A Multi-Axis Contour Error Controller for Free Form Curves*

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The growing need for higher precision and productivity in manufacturing industry has lead to an increased interest in computer numerical control (CNC) systems. It is well known fact that the cross-coupling controller (CCC) is an effective method for contouring applications. In this paper, a multi-axis contour error controller (CEC) based on a contour error vector using parametric curve interpolator is presented. The contour error vector is a vector from the actual tool position to the nearest point on the desired path. The contour error vector is the closest error model to the contour error. The simulation results show that the CEC is more accurate than the conventional CCC for a biaxial motion system. In addition, the experimental results on 3-axis motion system show that the CEC is simply applicable to 3-axis motions and contouring accuracy is significantly improved.

Key Words: Contour Error, Cross-Coupling Control, Multi-Axis Contour Error Controller, Curve Interpolator, Free Form Curve

1. Introduction

The higher precision in manufacturing field is demanded, the more accurate servo controller is needed. Recently, free form curve machining has gained popularity in manufacturing fields, especially in automobile and electronic industry. Hence the accuracy of the nonlinear contouring motion in CNC has come to forefront.

One of the important issues in servo mechanism system is reducing the contour error. Many researchers have studied techniques for reducing contouring error in CNC machines. Poo et al.⁽¹⁾ compared contour error of the symmetrical system with that of non-symmetrical system for linear and circular contours and concluded that the matched controller parameter reduces the contour error. Koren⁽²⁾ introduced the cross-coupling controller which reduce the contour error rather than diminish

the tracking error. Kulkarni and Srinivasan^{(3),(4)} estimated the contour error by using the tracking error of each axis and the instantaneous tangent of the desired contour. The contour error model is simple and easily applicable, but there is a considerable difference between the estimated contour error and the real contour error. This contour error model cannot solve the real contour error exactly for nonlinear contours. Chuang and Lui⁽⁵⁾ proposed a contour error model that has a linear relation between the feedrate and the contour error. Because of nonlinearity of servo system and desired contour, this model is not suitable to various nonlinear contouring. Koren and Lo^{(6),(7)} proposed the variable-gain CCC which has more precise nonlinear contouring. For nonlinear contouring, the variable-gain CCC estimates the contour error by using the instantaneous circle tangential to the reference point on the assumption that the axial errors are much smaller than the instantaneous radius of curvature. However, this CCC is not suitable in case of high-speed machining and a large curvature. Also it is difficult to apply this CCC to 3-axis CNC machine for nonlinear contouring motions. Syh-Shiuh Yeh and Pau-Lo Hsu⁽⁸⁾ estimated the contour error by using a vector from the actual position to the nearest point on the line that passes through the reference position tangentially. This method can be extended to

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multi-axis motion and is more efficient than the variable gain CCC. However when the tracking error is big, this method estimates the contour error poorly.

In the present work a multi-axis contour error controller (CEC) based on contour error vector using parametric curve interpolator is introduced. The proposed contour error model can find more accurate contour error from geometrical relationship between the actual tool position and the contour than other models. To verify the multi-axis CEC, the simulations and experiments were performed for various nonlinear contours. The results show that the multi-axis CEC improves the contouring accuracy considerably, and it can also be extended to 3-axis contouring motion.

2. Multi-axis Contour Error Controller

A new contour error model is shown in Fig. 1. The error component orthogonal to the desired trajectory is considered as the contour error. This contour error vector is the closest error model to the real contour error. In use of the curve interpolator, the function of curve is known and the error vector is found with it. The actual tool position P is obtained from the signal of encoder. The contour error vector $\vec{\epsilon}$ is defined as the vector from the actual tool position to the nearest point on the desired contour. The point Q is the meeting point with desired path when we draw the perpendicular line to desired path from the actual tool position P . The vector $\vec{Q}'$ is the differential of desired path about x or u (when desired path is parametric curve like Bezier, B-spline, and NURBS). The contour error vector $\vec{\epsilon}$ is expressed as

$$\vec{\epsilon} = \vec{Q} - \vec{P} \quad (1)$$

where $\vec{Q}$ is the vector from the origin O to the point Q , and $\vec{P}$ is the vector from origin to the point P . The contour error ϵ is defined as the magnitude of the contour error vector $\vec{\epsilon}$. The contour error ϵ is

$$\epsilon = \|\vec{\epsilon}\| = \sqrt{(Q_x - P_x)^2 + (Q_y - P_y)^2 + (Q_z - P_z)^2} \quad (2)$$

where, P_x, P_y, P_z, Q_x, Q_y and Q_z are, in turn, x -, y - and z -coordinates of the point P , x -, y - and z -coordinates of the point Q . If we find the coordinates of the point Q , we obtain the contour error vector $\vec{\epsilon}$. The coordinates of the point Q can be determined from the geometrical relationship shown in Fig. 1.

$$\vec{\epsilon} \cdot \vec{Q}' = 0 \quad (3)$$

Thus, the vectors are perpendicular. Substituting Eq. (1) into Eq. (3) yields

$$(\vec{Q} - \vec{P}) \cdot \vec{Q}' = 0 \quad (4)$$

The Eq. (4) is a polynomial of x or u (for parametric curve). We can solve the Eq. (4) by using the numerical methods, then we obtain the contour error ϵ .

Fig. 1 The contour error vector

Fig. 2 The multi-axis CEC

The structure of a Multi-axis contour error controller (CEC) is shown in Fig. 2. (R_x, R_y, R_z) and (P_x, P_y, P_z) denote the reference position and the actual position, respectively. (E_x, E_y, E_z) are the tracking errors of each axis. (K_{px}, K_{py}, K_{pz}) are the proportional gain of position loop of each axis. (D_x, D_y, D_z) are the disturbances of each axis. There are three proportional controllers for position loops and a PID controller for the multi-axis CEC. The contour error vector is calculated by solving Eqs. (1) and (4). The output of the PID controller is divided into three axial components as shown in Fig. 2. As the magnitude and direction of the contour error is known, its axial components are determined. The axial components are fed in real time to the individual loop of the x , y , and z axis, respectively. The purpose of CEC is to reduce the contour error rather than to diminish individual axial error. The CEC uses the feedback information from encoders of all axes and the function of the desired contour to calculate the contour error.

3. Simulation

To verify the performance of the multi-axis CEC, simulations are performed for a parabola, a Bezier curve and a curve in 3-axis motion. The system parameters used in the simulation are listed in Table 1. The parameters of experimental system are esti-

lated to show the trend of experimental results. The disturbances are due to friction forces in the linear motion guides. The gains of PID for CEC, K_p , K_i , and K_d are 7, 50, and 0.5, respectively. The feedrate is 60 mm/s and the sampling time is 1 msec.

The desired contour is shown in Fig. 3, and the equation of parabola is

$$y = \left(\frac{1}{15}\right)(x-15)^2 - 15 \quad (5)$$

The tracking ability of a parabolic contour motion for CEC and CCC is shown in Fig. 4. The multi-axis CEC provides a much better following ability for the desired contour than the variable gain CCC. The PID gains for CEC and CCC are the same as mentioned above. The contour errors of the variable gain CCC and the multi-axis CEC are compared

Table 1 The system parameters used in simulation

Parameter	Time constant τ	Open loop gain K	Position loop gain K_p	Disturbance D
X axis	35 msec	10.3 s^{-1}	1	0.75
Y axis	30 msec	10 s^{-1}	1	0.75
Z axis	35 msec	10.3 s^{-1}	1	0.75

Fig. 3 Parabolic contour

Fig. 4 Contouring of parabolic contour

in Figs. 5 and 6. The result of simulation by using the variable gain CCC is shown in Fig. 5. The dashed line is calculated by the contour error model of the variable gain CCC, and the solid line is calculated by the contour error model of the multi-axis CEC. To compare the contour errors, the dashed line is drawn with the absolute value of the calculated values. The variable gain CCC contour error model has negative values when the distance between the actual position of the tool and the origin of the instantaneous tan-

Fig. 5 Contour error by variable gain CCC

Fig. 6 Contour error by multi-axis CEC

gential circle is shorter than the radius of the circle. The solid line drawn with the magnitude of the contour error vector, however, always has positive values. The difference between the dashed line and the solid line is due to the difference of the contour error models. Figure 7 represents the contouring of parabola by using the variable gain CCC. Figure 7(a) shows actual tool positions and reference points from the 330th to the 370th turn. Figure 7(b) shows the 350th reference position and actual tool position, and instantaneous circle on this reference position. The coordinates of reference position and actual tool position are (13.738, -14.894) and (8.490, -12.118), respectively. The radius of the circle is 7.821 mm, and the contour error calculated by the contour error model of the variable gain CCC is 0.377 mm, the contour error calculated by the contour error vector of the multi-axis CEC is 0.043 mm. Actually, the instantaneous radius of the curvature is small, and the axial errors are relatively large. Thus, the conventional contour error model can not estimate the contour error exactly. When the feedrate is fast or the radius of curvature is small or the disturbance is big, the several contour error models depending on the assumption that the actual tool position is near by the reference point at the sampling time, as well as the contour error model of the variable gain CCC, provide poor estimation of the contour error. Because the contour error model of multi-axis CEC uses only the actual tool positions and the function of desired path, it always provides good estimation of the contour error.

The result of simulation by using the multi-axis CEC is shown in Fig. 6. The multi-axis CEC reduces the contour error by 0.0635 mm. The maximum, mean, and RMS of the contour errors of this simulation are presented in Table 2.

The second desired path is a Bezier curve. The

Fig. 7 (a) Delay of tool position to reference point
(b) Contour error model of variable gain CCC

curve with control points of (0, 0), (50, -200), (150, 200), and (200, 0) is shown in Fig. 8. The contour error of the simulation is shown in Fig. 9. The dashed line is the contour error with P controller without CEC, and the solid line is the contour error with the multi-axis CEC. The result shows that the multi-axis CEC can reduce the contour error significantly for the free form motion. Besides, the multi-axis CEC is easily applicable to parametric curves like Bezier, B-spline and NURBS curve. The maximum, mean, and RMS of the contour errors of this simulation are presented in Table 3.

The third desired path is a curve in 3-axis motion as shown in Fig. 10, and the equation of the curve is

$$\begin{aligned} x &= 3(4u-2) - (4u-2)^3 - 2 \\ y &= 3(4u-2)^2 - 12 \\ z &= -3(4u-2) - (4u-2)^3 - 14 \end{aligned} \quad (6)$$

$$[0 \leq u \leq 1]$$

Table 2 Contour error for parabolic contour [mm]

Controller	Max.	Mean	RMS
Variable-gain CCC	0.327	0.117	0.113
CEC	0.064	0.016	0.021

Fig. 8 Bezier curve

Table 3 Contour error for Bezier curve [mm]

Controller	Max.	Mean	RMS
P controller without CEC	0.403	0.119	0.101
CEC	0.017	0.002	0.004

Table 4 Contour error for 3-axis motion [mm]

Controller	Max.	Mean	RMS
P controller without CEC	0.244	0.061	0.068
CEC	0.033	0.008	0.009

Fig. 9 Contour error for Bezier curve

Fig. 10 Desired path for 3-axis motion

The contour errors of the simulation are shown in Fig. 11. The result shows that the multi-axis CEC can be directly extended to 3-axis motion. The maximum, mean, and RMS of the contour errors of this simulation are presented in Table 4.

4. Experiments

The experimental system is shown in Fig. 12. The three-axis desktop milling machine is composed of PC (Pentium III), three Mitsubishi servo motors and drivers, and DA/AD card, three linear motion guides,

Table 5 Experiment result for Bezier curve [mm]

Controller	Max.	Mean	RMS
P controller without CEC	1.347	0.434	0.316
CEC (Gain 0.5)	0.025	0.007	0.005

Fig. 11 Contour error for 3-axis motion

Fig. 12 Experimental system

and a spindle motor. The proportional gain of the position feedback loop is 0.001. The sampling time is 1 msec. To find contour error vector is a time critical task. A combination of bisection and Newton-Raphson method is used in order to meet this time critical task and to improve Newton-Raphson's global convergence properties. Experimental results for a Bezier curve shown in Fig. 8 are given in Fig. 13. The feedrate is 60 mm/s, and the proportional gain of CEC is 0.5. It is observed from Fig. 13 and Table 5 that the multi-axis CEC provides good performance in free from motion. In case of P-controller without CEC the maximum contour error is 1.347, however in case of CEC the maximum contour error is 0.025.

The multi-axis CEC method is also applicable to 3-axis motion. By adding the Z-axis component to

Table 6 Experiment result for 3-axis motion [mm]

Controller	Max.	Mean	RMS
P controller without CEC	1.622	0.494	0.453
CEC (Gain 0.5)	0.078	0.020	0.022

Fig. 13 Experimental results for Bezier curve

Fig. 14 Experimental results for 3-axis motion

the Eqs.(1) and (4), we apply CEC to the 3-axis motion. The desired path is shown in the Fig. 11, the arrow represents the direction of motion of the tool. The experimental results of 3-axis motion for the multi-axis CEC is compared to those for the proportional controller without CEC. It is seen from Fig. 14 and Table 6 that the contouring accuracy is improved remarkably. In case of P-controller without CEC the maximum contour error is 1.622, however in case of CEC the maximum contour error is 0.078.

5. Conclusions

A Multi-axis contour error controller which reduces the contour error significantly for various nonlinear contours has been proposed. The contour

error model proposed in the present estimates the contour error more accurately than the methods suggested in literature. The contour error vector is computed in real time by the combination of bisection and Newton-Raphson method. The multi-axis CEC satisfies 1msec sampling time. By using the actual tool positions and the information of desired path, the multi-axis CEC corrects the contouring effectively even though the tracking response is poor. This method can be easily applicable to the parametric curves like Bezier, B-spline and NURBS. The proposed method is also applicable to 3-axis motion without difficulty. Results of simulations and experiments show that the multi-axis CEC improves the contouring accuracy considerably.

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