can be formulated,

\[ \omega_r[k+1] = \omega_r[k] + V_e \frac{\Delta \phi}{\Delta T_c}, 0 \leq V_e \leq 1 \]  

(10)

Again, \( V_e = 1 \) characterizes a dead-beat design.

4.3 Tracking of harmonic signals

In principle it is possible to track harmonic signals, i.e., modulating the plant output \( y(t) \) not only with the base reference signal \( m \) but also with its multiples \( m_i = e^{j i \omega_r} \). Thus, not only the basic Fourier coefficient \( C_d \) but also the higher coefficients \( C_{d,i} \) can be determined. Still, the frequency tracking should only be performed for the base harmonic. However, simulations as well as measurements show this synchronous tracking of harmonics to be less robust than the independent tracking of harmonics by different TCOs.

5. Practical Realization

As mentioned above, the identification and compensation algorithm is to be performed not at the position control sample rate, but as a lower priority background task. Therefore, an advanced multitasking environment with automatic CPU sharing is required.

5.1 RTOS-UH task concept

RTOS-UH is a priority driven real-time operating system and first choice for advanced control applications. The distribution of the CPU time is based on the priority of every task. Extensive task synchronization methods are provided by semaphores variables and high-level commands for task activation, prevention and scheduling.

The frequency identification and TCO tracking can be performed in lower priority tasks and take several time steps of the position control. Newly determined parameters for the TCOs are handed over asynchronously when available.
5.2 Compensation scheduling

The scheduling of the compensation algorithm is based on three priority levels. The highest priority is given to the position control task, where besides the control itself the actual output values of the TCOs are being determined from the corresponding Fourier coefficient and frequency.

In a second level of lower priority, the TCO tracking is performed. Here, a sequence of the position signal is modulated with the reference signal $m(t)$ and low-pass filtered using a linear phase FIR filter.

At the lowest priority level the frequency determination is performed looking cyclically for new disturbances and scheduling TCOs to compensate detected ones. An arbitrary number of TCOs can be combined, each being attended by a separate task.

5.3 Smart oscillator surveillance

The chosen task concept is very flexible concerning different kinds of disturbances and allows for a surveillance of the oscillators. The high-level scheduler switches oscillators on when a new disturbance is detected and off when the disturbance disappears. Thus, it is possible to track a number of different independent disturbances as long as their frequency difference is not too small. The scheduler is as well able to decide about the significance of disturbances. When a new disturbance of large amplitude is detected, a TCO may be switched dynamically to the new frequency.

The parameters of the algorithm can easily be adapted to the specific problem. When a fast transient behavior is desired, a rougher frequency resolution for the DFT and a short TCO update time $\Delta T_e$ should be selected. Still, for low disturbance frequencies, $\Delta T_e$ must not be chosen too small as then the compensation steps interfere with the disturbance. For an improved long-time behavior $\Delta T_e$ should be chosen larger resulting in slower tracking.

6. AMG Measurement Results

To verify the proposed dynamic compensation algorithm, measurements have been performed on the AMG. The sample frequency of the position control was fixed to $f_s=5$ kHz. The compensation algorithm has been implemented for the vertical (z) degree of freedom of the AMG using eight independent TCOs and a TCO update time of $\Delta T_e=0.2$ s. The DFT was implemented using a 2 048-point FFT.

The scheduler was fixed to monitor a position amplitude of 0.5 $\mu$m for each frequency. After enabling a TCO, it was given four correction steps to reach a steady state before the next one was enabled. Thus, a robust transient behavior is achieved as the TCOs do not interfere with each other too much. For the same reason, the frequency correction according to Eq. (10) was enabled only every second correction step corresponding to Eq. (9).

The disturbances were applied via the guide's magnets. A constant disturbance acceleration amplitude of 5 m/s$^2$ has been chosen for all frequencies. Thus, the equivalent force is significantly higher compared to the force in the milling process of Fig. 4, as the resulting deflections in Fig. 6 - 9 are larger by a factor of 5 - 10.

6.1 Single frequency disturbances

Figure 6 shows the transient behavior of the algorithm when a disturbance signal of single frequency is applied. After a detection phase of about 0.25 s, where the high-level frequency identification is performed, a free TCO is enabled, which compensates the disturbance partially. A small frequency mismatch occurred in the identification and thus the amplitude grows again. In the next step, amplitude and phase are newly determined and the frequency corrected. The dead-beat characteristic of the correction is obvious, as in the third step amplitude and phase are matched as well. The position signal afterwards has the same level as before the disturbance was applied—the level of the sensor noise.

![Fig. 6 Measured compensation of a disturbance of single frequency $f_d=20$ Hz](image)

![Fig. 7 Measured compensation of a disturbance of two frequencies (20 Hz and 40 Hz)](image)
6.2 Multiple frequency disturbances

When disturbances consisting of multiple frequencies are applied, the algorithm compensates one frequency after another in the same way as in the single frequency case. Figure 7 shows the compensation of two frequencies. First the larger disturbance of \( f_1 = 20 \) Hz is compensated, then the other at \( f_2 = 40 \) Hz. The same is valid for four frequencies in Fig. 8. Here, the reaction to a suddenly disappearing disturbance is visible at \( t = 10 \) s: The oscillators are immediately switched off after one compensation step.

Since eight independent TCos have been implemented, eight different frequencies can be identified and the corresponding disturbances compensated, cp. Fig. 9. As the experiments show, between 0.7 s and 1.5 s is needed for each frequency, depending on the complexity of the disturbance signal. The settling time depends on the parametrization of the algorithm. When only few oscillators are used, a faster settling time can be achieved.

7. Conclusions

The measurements performed show the proposed algorithm to be very reliable, fast and robust. It can compensate periodic disturbances of any shape, and even several independent disturbances at the same time in a frequency band from 5 Hz to over 1000 Hz. The only limit for the number of tracking oscillators is the CPU power available, since in non-realtime simulations up to 32 TCos have been tested. The smart scheduler takes care that the largest disturbances, i.e. those resulting in the largest deflection, are cancelled first.

Further work has to include an extension to a multi-degree-of-freedom compensation, where first steps have already been taken. Then tests on a milling machine will be possible to prove the suitability of the algorithm for machining applications.

A new High Speed Cutting milling machine is currently under construction where an active magnetic guide will be implemented for one axis. With the results achieved so far the new machine axis will not only provide high damping but next to the already known infinite static stiffness also an infinite steady state stiffness even with periodic disturbances present.

References

(1) Tieste, K. D., Mehrgrößenregelung und Parameteridentifikation einer Linearmagnetführung, (in German), VDI-Fortschriftenberichte Reihe 8, Band 656, Düsseldorf, (1997).


