Analysis of Small Deformation of Helical Flagellum of Swimming *

Vibrio alginolyticus*

Yasunari TAKANO**, Kazuki YOSHIDA***, Seishi KUDO****,
Megumi NISHITOBA† and Yukio MAGARIYAMA††

The deformation of a flagellum of *Vibrio alginolyticus*, single-flagellate bacteria, is analyzed theoretically assuming the shape of the flagellum to be a circular helix. The viscous force exerted on the flagellum in aqueous fluid is estimated applying the resistive–force theory based on the Stokes flow. The moment of force in the flagellum are described in analytical expressions and also the curvature and the torsion of the deformed flagellum are expressed analytically according to the Kirchhoff rod model. The deformation of the flagellum is obtained numerically solving evolution equations which determine a space curve from the curvature and the torsion. Comparing variations of the pitch of helical flagella between the numerical solutions and the results of measurement, the flexural rigidity or the elastic bending coefficient for the flagellum of *Vibrio alginolyticus* is estimated.

** Key Words**: Swimming Bacterium, *Vibrio alginolyticus*, Flexible Flagellum, Flagellar Hydrodynamics, Kirchhoff Rod Model, Resistive–Force Theory

1. Introduction

Many bacteria swim in aqueous medium rotating helical flagella by use of rotary motors embedded in the cell surface(3). They move straight and sometimes change their direction by rotating the flagella in the opposite direction. The movement of bacteria as well as the mechanism of flagellar rotary motors is taken interest in by not only biophysicists but also engineers in the field of nanotechnology(3).

Recently, Magariyama et al. measured the swimming speed of *Vibrio alginolyticus*, single-flagellate bacteria, and found that the backward swimming speed were 1.5 times greater than the forward swimming speed on average(9). They considered such conceivable causes of the difference of the speed as asymmetric torque characteristics of a flagellar motor relating to the rotational direction and deformation of a flagellum due to viscous force exerted in the opposite direction of movement.

Takano and Goto carried out numerical analysis for the deformation of a flagellum rotating in aqueous environment(9). They estimated the speed of the forward and the backward movement, taking account of the deformation of the flagellum, and concluded that such a large difference of the velocity is hardly caused by the flagellar deformation.

Concerning the deformation of a flagellum, Hoshikawa and Kamiya analyzed the elongation of a flagellum subjected to fluid flow, modeling a flagellar filament as a chain of segments, and obtained a relation between the elongation and the flow rate(10). They determined the modulus of rigidity of the flagellar filament of Salmonella comparing the elongation between the theory and the experiment.

Recently, Kudo et al. have observed swimming

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** Faculty of Engineering, The University of Shiga Prefecture, 2500 Hassakacho, Hikone, Shiga 522-8533, Japan. E-mail: tkano@mech.shu.ac.jp
*** Graduate Student, The University of Shiga Prefecture
**** Faculty of Engineering, Toin University of Yokohama, 3614 Kuroganecho, Aobaku, Yokohama, Kanagawa 225-8502, Japan
† Graduate Student, Toin University of Yokohama
** National Food Research Institute, 2-1-2 Kannondai, Tsukuba, Ibaraki 305-8642, Japan

**Vibrio alginolyticus** using laser dark-field microscopy and measured a wavelength or a pitch of the helical flagellum which varies according to the forward and the backward swimming speed\(^6\).

In the previous investigation of Takano and Goto\(^9\), they developed a numerical procedure to calculate the moment of force in a rotating flagellum in arbitrary form and to simulate deformation of the flagellum caused by the moments. In the present investigation, we simplify the analysis of the flagellar deformation assuming the shape of the flagellum to be a circular helix. We begin with describing the moments of force in the flagellum in analytical expressions and also expressing variations of the curvature and the torsion along the flagellum in terms of the moment based on the Kirchhoff rod model\(^9\). It is straightforward to obtain flagellar deformation by solving evolution equations for a space curve. Comparing variations of the pitch of helical flagella between the numerical simulations and measurements of Kudo et al.\(^6\), we estimate the flexural rigidity of the flagellum, which is denoted as the elastic bending coefficient in the biological literature.

### 2. Formulation of Flagellar Propulsion

#### 2.1 Resistive-force theory of flagellar propulsion

We assume that a bacterium consists of an ellipsoidal cell body and a helical single flagellum, and moves at the velocity of \(V_F\) rotating the flagellum at the angular frequency of \(\Omega_F\) relative to the fluid at rest, as shown in Fig. 1. The velocity \(v\) of an element of the flagellum in the position \(r\) measured from the position of the flagellar motor embedded on the cell surface can be expressed in terms of \(V_F\) and \(\Omega_F\) as follows:

\[
v = V_F + \Omega_F \times r
\]  

(1)

We apply the resistive-force theory\(^6\) to estimate the force exerted on the flagellum. The velocity of the flagellum is decomposed into the tangential and the perpendicular component such as

\[
v = v_t + v_L
\]  

(2)

and the force per unit length of the flagellum is evaluated as

\[
\mathbf{f} = -C_t v_t - C_n v_L
\]  

(3)

where \(C_t\) and \(C_n\) are the tangential and the normal coefficient of resistance\(^6\), respectively, which are determined from the slender body theory of the Stokes flow. The values of the coefficients used in a numerical example will be written in the section of results and discussion.

Integrating \(\mathbf{f}\) and \(\mathbf{r} \times \mathbf{f}\) along the flagellum, the force \(\mathbf{F}_F(0)\) and the torque \(\mathbf{M}_F(0)\) which act on the cell body at the root of the flagellum are obtained as functions of \(V_F\) and \(\Omega_F\). They can be expressed as

\[
\begin{align*}
\mathbf{F}_F(0) & = -(C_t V_t + C_n V_L) \mathbf{r}_F \\
\mathbf{M}_F(0) & = -(C_t V_t + C_n V_L) r \mathbf{r}_F \times \mathbf{r}_F \\
\end{align*}
\]  

(4)

in terms of the length \(l\), the radius \(r\) and the pitch angle \(\beta\) of the helix. Here \(\mathbf{F}_F(s)\) and \(\mathbf{M}_F(s)\) are defined afterward in Eqs. (23) and (24), respectively. The torque \(\mathbf{M}_F(0)\) induces the counter rotation of the cell body to the flagellar rotation.

The force and the torque produced on the body in the form of an ellipsoid at the major diameter of \(2a\) and the minor diameter of \(2b\) are expressed as

\[
\begin{align*}
\mathbf{F}_S & = -6\pi C_F \mu a V_F \\
\mathbf{M}_S & = -6\pi C_M \mu b^2 \Omega_S
\end{align*}
\]  

(6)  

(7)

when it moves at the speed of \(V_F\) in the direction of the symmetric axis and rotates at the angular frequency \(\Omega_S\) around the symmetric axis. Here, \(\mu\) is the viscosity of fluid and \(C_F\) and \(C_M\) are the coefficients of resistance determined from the singularity method of the Stokes flow\(^6\). The values used in a numerical example will be written in the section of results and discussion.

#### 2.2 Propulsive speed of bacteria

As the fluid motion generated by bacteria is very small, the propulsive speed as well as the angular frequency of the flagellum is determined so that the force and the torque equilibrate on the whole, namely,

\[
\begin{align*}
\mathbf{F}_F(0) + \mathbf{F}_S & = 0 \\
\mathbf{M}_F(0) + \mathbf{M}_S & = 0
\end{align*}
\]  

(8)  

(9)

There is a relation among the rotation rate \(\Omega_S\) of the rotary motor, the angular frequency \(\Omega_F\) of the flagellum and angular frequency \(\Omega_S\) of the cell body as follows

\[
\Omega_S = \Omega_F - \Omega_B
\]  

(10)

Substituting Eqs. (4) - (7) into Eqs. (8) and (9), the propulsive speed \(V_F\) and the angular frequency \(\Omega_F\) of the flagellum are determined from \(\Omega_S\).

### 3. Formulation for Deformation of Flagellum

#### 3.1 Force and the moment of force in a flagellum

In order to obtain analytical expressions for the force and the moment of force in the cross section of a flagellum, we assume the shape of the flagellum to be a circular helix, neglecting the portion of the hook between the flagellar motor and the flagellar filament. Let the \(x_3\)-axis of the Cartesian coordinates \(O-x_1x_2x_3\)

![Fig. 1 Schematic of a swimming bacterium](image)


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be identified with the axis of such a left-handed helix as the helical flagellum of *Vibrio alginolyticus*, as shown in Fig. 2. Denoting the unit vectors of the Cartesian coordinates by $e_i$, $e_s$, and $e_\rho$, we express the position vector $r_f$ in terms of the unit vectors $e_i$ and $e_\rho$ used in the cylindrical polar coordinates as follows

$$r_f(s) = r_i(s) + \frac{\lambda}{2\pi} \frac{s}{\rho} e_\rho$$  \hspace{1cm} (11)

$$e_i(s) = e_i \cos \frac{s}{\rho} - e_\rho \sin \frac{s}{\rho}$$  \hspace{1cm} (12)

$$e_\rho(s) = e_i \sin \frac{s}{\rho} + e_\rho \cos \frac{s}{\rho}$$  \hspace{1cm} (13)

$$\rho = \sqrt{\rho^2 + (\lambda/2\pi)^2}$$  \hspace{1cm} (14)

where $\rho$ and $\lambda$ show the radius and the pitch of the helix, respectively, and $s$ indicates the arc length along the helical curve. We assume that the bacterium moves in the direction of the axis of a flagellar helix at the speed $V_f e_\rho$ rotating the flagellum at the angular frequency $\Omega e_\rho$ relative to the fluid at rest. The velocity $\mathbf{v}$ of the flagellum can be written from Eq. (1) as follows

$$\mathbf{v}(s) = V_f e_\rho + r_f \Omega e_\rho$$  \hspace{1cm} (15)

The Frenet triad consisting of the tangential unit vector $\mathbf{t}$, the normal unit vector $\mathbf{n}$ and the binormal unit vector $\mathbf{b}$ of the helical curve is derived from Eq. (11) as follows

$$\mathbf{t}(s) = \sin \beta e_\beta(s) + \cos \beta e_\rho$$  \hspace{1cm} (16)

$$\mathbf{n}(s) = -e_t(s)$$  \hspace{1cm} (17)

$$\mathbf{b}(s) = -\cos \beta e_\beta(s) + \sin \beta e_\rho$$  \hspace{1cm} (18)

$$\sin \beta = \frac{-r}{\rho}, \cos \beta = \frac{\lambda}{2\pi \rho}$$  \hspace{1cm} (19)

Separating the velocity vector $\mathbf{v}$ into the tangential component and its perpendicular component according to Eq. (2) and employ the relation (3), we obtain the force per unit length of the flagellum as follows

$$f(s) = f_t e_\beta(s) + f_\rho e_\rho$$  \hspace{1cm} (20)

$$f_t = -(C_t \sin^2 \beta + C_e \cos^2 \beta) r_\Omega^2$$  \hspace{1cm} (21)

$$f_\rho = -(C_t \cos \beta \sin \beta \cos \beta V_f)$$  \hspace{1cm} (22)

where $C_t$ and $C_e$ indicate the tangential and the normal force coefficients, respectively. The resultant internal force in the cross section of the flagellum at $r_f$ can be expressed as the integral of Eq. (20) from the position $s$ to the tip $(s=l)$ of the flagellum as follows

$$F_f(s) = \int_s^l f(s') ds'$$  \hspace{1cm} (23)

Integrating the moment of force as shown in the Fig. 3 from the position $s$ to the tip $(s=l)$ of the flagellum, the resultant internal moment in the cross section of the flagellum at $r_f$ can be expressed as

$$M_f(s) = \int_s^l [r_f(s') - r_f(s)] \times f(s') ds'$$  \hspace{1cm} (24)

Considering such relations derived from Eqs. (12) and (13) as

$$e_t = -\frac{\rho}{s} \frac{de_\rho}{ds}, \quad e_\rho = \frac{\rho}{s} \frac{de_\beta}{ds}$$  \hspace{1cm} (25)

and integrating by parts, we obtain the resultant force as well as the resultant moment of force in the flagellum as follows

$$F_f(s) = (l-s) [d e_t - f_\rho (e_t(l) - e_t(s))]$$  \hspace{1cm} (26)

$$M_f(s) = (l-s) \left[ rf_t e_\beta + rf_\rho e_\rho - \frac{\lambda}{2\pi} f_\rho e_\rho (l) \right]$$  \hspace{1cm} (27)

$$M_f(l)$$  \hspace{1cm} (28)

$M_f$ in Eq. (27) can be approximated as

$$M_f(l) \approx \left[ rf_t e_\beta + rf_\rho e_\rho - \frac{\lambda}{2\pi} f_\rho e_\rho (l) \right] \sin \beta$$  \hspace{1cm} (29)

because the terms proportional to $l-s$ dominate the others except in close vicinity to the flagellar tip, as will be verified in the section of results and discussion.

Making use of the scalar products between Eq. (28) and the unit vectors of Eqs. (16) - (18), the tangential, the normal and the binormal components of the resultant moment of force in the cross section of the flagellum can be written as follows,
\[ M_i(s) = (l - s) \left( r_f s \sin \beta + r_f \cos \beta \right) - \frac{\lambda}{2 \pi \mu} \left( \frac{d}{r} \right) \sin \beta \cos \frac{l - s}{\rho} \]  
\[ M_s(s) = (l - s) \frac{\lambda}{2 \pi \mu} s \sin \frac{l - s}{\rho} \]  
\[ M_\phi(s) = (l - s) \left( r_f s \sin \beta - r_f \cos \beta \cos \frac{l - s}{\rho} \right) \]  

where \( M_i \) denotes the torque, while \( M_s \) and \( M_\phi \) the bending moments.

### 3.2 Deformation of flagellum

The deformation of a flagellum is mainly produced by bending and torsion due to the moment acting on the cross section. Assuming that the flagellum is an isotropic elastic body, we apply the Kirchhoff rod model to obtain the constitutive relation for the moment of force as follows

\[ M_i = E I (\kappa_\alpha - \kappa_\beta) q + E I k_\alpha p + G J (\omega - \tau) t \]  

where \( E \) is Young’s modulus and \( G \) is the modulus of rigidity. \( I \) and \( J \) are the moment of inertia of cross section. For circular rod of the diameter \( d \), the moment of inertia is given as

\[ I = \frac{1}{2} \frac{\pi d^4}{64} \]  

The curvature \( \kappa \) and the torsion \( \tau \) of the flagellum without deformation are written as respectively

\[ \kappa = \frac{r}{\rho}, \quad \tau = -\frac{\lambda/2}{\rho} \]  

where \( r \) and \( \lambda \) are the radius and the pitch of the circular helix, respectively, and \( \rho \) is defined in Eq.(14). We consider the orthogonal triad \( t, p \) and \( q \) which is identical with the Frenet triad \( t, n \) and \( b \) of the flagellum without deformation. The orthogonal triad \( t, p \) and \( q \) satisfies such evolution equations as

\[ \frac{d}{ds} \begin{bmatrix} t \, p \, q \end{bmatrix} = \begin{bmatrix} 0, \kappa, \omega \kappa, 0, \omega \kappa, 0 \end{bmatrix} \begin{bmatrix} t \, p \, q \end{bmatrix} \]  

In case of \( M_i = 0 \), Eq.(32) yields \( \kappa_\alpha = \kappa, \kappa_\beta = 0, \omega = \tau \). Consequently, the Eq.(35) is reduced to the Frenet-Serret equation, and the orthogonal triad \( t, p, q \) becomes identical with the Frenet triad \( t, n, b \).

From Eq.(32), \( \kappa_\alpha, \kappa_\beta, \omega \) can be written as

\[ \kappa_\alpha = \kappa + \frac{M_i(s)}{E I} \]  
\[ \kappa_\beta = \frac{M_s(s)}{E I} \]  
\[ \omega = \tau + \frac{M_\phi(s)}{G J} \]  

in terms of the moment of force which is expressed analytically in Eqs.(29) – (31). The tangential vector \( t \) of the deformed flagellum is obtained from Eq.(35). Besides, we obtain the shape of the deformed flagellum integrating \( t \) as follows

### Table 1 Parameter of bacterium

| Symbol | Parameter | \( \mu \) m
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2a</td>
<td>Cell length</td>
<td>2</td>
</tr>
<tr>
<td>2b</td>
<td>Cell width</td>
<td>1</td>
</tr>
<tr>
<td>d</td>
<td>Diameter of flagellar filament</td>
<td>0.032</td>
</tr>
<tr>
<td>l</td>
<td>Length of flagellar filament</td>
<td>5.5</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Pitch of flagellar helix</td>
<td>1.27</td>
</tr>
<tr>
<td>r</td>
<td>Radius of flagellar helix</td>
<td>0.233</td>
</tr>
</tbody>
</table>

### Table 2 Results of measurement of flagella

<table>
<thead>
<tr>
<th>Speed ( V_f (\mu m/s) )</th>
<th>Pitch ( \lambda (\mu m) )</th>
<th>Radius ( r (\mu m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.27</td>
<td>0.233</td>
</tr>
<tr>
<td>100</td>
<td>1.23</td>
<td>0.232</td>
</tr>
<tr>
<td>-100</td>
<td>1.31</td>
<td>0.235</td>
</tr>
</tbody>
</table>

\[ r_f(s) = \int_0^s t(s')ds' + r_f(0) \]  

It is straightforward to solve evolution Eq.(35) by using a mathematical software such as Mathematica (Wolfram Research, Inc.).

### 4. Results and Discussion

Table 1 shows the size of a bacterium used in a numerical example of the analysis. We consider water as the aqueous medium in which the bacteria swim and use \( \mu = 1.3 \times 10^{-3} \) kg m\(^{-1}\) s\(^{-1}\) for the viscosity coefficient so that we can obtain the tangential and the normal coefficients of resistance for the flagellum \( C_{ri} = 1.38 \) and \( C_{r\|} = 2.25 \) as well as the drag and the torque coefficients \( C_{ri} = 0.60 \) and \( C_{r\theta} = 0.81 \) for calculations of the flagellar propulsion of the bacterium.

Kudo et al. observed helical flagella of swimming Vibrio alginolyticus and measured the pitch \( (\lambda) \) and the radius \( (r) \) of flagellar helix at about one turn from the tip\(^{16} \). Table 2 shows a typical relation between the size of bacteria and the swimming speed obtained from a regression analysis where the positive and the negative value of \( V_f \) indicate the forward and the backward movement, respectively.

### 4.1 Results for forward swimming by CCW flagellar rotation

In the following, results are shown for the case of the angular velocity \( \Omega_{ei}/2\pi = 700 \) rps of a flagellar motor which rotates in the counterclockwise observed from the outside of the cell body. The cell body of the bacterium rotates at \( \Omega_{e}/2\pi = -64 \) rps relative to stationary fluid in the opposite direction to the flagellum, and consequently the flagellum rotates at \( \Omega_{e}/2\pi = 636 \) rps relative to the fluid. The bacterium moves forward at the velocity of \( V_f = -103.1 \mu m/s \). The resistive force per 1 \( \mu m \) of the flagellum is estimated as \( f_h = -2.96 \times 10^{-5} \) pN and \( f_h = -0.276 \) pN.

Figure 4 shows the moment of force on the cross
Fig. 4 Distributions of the moment of force along flagellum

Fig. 5 Variations of tangential vector along flagellum rotating counterclockwise (solid lines: deformed flagellum; dashed lines: without deformation)

Fig. 6 Shape of flagellum rotating counterclockwise (solid lines: deformed flagellum; dashed lines: without deformation)

Fig. 7 Variations of tangential vector along flagellum rotating clockwise (solid lines: deformed flagellum; dashed lines: without deformation)

section of the flagellum in the unit of pN μm (=10^{-18} Nm). Here, M_s, M_R, and M_l of dashed lines are given by Eq.(28) or Eqs.(29) – (31), whereas those of thin solid lines are given by Eq.(27). The comparison indicates that both results overlap and verifies that the moment of force on the cross section of the flagellum can be precisely approximated by Eq.(28).

We assume the value of the flexural rigidity or the elastic bending coefficient EI=1.5×10^{-19} Nm^2 (=15 pN(μm)^2) to obtain the flagellar deformation which is comparable with the results of measurement by Kudo et al.(6) The torsional rigidity coefficient is calculated using such the relation of isotropic material as GI=EI/(1+ν) and Poisson’s ratio ν=1/3.

Figure 5 shows the variations of the tangential vector t=(t_s, t_R, t_l) of the flagellum where s denotes the length along the flagellum. The solid lines illustrate the components of the tangential vector of the deformed flagellum and the dashed lines show those of the flagellum without deformation. The wavelength of sinusoidal form of t_s and t_R of the deformed flagellum becomes smaller but t_l which relates to the pitch angle remains almost constant.

Figure 6 illustrates the shape of the deformed flagellum compared with that without deformation. The pitch as well as the radius of the flagellum becomes shortened but its axial length is unchanged probably because the pitch angle remains constant. Besides, the number of turns of the flagellum increases from 2.84 to 2.92. In short, the resistive force exerted on the flagellum of CCW rotation acts to wind up the flagellum.

4.2 Results for backward swimming by CW flagellar rotation

Results of flagellar rotating at 700 rps in clockwise are shown in the following. In this case, the bacterium moves backward at the speed of 103.1 μm/s. The moment of force acting on the cross section of the flagellum is identical with the results of Fig. 4 multiplied with −1, namely M_s, M_R, and M_l are replaced by −M_s, −M_R, and −M_l.

Figure 7 shows the variations of the tangential vector of the flagellum. The pitch of the deformed flagellum is shown to increase in contrast with the case of CCW rotation. Also, the pitch angle remains almost constant as is the same as the case of CCW rotation.

Figure 8 illustrates the shape of the deformed
flagellum compared with that without deformation. The pitch and the radius of the flagellum are extended but the axial length of the flagellum does not change either. Also, the number of turns of the flagellar helix decreases from 2.84 to 2.77. In this case, the resistive force exerted on the flagellum of CW rotation tends to wind off the flagellum.

4.3 Variations of the pitch of the deformed flagellum

We calculate shapes of deformed flagellum in the range of \( \Omega_n/2\pi = 0 \sim 1,000 \text{ rps} \) at every 100 rps interval and determine the pitches from intersections between the waveform of \( x(x_i) \) and the \( x_i \)-axis from the root to the tip of the flagellum. Figure 9 shows a relation between the pitch and the angular velocity, where \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) denote the pitches corresponding to zero to one turn, 0.5 to 1.5 turns, one to two turns and 1.5 to 2.5 turns, respectively. Also CCW rotation is shown in positive range of \( \Omega_n/2\pi \), CW in negative range of \( \Omega_n/2\pi \). The straight lines are drawn by the least-squares method.

Fig. 8 Shape of flagellum rotating clockwise (solid lines: deformed flagellum; dashed lines: without deformation)

Fig. 9 Pitches at several locations of flagellum

\[ \frac{\Omega_n}{2\pi} = \pm 700 \text{ rps} \]

Variations of the pitch near the root such as \( \lambda_1 \) and \( \lambda_2 \) are proportional to the angular velocity of the flagellar motor, \( \Omega_n/2\pi \), but variations of \( \lambda_3 \) and \( \lambda_4 \) deviate from the linear dependence on the angular velocity. It should be mentioned that \( \lambda_1 \) and \( \lambda_2 \) converge each other for CCW rotation \( (\Omega_n > 0) \) and \( \lambda_3 \) becomes larger than \( \lambda_3 \) for CW rotation \( (\Omega_n < 0) \) for this size of flagellum. This is probably due to a combined effect of deformation of the helix and inclination of the axis. We consider that the inclination is caused by unbalance of the moment of force but the detailed mechanism of inclination is not clear. Also there is a problem in the definition for \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) affected by the inclination of the axis.

4.4 Effect of the flexural rigidity of the flagellum on the pitch

We calculate shapes of flagellum rotating at \( \Omega_n/2\pi = 700 \text{ rps} \) and determine the pitches from the root to the tip of the flagellum, changing values of the flexural rigidity or the elastic bending coefficient \( EI \). Figure 10 shows a relation between the variations of the pitches and the flexural rigidity. There are some deviations for \( \lambda_1 \) and \( \lambda_2 \) as is mentioned above. \( \lambda_4 \) is occasionally larger than \( \lambda_3 \) for CW rotation and they overlap for CCW rotation.

The pitch of deformed flagellum of swimming *Vibrio alginolyticus* as shown in Table 2 corresponds to \( \lambda_4 \). These experimental data of 1.31 \( \mu \text{m/s} \) and 1.23 \( \mu \text{m/s} \) are also included as horizontal lines in Fig. 10. Comparing the experimental results with the numerical results of the analysis, we could estimate \( EI = 10 \sim 15 \text{ pN(\mu m)^2} \) for flagella of *Vibrio alginolyticus*.

5. Conclusions

Analysis has been made for small deformation
produced in the flagellum that single-flagellate bacteria rotate to swim in aquatic environment. Resistive force exerted on the flagellum produces its deformation. In the present investigation, we assumed the shape of the flagellum to be a circular helix and described the force and the moment of force in the cross section of the flagellum in analytical expressions. Employing the Kirchhoff rod model, the variations of the curvature and the torsion of the flagellum due to the moment of the force can be also written in analytical expressions. We obtained deformed shape of the flagellum numerically solving the evolution equations which determine a space curve from the curvature and the torsion. Comparing variations of the pitch of helical flagella between the numerical solutions and the results of measurement, we estimate the flexural rigidity or the elastic bending coefficient of the flagellum in the order of 10 pN(μm)².

References


