Active Progressive Wave Control of a Simply Supported Plate*

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This paper deals with the active control of progressive wave of a simply supported, lossless, thin distributed-parameter planar structure. It is the purpose of this paper to present a novel active wave control method for blocking the progressive wave, and thus a quiet zone (a specific region where vibration is to be prohibited) is produced. It is also the purpose of this paper to present a means to visualize wave propagation of the structure, whereby control effect for suppressing power flow in terms of vibration intensity, progressive wave, reflected wave and the combination of thereof becomes evident.

Key Words: Active Vibration Control, Active Wave Control, Simply Supported Plate, Progressive Wave Control, Vibration Energy, Quiet Zone

1. Introduction

From a viewpoint of a wave control method, this paper addresses the active vibration control of a simply supported, lossless, thin distributed-parameter planar structure. Unlike a conventional modal-based vibration control method⁽¹⁾⁻⁽³⁾, this method falling on the category of a power flow control has the potential for suppressing all the vibration modes of the structure. It is a common practice of the conventional power flow control⁽⁴⁾⁻⁽⁸⁾ to pursue the minimization of vibration intensity appearing on the structure. However, when the vibration intensity becomes zero as an extreme state of the control, two phenomena give rise to; one is that vibration energy accumulated in the structure is completely suppressed, and the other is that standing wave is completely dominant over the structure and hence the structural modes are thor-

oughly excited. As a result, the final goal in an attempt to suppress the vibration intensity becomes obscure; it might result in the vibration suppression or vibration excitation of the structure.

To cope with this problem, this paper presents a progressive wave control approach; minimization of the progressive wave element always leads to the minimization of structural modes. Particular emphasis is placed on the confinement of external energy, a cause of exciting vibration modes, into a restricted area. This paper begins by deriving the solution of wave element of a distributed-parameter planar structure. With the solution of wave element, progressive waves, reflective waves and the interfered waves are discussed, and the modal excitation mechanism induced by the interfered waves is shown. Relevance between the vibration energy and progressive wave element is clarified, and the minimization of progressive waves element is found to be the minimization of structural vibration. Next, this paper presents a new method for blocking the progressive wave, and thus a quiet zone (a specific region where vibration is to be prohibited) is produced. Applying the active wave control method, progressive waves are significantly suppressed in the quiet zone, and hence the structural modes cannot be excited due to the imbalance between the structural mode shape and the standing

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wave shape. Finally, this paper shows the dynamical compliance at a specific point in the quiet zone in case the progressive wave is blocked, demonstrating 23 vibration modes in the frequency range up to 500 Hz are completely suppressed.

2. Solution of Wave Element

2.1 Derivation of the solution of wave element

Consider a simply supported, thin, lossless, steel rectangular plate of $L_x (=0.612 \text{ m})$ and $L_y (=1.212 \text{ m})$ in the x and y directions, respectively, and of thickness $h (=0.0042 \text{ m})$. If a displacement $w(\mathbf{r})$ at the location $\mathbf{r}=(x, y)$ of a plate subjected to a primary force f_d , acting at $\mathbf{r}_d=(x_d, y_d)$ can be written in terms of (m, n) vibration mode as

$$w(\mathbf{r}, t) = w(\mathbf{r})e^{j\omega t} = f_d W_d(\mathbf{r})e^{j\omega t} \quad (1)$$

where

$$W_d(\mathbf{r}) = \sum_{m,n} w_{d,mn} \sin \alpha_m x \sin \beta_n y \quad (2)$$

$$w_{d,mn} = \frac{4}{\rho h L_x L_y (\omega_{mn}^2 - \omega^2)} \sin \alpha_m x_d \sin \beta_n y_d \quad (3)$$

$$\alpha_m = \frac{m\pi}{L_x}, \quad \beta_n = \frac{n\pi}{L_y} \quad (4)$$

$$\omega_{mn} = \sqrt{\frac{D}{\rho h} (\alpha_m^2 + \beta_n^2)} \quad (5)$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (6)$$

and where ρ is the material density; ω_{mn} , ω , m and n are (m, n) modal resonance frequency, excitation frequency and modal indices in the x and y . The resonance frequencies in the range up to 500 Hz are listed in Table 1; D , E and ν are the bending stiffness of the panel, Young's modulus and Poisson's ratio.

Deriving of a solution of wave element, we consider a displacement w , and it is rewritten as

$$w(\mathbf{r}) = f_d \sum_{m,n} w_{d,mn} \sin \beta_n y \frac{j(e^{-j\alpha_m x} - e^{j\alpha_m x})}{2} \quad (7)$$

where $e^{-j\alpha_m x}$ and $e^{j\alpha_m x}$ are considered to be elements of a progressive wave and a reflective wave, respectively

in the x direction. Then, the solutions of the progressive wave element and the reflective wave element in the x direction are defined as

$$\phi_{+x}(\mathbf{r}) = f_d \eta_{+x,d}(\mathbf{r}) \quad (8)$$

$$\phi_{-x}(\mathbf{r}) = f_d \eta_{-x,d}(\mathbf{r}) \quad (9)$$

where

$$\eta_{+x,d}(\mathbf{r}) = \sum_{m,n} w_{d,mn} \sin \beta_n y \cdot e^{-j\alpha_m x} \quad (10)$$

$$\eta_{-x,d}(\mathbf{r}) = - \sum_{m,n} w_{d,mn} \sin \beta_n y \cdot e^{j\alpha_m x} \quad (11)$$

and the solution of each wave element in the y direction is defined by the same way. Using these solutions of wave element in the x and y directions, the progressive wave vector and the reflective wave vector of the plate can be defined as

$$\phi_+(\mathbf{r}) = [\phi_{+x}(\mathbf{r}), \phi_{+y}(\mathbf{r})]^T \quad (12)$$

$$\phi_-(\mathbf{r}) = [\phi_{-x}(\mathbf{r}), \phi_{-y}(\mathbf{r})]^T \quad (13)$$

Next, the solution of wave element with n control forces, which act at $\mathbf{r}_c=(x_{cn}, y_{cn})$, are given by

$$\phi_{+x}(\mathbf{r}) = \mathbf{f} \boldsymbol{\eta}_{+x}^T(\mathbf{r}) \quad (14)$$

$$\phi_{+y}(\mathbf{r}) = \mathbf{f} \boldsymbol{\eta}_{+y}^T(\mathbf{r}) \quad (15)$$

$$\phi_{-x}(\mathbf{r}) = \mathbf{f} \boldsymbol{\eta}_{-x}^T(\mathbf{r}) \quad (16)$$

$$\phi_{-y}(\mathbf{r}) = \mathbf{f} \boldsymbol{\eta}_{-y}^T(\mathbf{r}) \quad (17)$$

where

$$\boldsymbol{\eta}_{\pm x}(\mathbf{r}) = [\eta_{\pm x,d}(\mathbf{r}) \quad \eta_{\pm x,c1}(\mathbf{r}) \quad \cdots \quad \eta_{\pm x,cn}(\mathbf{r})] \quad (18)$$

$$\boldsymbol{\eta}_{\pm y}(\mathbf{r}) = [\eta_{\pm y,d}(\mathbf{r}) \quad \eta_{\pm y,c1}(\mathbf{r}) \quad \cdots \quad \eta_{\pm y,cn}(\mathbf{r})] \quad (19)$$

$$\mathbf{f} = [f_d \quad f_{c1} \quad \cdots \quad f_{cn}] \quad (20)$$

Each control force f_{cn} formulated by a feedforward control method is written as

$$f_{cn} = \alpha_{cn} e^{j\phi_{cn}} f_d \quad (21)$$

where α_{cn} and ϕ_{cn} mean the gain and phase of the controller, respectively.

2.2 Distribution of the wave element

Using the solutions of wave element, this section shows the characteristics of the each wave elements by visualizing wave propagation of the structure. Figure 1 shows the distribution patterns of the progressive wave vector (a), the reflective wave vector (b) and the interfered wave vector (c) over the plate, where the disturbance force f_d [10 N, 35 Hz: resonance frequency of (1, 1) mode] acts at the location $\mathbf{r}_d=(0.163 \text{ m}, 0.273 \text{ m})$. As shown in Fig. 1(a) and (b), the progressive wave moves from center to edge of the plate, while the reflective wave moves from the edge to the center of the plate. As shown in Fig. 1(c), the interfered wave, a combination of two waves, vanishes all over the plate. It means there is no transfer element of wave, and standing waves completely dominate over the plate.

2.3 Wave solution and vibration energy

Consider the control effect for suppressing the vibration of the plate from the viewpoint of vibration energy stocked in the plate and wave elements. Moreover, the time average vibration energy V , ac-

Table 1 Mode number and modal frequencies of the pinned panel with the dimension of $0.612 \text{ m} \times 1.212 \text{ m} \times 0.0042 \text{ m}$

Mode number	Mode	Modal frequency Hz	Mode number	Mode	Modal frequency Hz
1	(1, 1)	34.49	13	(2, 5)	285.12
2	(1, 2)	55.51	14	(3, 3)	310.42
3	(1, 3)	90.55	15	(3, 4)	359.47
4	(2, 1)	116.94	16	(2, 6)	362.20
5	(2, 2)	137.96	17	(1, 7)	370.85
6	(1, 4)	139.60	18	(3, 5)	422.54
7	(2, 3)	173.00	19	(4, 1)	446.74
8	(1, 5)	202.67	20	(2, 7)	453.30
9	(2, 4)	222.05	21	(4, 2)	467.76
10	(3, 1)	254.36	22	(1, 8)	475.97
11	(3, 2)	275.38	23	(3, 6)	499.62
12	(1, 6)	279.75

Fig. 1 Wave element distribution at 35 Hz. (a) Progressive wave, (b) Reflective wave, (c) Interfered wave

accumulated in the plate, is obtained by the superposition of kinetic energy V_k and potential energy V_p

$$V = V_k + V_p \quad (22)$$

where

$$V_k = \frac{1}{2} \iint_D \rho h \{ \dot{w}(\mathbf{r}, t) \} \{ \dot{w}^*(\mathbf{r}, t) \} d\mathbf{r} \quad (23)$$

$$V_p = \frac{1}{2} \iint_D \left\{ M_x(\mathbf{r}, t) \frac{\partial^2 w^*(\mathbf{r}, t)}{\partial x^2} + M_y(\mathbf{r}, t) \frac{\partial^2 w^*(\mathbf{r}, t)}{\partial y^2} + 2T_{xy}(\mathbf{r}, t) \frac{\partial^2 w^*(\mathbf{r}, t)}{\partial x \partial y} \right\} d\mathbf{r} \quad (24)$$

Figure 2 shows the control effect for suppressing the wave elements at the disturbance point (a) and the vibration energy (b), plotted as a function of control force phase difference ϕ varying from 0 to 360 degree while the gain α is fixed at 10 N. Where the control force f_c acts at a point symmetrical about the disturbance point is given as Eq. (21).

As shown in Fig. 2(a), at the phase difference 0, 180 and 360 degree, interfered wave element is zero; it means that there is no transfer element of vibration, and the standing wave is thoroughly dominant. In the

Fig. 2 Wave elements and vibration energy versus the phase difference of feedforward control at 35 Hz. (a) Wave elements, (b) Vibration energy

end, there is no vibration intensity over the plate. However, Fig. 2(b) shows that the vibration energy accumulated in the plate has its maximum value at the phase difference 0 and 360 degree. It shows that the vibration intensity becoming zero leads to two phenomena; one is that vibration energy accumulated in the structure is completely suppressed, and the other is that standing wave is completely dominant over the structure and hence the structural modes are thoroughly excited. However, as shown in Fig. 2(a) again, progressive wave element and reflective wave element are excited at 0 and 360 degree, and suppressed at 180 degree and hence the minimization of the progressive wave always leads to the minimization of the vibration energy.

3. The Blocking Progressive Wave Method

3.1 Proposition of the blocking progressive wave method

If the progressive waves crossing a designated boundary that divides off the region in the plate are eliminated, there is no progressive wave in a region across the boundary. As suppressing progressive waves element always leads to suppressing vibration energy, a region where vibration is to be prohibited is produced, and we call this region a quiet zone.

3.2 Derivation of the blocking progressive wave method

Figure 3 illustrates the simply supported plate model. Disturbance force f_d and control forces f_c act on the opposite of a quiet zone, and the control boundary is located at $y=y_{cs}$. Then the evaluation function J is defined as an integral of square of the progressive wave element in the y direction at the control boundary, and is written as

$$J = \int_0^{L_x} \psi_{+y}(\mathbf{r}_{cs}, t) \psi_{+y}^*(\mathbf{r}_{cs}, t) dx \quad (25)$$

Using Eq.(15), Eq.(25) is rewritten as

$$J = \mathbf{f} \cdot \mathbf{A} \cdot \mathbf{f}^H \quad (26)$$

$$\begin{aligned} \mathbf{A} &= \int_0^{L_x} \boldsymbol{\eta}_{+y}^T(\mathbf{r}_{cs}) \boldsymbol{\eta}_{+y}^*(\mathbf{r}_{cs}) dx \\ &= \int_0^{L_x} \begin{bmatrix} \eta_{+y,d}(\mathbf{r}_{cs}) \eta_{+y,d}^*(\mathbf{r}_{cs}) & \cdots \\ \vdots & \ddots \\ \eta_{+y,cn}(\mathbf{r}_{cs}) \eta_{+y,cn}^*(\mathbf{r}_{cs}) & \cdots \\ \eta_{+y,d}(\mathbf{r}_{cs}) \eta_{+y,cn}^*(\mathbf{r}_{cs}) & \cdots \\ \vdots & \ddots \\ \eta_{+y,cn}(\mathbf{r}_{cs}) \eta_{+y,cn}^*(\mathbf{r}_{cs}) \end{bmatrix} dx \end{aligned} \quad (27)$$

and where the (i, j) element of matrix $\mathbf{A}$ of Eq.(27) is given by

$$\begin{aligned} &\int_0^{L_x} \eta_{+y,i}(\mathbf{r}_{cs}) \eta_{+y,j}^*(\mathbf{r}_{cs}) dx \\ &= \int_0^{L_x} \sum_{m,n,k,l} w_{i,mn} w_{j,kl} e^{-j(\beta_n - \beta_l)y_{cs}} \\ &\quad \times \sin a_m x \sin a_k x dx \end{aligned} \quad (28)$$

Since the integral terms of matrix $\mathbf{A}$ become zero when $m \neq k$, Eq.(28) is rewritten as

$$\begin{aligned} &\int_0^{L_x} \eta_{+y,i}(\mathbf{r}_{cs}) \eta_{+y,j}^*(\mathbf{r}_{cs}) dx \\ &= \frac{L_x}{2} \sum_{m,n,l} w_{i,mn} w_{j,ml} e^{-j(\beta_n - \beta_l)y_{cs}} \end{aligned} \quad (29)$$

Using Eq.(26), the partial derivatives of the evaluation function J with respect to the real and imaginary components of the several control forces yield

$$\frac{\partial \mathbf{f} \mathbf{A} \mathbf{f}^H}{\partial f_{cn,r}} = \mathbf{a}^{(i+1)} \mathbf{f}^H + \mathbf{f} \mathbf{a}_{i+1} \quad (30)$$

$$\frac{\partial \mathbf{f} \mathbf{A} \mathbf{f}^H}{\partial f_{cn,i}} = j \cdot \mathbf{a}^{(i+1)} \mathbf{f}^H - j \cdot \mathbf{f} \mathbf{a}_{i+1} \quad (31)$$

Fig. 3 Simply supported panel with disturbance force and control forces

respectively, where

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}^{(1)} \\ \vdots \\ \mathbf{a}^{(n+1)} \end{bmatrix} = [\mathbf{a}_1 \quad \cdots \quad \mathbf{a}_{n+1}] \quad (32)$$

The optimal force is found by setting that both of these derivatives are zero: that is,

$$\frac{\partial \mathbf{f} \mathbf{A} \mathbf{f}^H}{\partial f_{cn,r}} + j \frac{\partial \mathbf{f} \mathbf{A} \mathbf{f}^H}{\partial f_{cn,i}} = 0 \quad (33)$$

Therefore

$$\mathbf{f} \mathbf{a}_{i+1} = f_d \mathbf{a}_{i+1,1} + \mathbf{f}_c^{opt} \begin{bmatrix} a_{i+1,2} \\ \vdots \\ a_{i+1,n+1} \end{bmatrix} = 0 \quad (34)$$

Using Eq.(34), the optimum force vector $\mathbf{f}_c^{opt}$ is given by

$$\mathbf{f}_c^{opt,T} = - \begin{bmatrix} a_{2,2} & \cdots & a_{2,n+1} \\ \vdots & \ddots & \vdots \\ a_{n+1,2} & \cdots & a_{n+1,n+1} \end{bmatrix}^{-1} \begin{bmatrix} a_{2,1} \\ \vdots \\ a_{n+1,1} \end{bmatrix} f_d \quad (35)$$

3.3 Simulation of the blocking progressive wave method

Consider the control effect for suppressing the vibration using the proposed method. This section presents an example with the specification as follows; the location of the disturbance force f_d [10 N, 35 Hz: resonance frequency of (1, 1) mode] acts at $\mathbf{r}_d = (0.163 \text{ m}, 0.273 \text{ m})$, while five control forces act at $\mathbf{r}_c = (0.106 \text{ m}, 0.375 \text{ m})$, $(0.206 \text{ m}, 0.375 \text{ m})$, $(0.306 \text{ m}, 0.375 \text{ m})$, $(0.406 \text{ m}, 0.375 \text{ m})$, $(0.506 \text{ m}, 0.375 \text{ m})$, and the control boundary is designated at $y_{cs} = 0.930 \text{ m}$.

Figure 4 shows the analytical result of a distribution pattern of progressive wave element over the

Fig. 4 Distribution of progressive wave elements. (a) before control and (b) after control driven at 35 Hz

plate without control (a) and with control (b). As seen from this figure, the progressive waves excite around disturbance force area, however do not cross over the control boundary. The blocking progressive wave at the control boundary appears to be achieved. The maximum amplitude of progressive wave element under control is 0.58 mm corresponding to 2.3% of the case without control, moreover, in the quiet zone, it is 1.38 μm (0.2% of the case with out control).

Figure 5 illustrates the three-dimensional image of the absolute wave distribution, showing the generation of the quiet zone. The maximum amplitude is suppressed from 0.57 mm to 11.17 μm (1.9%), and the maximum amplitude in the quiet zone is reduced to 0.9 μm corresponding to 0.2% of the case without control. To see this more clearly, the shape of distribution

Fig. 5 Envelope of absolute displacement of a panel. (a) before control and (b) after control at 35 Hz

pattern is unbalanced obviously, and hence the structural modes cannot be excited due to the imbalance between the structural mode shape and the standing wave shape.

Figure 6 shows (a) the driving point compliance and (b) the dynamical compliance at specific point $r_m (=0.277 \text{ m}, 1.049 \text{ m})$ in the quiet zone. As shown in these figures, in the frequency range up to 500 Hz, 23 vibration modes are completely suppressed using the proposed method.

4. Conclusion

From a viewpoint of modal structural mechanism, aiming at a wave element, this paper presented two results as follows:

1. This paper presents the solution of wave element induced from the displacement of a simply supported, lossless, thin plate. Analyzing this wave element, the modal excitation mechanism induced by the interfered wave element is shown. Relevance between the vibration energy and progressive wave element is clarified, and the minimization of progressive waves element is found to be the minimization of structural vibration.

2. The blocking progressive wave method is presented. It is to suppress the progressive wave elements cross the location boundary of the plate. By using the solution of progressive wave element, the control law for this method is derived.

3. From an analytical point of view, the characteristics of this method are investigated. The quiet zone is produced at the designated location region, and hence the structural modes cannot be excited due to the imbalance between the structural mode shape and the standing wave shape.

Fig. 6 Frequency response of a panel before and after control. (a) Driving point compliance at $r_d=(0.163 \text{ m}, 0.273 \text{ m})$, (b) Compliance at $r_m=(0.277 \text{ m}, 1.049 \text{ m})$

4. In the frequency range up to 500 Hz, the control effect of this method is demonstrated using the dynamical compliance at a point in the quiet zone.

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