A Probabilistic Model for Crack Formation in Laser Cutting of Ceramics*

Scoung Hwan LEE** and Sun-Eung AHN***

Ceramics are being increasingly used in industry due to their outstanding physical and chemical properties. But these materials are difficult to machine by traditional machining processes because they are hard and brittle. Recently, as one of the various alternative processes, laser-beam machining is widely used in the cutting of ceramics. Although the use of lasers presents a number of advantages over other methods, one of the problems associated with this process is the uncertain formation of cracks that result from the thermal stresses. This paper presents a Bayesian probabilistic modeling of crack formation over thin alumina plates during laser cutting. The proposed model can predict the critical cutting front angle which is directly related to the initiation of crack formation.

**Key Words:** Laser Cutting, Crack Formation, Bayesian Probabilistic Model, Alumina

1. Introduction

Ceramics are being increasingly used in industry due to the outstanding mechanical and chemical properties such as high strength and hardness, chemical stability, and high wear resistance. Despite all of the advantages, the usage is still limited in many applications, mainly because of the difficulties and high cost of the manufacturing and processing of the hard and brittle materials.

Recently, laser beam machining has become a viable alternative to conventional machining such as cut-off grinding for the flexible processing of ceramic plates. As laser machining is a tool free, non-contact process, it has several advantages over conventional machining, such as no tool wear and deflections, and no vibrational effects. However, since laser machining is a thermal process the temperature gradients during the cutting process is unavoidable and this may induce the formation of cracks, especially for brittle materials including ceramics.

There has been a lot of research on this phenomenon. Smith et al. conducted a series of experiments with various laser power, cutting speed and plate thickness to investigated the onset of specimen fracture during the laser cutting of alumina. They related the formation of cracks to the steep temperature variations by using a simplified two dimensional model and numerical simulations, but the thermal stress field was not considered. As the temperature gradient and subsequent thermal stresses during the laser cutting are directly related to the occurrence of cracks, many attempts have been focused on this subject. Glass et al. used the three dimensional finite difference method to simulate the thermal stresses and temperature variations numerically during the cutting of metallic glasses. Also, Li et al. analyzed the laser cutting process of ceramics considering the temperature and thermal stresses distributions with experiments and FEM simulations. In actual processing, the direct influence of laser process parameters on the formation of cracks are of importance. Lu et al. used dimensional analysis to formulate an empirical formula for the formation of cracks during the laser cutting of ceramics with varying laser power, cutting speed and material thickness.

From the above results, it has been found that the
formation of cracks are mainly dependent on the laser cutting conditions such as laser power and scanning speed and the crack initiation conditions can be expressed in terms of the conditions, as well. However, the formation of cracks are still uncertain near the crack initiation boundary points even with the identical machining conditions.

In this study, with the assumption that the uncertainty of crack formations near the crack initiation points is caused by the undulation of absorptivity of lasers and the undulations are directly related to the laser cutting front angle, a probabilistic model which expresses the critical cutting front angles at the crack formation initiation conditions (power and scanning speed) is proposed.

2. Theory of Crack Formation

As laser machining is a thermal process, steep thermal gradients and subsequent thermal stresses near the cutting front are inevitable. In this section, theories of crack formation in ceramics, due to the thermal stresses during laser machining, are briefly explained.

2.1 Thermal field

For a straight laser cutting with a constant scanning speed and a very small focused beam radius (Fig. 1), the process can be considered as the steady state movement of a point heat source with respect to the coordinate system fixed to the laser head. The thermal stress field \( T(x) \) in an uncut infinite elastic plate at \( y = 0 \) is expressed as:

\[
T(x) = T(x, 0) = \frac{q}{2\pi hk} \exp(\lambda x)K_0(\lambda x)
\]

(1)

where \( V \) is the moving speed of the heat source (laser scanning speed), \( D \) is the thermal diffusivity, \( \lambda = V/2D \), \( q \) is the heat intensity, \( k \) is the thermal conductivity, \( h \) is the thickness of the plate, \( K_0(\cdot) \) is a zeroth-order modified Bessel function of the second kind. The temperature distribution has the maximum (theoretically infinity) near the heat source \( x = 0 \) and decays fast to the asymptotic values as \( x \to \pm \infty \).

2.2 Thermal stresses

By using Eq. (1), the quasi-static stress distribu-

\[
\sigma(x) = \frac{qum}{2\pi hk}\left[ \text{sign}(x)K_1(\lambda|x|) - K_0(\lambda|x|) - \frac{\lambda}{\lambda x} \right]
\]

(2)

where \( \mu \) is the shear modulus, \( m = (1 + \nu)\alpha, \alpha \) is the coefficient of thermal expansion, \( \nu \) is Poisson's ratio, and \( K_1(\cdot) \) is the first-order modified Bessel function of the second kind.

Figure 2 shows the stress distribution near a positive \( (q > 0) \) heat source. A tensile zone ahead of the thermal source and a compressive zone around the thermal source and behind it are shown. From the asymptotic expansion of Eq. (2) the maximum thermal stress (in the tensile zone) can be written as:

\[
\sigma_{\text{max}}(x) \propto \frac{qum}{2\pi hk}
\]

(3)

where \( \propto \) denotes "proportional to".

The thermal crack forms when the maximum tensile thermal stress exceeds the ultimate tensile stress of a material.

As mentioned earlier, it is known from previous studies that the formation of cracks during the laser cutting of ceramics is dependent upon the laser operating parameters, mainly the laser power and the cutting speed.

Figure 3 shows a data fitting line form laser cutting data with varied power and speed. The fitting line is the boundary of the crack formation, above which is the cracking zone and below which is the non-cracking zone. Though a rough crack initiation criterion—the lower the laser power or the higher the cutting speed—is shown in the figure, it is still unclear whether cracks occur with the machining conditions (power and speed) which are on the critical line. This is due to the inconsistent operating conditions—materials, inert gas, focusing, etc.—which also cause the
variation of the cutting front angle and the occurrence of the cracking.

In this study, first, a crack initiating critical point (power and speed) was determined experimentally, then a probability model was proposed which predicts the variation of the cutting front angle and the probability of the occurrence of the cracking under the critical machining conditions. For the probability model, a mathematical model of cutting front in terms of machining conditions and geometry is introduced in the following section.

3. Formation of Cutting Front

In a steady state laser fusion cutting, the material is melted and removed continuously at the cutting front (Fig. 4), the variation of which is directly related to the laser absorptivity. Powell et al. (1993) formulated a laser cutting front model based on a balance between the absorbed laser power, the power for melting material, the power for heating material, and conduction heat losses.

\[
A(z, \beta)P_{\text{laser}} = \cos \delta A_s (n, \beta) I(z, \beta) \quad (4)
\]

where, \(AP_{\text{laser}}\) is the absorbed beam intensity, \(\beta\) is the circular angle, \(z\) is the depth from the top surface, \(A_s\) is the absorptivity of the erosion front, \(n\) is the refractive index, \(\delta\) is the angle of the laser beam on the erosion front (cutting front angle), and \(I\) is the beam intensity. On the assumption that material removal occurs through melting only, the cutting front shape can be determined based on a power balance between laser power and the rates of heating, melting, and the thermal loss.

\[
AP_{\text{laser}} = P_{T} + P_{m} + P_{r} \quad (5)
\]

\[
P_{T} = \frac{b_s V \rho c_m}{T_0 - T_a} \quad (6)
\]

\[
P_{m} = \frac{\pi k(T_a - T_0) \sqrt{b_s}}{\arctan \left( \frac{16D}{Vs} \right)} \exp \left( - \frac{Vb}{2D} \right) \quad (7)
\]

\[
P_{r} = P_m \quad (8)
\]

where, \(P_{\text{laser}}\) is the total laser power, \(P_{T}\) is the heating power, \(P_{m}\) is the melting power, \(P_{r}\) is the heat conduction loss, \(b_s\) is the kerf width, \(s\) is the cutting depth, \(\rho\) is the density, \(c\) is the specific heat, \(T_a\) is the melting temperature, \(T_0\) is the ambient temperature, \(\varepsilon_s\) is the heat of fusion, and \(D\) is the thermal diffusivity. If the relative motion between the workpiece and laser beam is constant, the cutting process can be regarded as steady state with respect to the coordinate system fixed to the laser beam. In that case, the cutting front can be considered as a set of linear surface elements with infinitesimal area \(dA\) and surface orientation \(n\). Figure 5 shows a geometrical representation of the surface element (14)-(16).

As illustrated in Fig. 5, the infinitesimal area \(dA\) is defined by the vectors \(\mathbf{p}\) and \(\mathbf{q}\), which can be described by the inclination angles \(\theta\) and \(\Phi\).

\[
\hat{p} = dx(i - \tan \theta \cdot \hat{k}) \quad \hat{q} = dy(j - \tan \Phi \cdot \hat{k}) \quad (9)
\]

Also, the normal orientation vector \(\mathbf{n}\) can be expressed as

\[
\mathbf{n} = \frac{\hat{p} \times \hat{q}}{|\hat{p} \times \hat{q}|} \quad (10)
\]

By using Eqs. (9) and (10) \(\mathbf{n}\) can be rewritten as

\[
\mathbf{n} = \frac{-\tan \theta \hat{i} + \tan \Phi \hat{j} + \hat{k}}{\sqrt{1 + \tan^2 \theta + \tan^2 \Phi}} \quad (11)
\]

The energy balance based cutting front shape can be constructed using Eqs. (4) and (11) along with the heat conduction (Fourier) Eq.(14). Figure 6 illustrates a simulation result.

In the laser cutting model, the overall (two dimensional) shape of the cutting front is determined by the
cutting front angle ($\delta$), which is defined as the angle between the vertical line ($z$ axis) and the normal vector $n$. The angle is assumed to have a constant value if the laser power and the scanning speed are fixed. However, in reality, the angle varies due to some uncertain conditions such as the inert gas, and the inhomogeneity of materials. This explains the uncertainty of formation of cracks at the critical machining conditions described in Fig. 3. The critical cutting front angle varies; if $\delta$ becomes smaller, the amount of absorbed energy will be increased (no cracking), and if $\delta$ becomes larger, the probability of crack formation will be increased (Fig. 7). That is, Crack forms if $\delta > \delta^*$ and no crack forms otherwise, where, $\delta^*$ is the lower bound of the crack-forming cutting front angle.

4. Probabilistic Modeling Procedure

Due to the variability in the cutting front angle causing cracks during laser cutting, a probabilistic model is proposed for the prediction of their formations. That is, the probability model proposed in this study is a function of cutting front angle, which has a physical meaning, and it can be used for predicting the formation of cracks during laser cutting. When a finite number of samples or observed data are available, we make use of them in updating the probability of crack formation. Since such available data is always finite, a probability model should not be a statistically asymptotic model, at least in the beginning of the modeling stage. A Bayesian parametric modeling approach is adopted in this study. Parametric modeling is a representation of the probabilistic behavior of a system and has flexibility in the prediction of the uncertain behavior simply by changing the values (or distributions) of the relevant parameter. In this framework, the Bayesian modeling approach is a powerful inference method for such prediction and is well-equipped with an updating procedure using the observed data. The following is the notation used in the probability modeling procedure:

$\delta^*$: Cutting front angle which ranges over $(\delta_0, \delta_1)$
$\delta^*$: Lower bound of cutting front angle resulting in a crack
$L$: Sampling length of a specimen
UCP: Unit cutting path
$n$: Number of UCPs within a specimen, or the smallest integer of $L//$
$X_i$: Random variable indicating that a crack forms in $i$th UCP ($X_i=1$ if a crack forms and $X_i=0$ otherwise)
$s$: Number of cracks within a specimen, or $\sum_{i=1}^{n} X_i$

Unit cutting paths (UCPs) are the objects to be measured for depicting the random formations of cracks resulting from laser cutting. In this study, the length of a UCP is empirically determined to be 0.5 mm. It can be regarded as the minimal length in which a crack forms. Under a specific cutting condition, the observed data relevant to the number of UCPs including cracks will be used to update the probability of crack formation.

4.1 Probability distribution of cutting front angle

During crack formation, uncertainty exists in the value of the cutting front angle caused by the inhomogeneity of material properties and the flow fluctuations of inert gases. Hence a probability distribution of cutting front angle is necessary to represent its uncertainty.

We assume that the variation in cutting front during the crack formation process is continuous and the slope is linear. According to the assertion in section 3 regarding the uncertain formation of the
cutting front angle, we define the probability, \( p \), that a crack forms in a unit cutting path (UCP) as follows:

\[
p = Pr\{\text{a crack forms} \} = Pr\{\delta > \delta^*\}
\]
or equivalently,

\[
1 - p = Pr\{\text{no crack forms} \} = Pr\{\delta < \delta^*\}
\]  \hspace{1cm} (12)

In order to evaluate the probability \( p \) we need the probability density of cutting front angle, \( p(\delta) \). That is, we calculate \( p \) by way of the following equation

\[
p = Pr\{\delta > \delta^*\} = \int_{\delta^*}^{\infty} p(\delta)\,d\delta
\]  \hspace{1cm} (13)

Under the supposition that there is no prevailing cutting front angle over the prespecified range \((\delta_0, \delta_i)\), one way to express the uncertainty about the cutting front angle is the application of uniform probability density over \((\delta_0, \delta_i)\). That is, the probability density of cutting front angle, \( p(\delta) \), is of the form

\[
p(\delta) = \frac{1}{\delta_i - \delta_0}, \quad \delta \in (\delta_0, \delta_i)
\]  \hspace{1cm} (14)

Then, from Eq.(13), the crack-formation probability, \( p \), becomes

\[
p = \frac{\delta_i - \delta^*}{\delta_i - \delta_0}
\]  \hspace{1cm} (15)

Even though we have a lower bound for the cutting front angle resulting in crack-formation, we still do not have its exact value. Hence a probabilistic expression for \( \delta^* \) should be given in this case. Since there is no information about its value until experiments are performed, we again apply the same uniform probability density to the lower bound as to the cutting front angle itself. It follows that the crack-formation probability, \( p \), is regarded as a parameter which has a probability density. According to the foregoing statements, \( \delta^* \) is uniformly distributed and its density \( \pi(\delta^*) \) becomes

\[
\pi(\delta^*) = \frac{1}{\delta_i - \delta_0}
\]  \hspace{1cm} (16)

Notice that the probability density Eq.(16) is a special case of the Beta probability density. Because the crack-formation probability, \( p \), takes its value over \((0, 1)\), the Beta probability density of \( p \) with parameters \( a \) and \( b \) has the form

\[
p(a, b) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} p^{a-1}(1-p)^{b-1},
\]  \hspace{1cm} (17)

where \( a, b > 0 \) and \( \Gamma(\cdot) \) is the gamma function. When the parameters \( a = b = 1 \), the Beta probability density becomes a uniform density. By rescaling and relocating \( p \) through the transformation \( \delta^* = \delta_i - (\delta_i - \delta_0)p \), \( \delta^* \) has a uniform density over \((\delta_0, \delta_i)\).

4.2 Updating the procedure of the crack-formation probability

Since we judge that the random variables \( X_i \)'s are exchangeable\(^{(19)}\) conditional on the crack-formation probability, using Eq.(15), we have the following probability distribution of the number of cracks, \( s \), given \( n \) UCPs.

\[
p(s \mid n, \frac{\delta_i - \delta^*}{\delta_i - \delta_0}) \propto \left( \frac{\delta_i - \delta^*}{\delta_i - \delta_0} \right)^s \left( \frac{\delta^* - \delta_0}{\delta_i - \delta_0} \right)^{s-n}, \hspace{1cm} (18)
\]

where \( \propto \) denotes "proportional to". By using Bayes' formula with Eqs.(16) and (18), we can update the crack-formation probability as follows:

\[
p \left( \frac{\delta_i - \delta^*}{\delta_i - \delta_0} \mid s, n \right) \propto \left( \frac{\delta_i - \delta^*}{\delta_i - \delta_0} \right)^s \left( \delta^* - \delta_0 \right)^{s-n} \left( \frac{1}{\delta_i - \delta_0} \right) \hspace{1cm} (19)
\]

Equation (19) represents the updated probability density of the crack-formation probability when \( s \) cracks are observed out of \( n \) UCPs. Additionally, after observing \( t \) more cracks from the additional \( m \) sampled UCPs, we can keep updating by following the above procedure, wherein Eq.(19) plays the same role as Eq.(15) in the previous updating. The resulting updated probability distribution is

\[
p \left( \frac{\delta_i - \delta^*}{\delta_i - \delta_0} \mid s, n, t, m \right) \propto \left( \frac{\delta_i - \delta^*}{\delta_i - \delta_0} \right)^{s+t} \left( \delta^* - \delta_0 \right)^{s+n-m-t} \hspace{1cm} (20)
\]

5. Experiments

For the experiments, a continuous wave type Nd:YAG laser with the maximum power 350 W, the beam radius 0.3 mm, and the focal length 100 mm was used. Also, Nitrogen gas (8 bar) was used as the inert gas for the cutting of 1 mm thick alumina plates. After the cutting, each specimen (Fig.8) was investigated with an optical microscope with 0.005 mm resolution.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Properties of alumina</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical Properties</td>
<td>value</td>
</tr>
<tr>
<td>Alumina content</td>
<td>96%</td>
</tr>
<tr>
<td>Bulk density</td>
<td>3.8</td>
</tr>
<tr>
<td>Water absorption</td>
<td>0%</td>
</tr>
<tr>
<td>Bending strength</td>
<td>3,000 kg/cm²</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>20,000 kg/cm²</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>3.05 × 10¹¹ kg/cm²</td>
</tr>
<tr>
<td>Thermal Properties</td>
<td>value</td>
</tr>
<tr>
<td>Linear expansion coeff. at 25-400°C</td>
<td>7.1 × 10⁻⁶°C</td>
</tr>
<tr>
<td>at 25-800°C</td>
<td>8.2 × 10⁻⁴°C</td>
</tr>
<tr>
<td>Heat conductivity(25°C)</td>
<td>0.55 cal/cm sec°C</td>
</tr>
<tr>
<td>Specific heat (25°C)</td>
<td>0.185 cal/g°C</td>
</tr>
</tbody>
</table>

Fig. 8 Schematic of specimen
Table 2  Machining conditions

<table>
<thead>
<tr>
<th>Laser power [W]</th>
<th>350</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed rate [mm/s]</td>
<td>18, 18.5, 19, 19.5, 20, 20.5, 21, 21.5, 22</td>
</tr>
</tbody>
</table>

Fig. 9  Typical crack formation

For the detection of the occurrence of cracks during the steady state cutting, the observations were made for the mid-span (3 cm to 8 cm from the cut start point) of the specimen (out of total 10 cm span). Table 1 shows the mechanical properties of alumina.

To find out the crack initiation point, experiments were conducted with the machining conditions given in Table 2, in which the power was fixed as 350 W and the feed rates were varied.

As the initiation of cracking (Fig. 9) was observed at the feed rate of 20.5 mm/s (laser power 350 W), this point was selected as the critical machining conditions for the occurrence of cracks. To update the lower bound distribution of the cutting front angle, experiments were repeated three times at the critical condition. For the verification of the proposed probability model, experiments with a slightly higher speed (21 mm/s) were performed with the same laser power.

6. Experimental Results

With the critical machining conditions, the cracking occurred 14 UCPs, 18 UCPs, 13 UCPs, respectively, in 5 cm (100 UCPs) spans. Equations (19) and (20) were used for updating the probability density of the lower bound of cutting front angle resulting in cracks.

Figure 10 shows the densities updated from the initially uniform density by using the three times experimental results. In the process of repeating the experiment, the lower bound values were updated as 82°, 83°, 84°, respectively. Therefore, the lower bound value of the critical cutting front angle for the cracking from this probability model is 84°. The increase in the probability density with updating confirms the relationship between cutting front angle and crack formation.

In another test series, with the increased scanning speed (21 mm/s), the cracking occurred 21 UCPs, 25 UCPs, 36 UCPs, respectively, for the same length span. Again, Eq. (20) was used for updating the probability density of the lower bound of cutting front angle and Fig. 11 shows the results. From Fig. 11, we observe a shift in the probability densities to the left (64° to 79°) as the scanning speed increases. This implies that the lower bound of cutting front angle causing cracks decreases as the scanning speed increases. From the energy balance based simulation (Fig. 6), the cutting front angle was calculated as 79.9° for the speed 20.5 mm/s, and 79.3° for the speed 21 mm/s with the fixed laser power (350 Watt). This shows an identical tendency between the predictions.
using the probability model and the physics based simulation.

7. Conclusions

Though the critical crack formation boundaries during the laser cutting of alumina can be determined by the laser power and the cutting speed, the occurrence of cracks are still uncertain at the critical conditions due to the undulations of the cutting front angle which is affected by environmental operating conditions such as materials, inert gas and laser focusing.

In this study, the Bayesian probability modeling approach was used for explaining the uncertain physics of a phenomenon relevant to the crack formations during laser cutting. The 0-1 type random variable which indicates crack formation was used in the modeling. As the experiments were repeated, the increase in the probability density with updating confirms the relationship between the cutting front angle and the crack formation.

In another test series in which the workpiece was machined at a higher scanning speed, the lower bound of cutting front angle causing cracks decreases as in the energy balance simulation. This validates the probability model in the physical sense and the usefulness of the proposed scheme.

Acknowledgements

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References