Finite Element Analysis of the Misalignment Effects on the Dynamic Force Coefficients of Spiral Groove Gas Face Seals*

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This paper presents a finite element procedure specially devised to analyze the misalignment effects on the behavior of spiral groove gas face seals operating at high speeds. In this study, the seal stationary face is slightly misaligned and the flexibly mounted face is perfectly aligned. Predictions of some steady-state and dynamic performance characteristics versus misalignment angle are presented for spirally grooved gas seals operating under stringent conditions. Curves of dynamic force coefficients versus the static misalignment angle of the seal face indicate that the seal misalignment affects considerably the dynamic response of gas lubricated face seals. At high speeds, the static seal misalignment not only results in increased stiffness coefficients but also leads to negative damping coefficients, which may be a sign of the seal susceptibility to excessive angular motions.

Key Words: Gas Seals, Face Seals, Seal Misalignment, Spiral Grooves, Grooved Seals, Spiral Groove Seals, Finite Element

1. Introduction

The development of novel gas sealing technology for industrial turbomachinery employed in the chemical, petrochemical, pulp and paper and food processing industries must not only lead to more efficient seal designs but also meet the low fluid emission strict requirements imposed by modern environmental laws. Grooves have been introduced into the noncontacting seal faces to increase the seal opening force and decrease fluid leakage⁽¹⁾. The spiral grooved (whipple groove) face type of seal is one of the most successful seal geometries used in noncontacting gas face seals and has been widely employed in both single and double seal arrangements for the process industry⁽²⁾⁻⁽⁴⁾.

Demands for increasing productivity in the process industry have required the development of high speed mechanical face seals, which are subjected to stringent operating conditions. The seal operating

conditions are limited by temperature and pressure at which the seal components and face materials are running⁽¹⁾. The mating faces of gas lubricated mechanical seals must be completely separated during the operation to avoid potential catastrophic contact. Furthermore, gas seals are unable to dissipate heat generated by the friction of their sliding plates⁽⁵⁾. Varying high pressure ratios combined with almost inevitable geometric imperfections can induce instantaneous angular motions on the face seal plates, which can drastically change the seal performance⁽⁶⁾.

Advancements in gas seal technology have been accompanied by the development of efficient computational procedures based on the finite element method (FEM) to analyse the behaviour of grooved gas face seals. Most of the FEM studies performed on spiral groove gas face seals (SGGFSs) and thrust bearings analyze some steady-state and dynamic performance characteristics of seals operating at low and medium speeds and are based on the assumption that the seal mating faces are perfectly aligned⁽⁷⁾⁻⁽⁹⁾. Tournier et al.⁽¹⁰⁾ present one of the few FEM analyses of gas lubricated face seals accounting for the face misalignment and predict some steady-state performance characteristics of misaligned SGGFSs

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operating at low speeds.

The importance of seal dynamics on the analysis of sealing systems employed in high speed rotating machines has been demonstrated by numerous analyses of liquid lubricated misaligned plain face seals^{(11),(12)}. These analyses take into account the hydrodynamic effects caused by the angular misalignment, however they are based on the narrow seal approximation on the Reynolds equation for incompressible fluids, which relies on the simplifying assumption of an infinite number of grooves. On the other hand, the technical literature lacks both experimental and numerical data about the influence of the angular misalignment on the behavior of gas lubricated face seals. The prediction of the dynamic force coefficients of gas seals can provide useful information for the rotordynamics analysis of industrial turbomachines, such as centrifugal compressors, turbines, pumps, agitators and mixers.

This paper presents a finite element procedure specially devised to analyse some steady-state and dynamic performance characteristics of spiral groove gas face seals (SGGFSs) accounting for the static face misalignment. This computational procedure, based on a high-order Galerkin weighted residual method⁽¹³⁾, is able to render numerically stable results for gas face seals operating at high speeds. A linearized perturbation procedure⁽¹⁴⁾ is applied on the classical Reynolds equation for compressible fluids to render the zero-th and first order lubrication equations, which allow the prediction of, respectively, the opening force and the dynamic force coefficients for misaligned noncontacting gas face seals. The opening force, the leakage flow, the stiffness and damping coefficients are evaluated for statically misaligned gas face seals operating at high speeds. The results show the influence of the misalignment on the static and dynamic response of high speed SGGFSs.

2. Geometry and Equations of a SGGFS

Figure 1 presents the geometry and parameters describing a gas-lubricated spiral groove face seal (SGGFS). The seal geometry includes the ridge clearance c , the groove depth c_g , the ridge width w_r , the groove width w_g , the number of grooves N_g , the groove angle β , the seal inner radius r_i , the seal outer radius r_o , the seal grooved portion inner radius r_{gi} and the seal grooved portion outer radius r_{go} . The groove depth ratio $\delta = c_g/c$, the groove width ratio $\alpha_g = w_g/(w_r + w_g)$ and the seal dam extent $l = (r_{gi} - r_i)/(r_o - r_i)$ are dimensionless geometric parameters widely used in the description of the seal geometry. The seal configuration is described in relation to a coordinate system attached to the grooved seal face. The (r, θ, z)

Fig. 1 Geometry and physical parameters of a SGGFS with rotating and stationary grooved surfaces

z) coordinate system is attached to the rotating grooves at speed Ω . The (r, Φ, z) coordinate system is attached to the stationary face. The relation between the two coordinate systems is given by $\Phi = \theta + \Omega t$. The equation governing the logarithmic spiral contour of the grooves is $r = r_{gi} \cdot e^{\theta \tan(\beta)}$ (15).

Grooves are etched either on the rotating surface or on the stationary surface of the seal. The relative motion between the grooved and smooth surfaces causes a pumping action in the fluid. Pumping-in (inwardly pumping) and pumping-out (outwardly pumping) seal designs usually find applications in industrial rotating machinery⁽¹⁵⁾. Sealing pressure dams are generally employed in SGGFS configurations either to minimize leakage or increase axial stiffness. Inner and outer pressure dams are introduced into the grooved surface to enhance the seal performance.

The laminar flow of an isothermal, isoviscous ideal gas within the film lands of a spirally grooved face seal⁽¹³⁾ is described by

$$\frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{ph^3}{12\mu} \frac{1}{r} \frac{\partial p}{\partial \theta} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{ph^3}{12\mu} r \frac{\partial p}{\partial r} \right) = \frac{B\Omega}{2r} \frac{\partial}{\partial \theta} (rph) + \frac{\partial(ph)}{\partial t} \quad (1)$$

Here, p represents the hydrodynamic pressure, h is the fluid film thickness, and μ is the fluid viscosity. B is a seal parameter that describes the groove rotation direction. Table 1 shows the values of B for the different seal configurations⁽¹³⁾.

The seal is subjected to pressures p_{in} and p_{out} at its inner and outer radii r_i and r_o , respectively. Typically $p_{out} > p_{in}$. In a seal, the inner and outer radii sections are open allowing radial flow inward and outward. The pressure distribution is periodic over the seal/

Table 1 The groove rotation direction parameter with seal configuration

Groove Angle (β) Range	Stationary Grooves	Rotating Grooves
$0^\circ < \beta < 90^\circ$	Outward Pumping ($B = -1$)	Inward Pumping ($B = 1$)
$90^\circ < \beta < 180^\circ$	Inward Pumping ($B = 1$)	Outward Pumping ($B = -1$)

Fig. 2 Equilibrium position of a misaligned moving surface

bearing circumferential length.

The fluid film thickness is described accounting for the static misalignments of the seal surface⁽¹⁶⁾. The seal moving surface has three degrees of freedom, schematically shown in Fig. 2. The film thickness h_o at an equilibrium position can be written as

$$h_o(r, \theta) = c + r\phi_{y0} \cos \theta - r\phi_{x0} \sin \theta, \quad \text{in the ridge region} \quad (2)$$

$$h_o(r, \theta) = c + c_g + r\phi_{y0} \cos \theta - r\phi_{x0} \sin \theta, \quad \text{in the groove region} \quad (3)$$

ϕ_{x0} and ϕ_{y0} describe the angular position of the moving surface at equilibrium, i.e., are small angular displacements representing the face misalignment about axes X and Y , respectively.

3. Lubrication Equations

A linearized perturbation procedure⁽¹⁴⁾ is performed on the Reynolds equation (Eq. (1)) to render the zeroth- and first-order lubrication equations. Small dynamic perturbations ΔZ , $\Delta \phi_x$, and $\Delta \phi_y$ at excitation frequency ω about an equilibrium position of the moving rotating face cause perturbations in the film thickness and pressure distributions. The perturbed film thickness $h(r, \theta, t)$ is given by

$$h(r, \theta, t) = h_o(r, \theta) + r \cos \theta \cdot \Delta \phi_y e^{i\omega t} - r \sin \theta \cdot \Delta \phi_x e^{i\omega t} + \Delta Z \cdot e^{i\omega t} \quad (4)$$

where $i = \sqrt{-1}$. The perturbed pressure field is expressed as

$$p(r, \theta, t) = p_o(r, \theta) + p_z(r, \theta) \Delta Z e^{i\omega t} + p_{\phi_x}(r, \theta) \Delta \phi_x e^{i\omega t} + p_{\phi_y}(r, \theta) \Delta \phi_y e^{i\omega t} \quad (5)$$

where p_o represents the zeroth-order pressure field, and p_z , p_{ϕ_x} , and p_{ϕ_y} are the first-order pressure distributions caused by the small perturbations. Expressions for the linearized zeroth- and first-order lubrication equations are obtained by substituting Eqs. (4) and (5) into Eq. (1). The zeroth-order lubrication equation has the following form.

$$\frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{p_o h_o^3}{12\mu} \frac{1}{r} \frac{\partial p_o}{\partial \theta} - \frac{B\Omega r}{2} p_o h_o \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{p_o h_o^3}{12\mu} r \frac{\partial p_o}{\partial r} \right) = 0 \quad (6)$$

The first-order equation for axial displacement Z is expressed as

$$\frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{3h_o^2}{12\mu r} p_o \frac{\partial p_o}{\partial \theta} + \frac{h_o^3}{12\mu r} \frac{\partial(p_o p_z)}{\partial \theta} - \frac{B\Omega r}{2} (p_z h_o + p_o) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\frac{3h_o^2}{12\mu} p_o \frac{\partial p_o}{\partial r} + \frac{h_o^3}{12\mu} \frac{\partial(p_o p_z)}{\partial r} \right) \right] = i\omega (p_z h_o + p_o) \quad (7)$$

The first-order equations for the dynamic angular displacements ϕ_x and ϕ_y around the X and Y axes are given by

$$\frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{-3h_o^2 r \sin \theta}{12\mu r} p_o \frac{\partial p_o}{\partial \theta} + \frac{h_o^3}{12\mu r} \frac{\partial(p_o p_{\phi_x})}{\partial \theta} - \frac{B\Omega r}{2} (p_{\phi_x} h_o - p_o r \sin \theta) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\frac{-3h_o^2 r \sin \theta}{12\mu} p_o \frac{\partial p_o}{\partial r} + \frac{h_o^3}{12\mu} \frac{\partial(p_o p_{\phi_x})}{\partial r} \right) \right] = i\omega (p_{\phi_x} h_o - p_o r \sin \theta) \quad (8)$$

$$\frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{3h_o^2 r \cos \theta}{12\mu r} p_o \frac{\partial p_o}{\partial \theta} + \frac{h_o^3}{12\mu r} \frac{\partial(p_o p_{\phi_y})}{\partial \theta} - \frac{B\Omega r}{2} (p_{\phi_y} h_o + p_o r \cos \theta) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\frac{3h_o^2 r \cos \theta}{12\mu} p_o \frac{\partial p_o}{\partial r} + \frac{h_o^3}{12\mu} \frac{\partial(p_o p_{\phi_y})}{\partial r} \right) \right] = i\omega (p_{\phi_y} h_o + p_o r \cos \theta) \quad (9)$$

4. Finite Element Procedure

An efficient and accurate finite element procedure is implemented for the solution of the perturbed lubrication equations. The solution procedure is based on the Galerkin weighted residual method and uses high-order shape functions $\{\psi_j^e\}_{j=1,2,3,4}$, which are derived from an approximate solution to the non-linear Reynolds equation within an element^{(13),(17)}. The fluid flow domain within the seal lands is divided into four-node isoparametric finite elements Ω_e with boundary given by Γ_e . The zeroth-order pressure field is interpolated over the element domain in the following form.

$$p_o^e = \sum_{i=1}^4 \psi_i^e p_{oi}^e \quad (10)$$

where p_{oi}^e are the nodal values of pressure on the element (e). By employing the Galerkin weighted residual method and substituting Eq. (10) into Eq. (6) render the following zeroth-order finite element system of asymmetric equations

$$K_{ji}^e \cdot p_{oi}^e = q_j^e \quad (i, j=1, 2, 3, 4) \quad (11)$$

where

$$K_{ji}^e = \iint_{\Omega_e} \left[\frac{p_o h_o^3}{12\mu} \left(\frac{\phi \psi_i^e}{\partial r} \frac{\partial \psi_j^e}{\partial r} + \frac{1}{r^2} \frac{\partial \psi_i^e}{\partial \theta} \frac{\partial \psi_j^e}{\partial \theta} \right) - \frac{B\Omega h_o}{2} \psi_i^e \frac{\partial \psi_j^e}{\partial \theta} \right] d\Omega_e \quad (12)$$

$$q_j^e = - \oint_{\Gamma_e} \psi_j^e \dot{m}_n d\Gamma_e \quad (13)$$

$\dot{m}_n$ is the zeroth-order mass flow rate normal to the element boundary. A successive substitution procedure is implemented to solve iteratively the system of zeroth-order finite element equations⁽¹³⁾. The initial guess for pressure is a linear pressure distribution determined from the pressure boundary conditions. The iterative process ends when $|F_{\text{new}} - F_{\text{old}}|/F_{\text{old}} \leq 0.001$, where F_{new} is the seal opening force computed at iteration (n) and F_{old} is the force at the previous iteration ($n-1$).

The finite element equations for the first-order pressures are obtained in a similar way. The pressures p_z , $p_{\varphi x}$, and $p_{\varphi y}$ are interpolated within an element domain in the following form.

$$p_z^e = \sum_{i=1}^4 \psi_i^e p_{zi}^e \quad (14)$$

$$p_{\varphi x}^e = \sum_{i=1}^4 \psi_i^e p_{\varphi xi}^e \quad (15)$$

$$p_{\varphi y}^e = \sum_{i=1}^4 \psi_i^e p_{\varphi yi}^e \quad (16)$$

Then the finite element first-order equation for axial displacements Z is expressed as

$$K_{zi}^e \cdot p_{zi}^e = q_{zi}^e + f_{zi}^e \quad (i, j=1, 2, 3, 4) \quad (17)$$

where

$$K_{zi}^e = \iint_{\Omega_e} \left[\frac{p_o h_o^3}{12\mu} \left(\frac{\partial \psi_j^e}{\partial r} \frac{\partial \psi_i^e}{\partial r} + \frac{1}{r^2} \frac{\partial \psi_j^e}{\partial \theta} \frac{\partial \psi_i^e}{\partial \theta} \right) + \frac{h_o^3}{12\mu} \left(\frac{\partial \psi_j^e}{\partial r} \frac{\partial p_o}{\partial r} + \frac{1}{r^2} \frac{\partial \psi_j^e}{\partial \theta} \frac{\partial p_o}{\partial \theta} \right) \psi_i^e - \frac{B\Omega h_o}{2} \frac{\partial \psi_j^e}{\partial \theta} \psi_i^e + i\omega h_o \psi_j^e \psi_i^e \right] d\Omega_e \quad (18)$$

$$q_{zi}^e = - \oint_{\Gamma_e} \psi_j^e \dot{m}_{zn} d\Gamma_e \quad (19)$$

$$f_{zi}^e = \iint_{\Omega_e} \left[\frac{-3h_o^2}{12\mu} p_o \left(\frac{\partial \psi_j^e}{\partial r} \frac{\partial p_o}{\partial r} + \frac{1}{r^2} \frac{\partial \psi_j^e}{\partial \theta} \frac{\partial p_o}{\partial \theta} \right) + \frac{B\Omega_o}{2} \frac{\partial \psi_j^e}{\partial \theta} \psi_i^e - i\omega p_o \psi_j^e \psi_i^e \right] d\Omega_e \quad (20)$$

The first-order mass flow rate outward the element boundary is given by $\dot{m}_{zn}$.

The finite element systems of equations for the first-order equations for angular displacements φ_x and φ_y are given by

$$K_{\varphi xi}^e \cdot p_{\varphi xi}^e = q_{\varphi xi}^e + f_{\varphi xi}^e \quad (21)$$

$$K_{\varphi yi}^e \cdot p_{\varphi yi}^e = q_{\varphi yi}^e + f_{\varphi yi}^e \quad (22)$$

where

$$K_{\varphi xji}^e = K_{zji}^e \quad (23)$$

$$K_{\varphi yji}^e = K_{zji}^e \quad (24)$$

$$q_{\varphi xj}^e = - \oint_{\Gamma_e} \psi_j^e \dot{m}_{xn} d\Gamma_e \quad (25)$$

$$q_{\varphi yj}^e = - \oint_{\Gamma_e} \psi_j^e \dot{m}_{yn} d\Gamma_e \quad (26)$$

$$f_{\varphi xj}^e = \iint_{\Omega_e} \left[\frac{3h_o^2 r \sin \theta}{12\mu} p_o \left(\frac{\partial \psi_j^e}{\partial r} \frac{\partial p_o}{\partial r} + \frac{1}{r^2} \frac{\partial \psi_j^e}{\partial \theta} \frac{\partial p_o}{\partial \theta} \right) - \frac{B\Omega r \sin \theta}{2} p_o \frac{\partial \psi_j^e}{\partial \theta} + i\omega p_o r \sin \theta \psi_j^e \right] d\Omega_e \quad (27)$$

$$f_{\varphi yj}^e = \iint_{\Omega_e} \left[\frac{-3h_o^2 r \cos \theta}{12\mu} p_o \left(\frac{\partial \psi_j^e}{\partial r} \frac{\partial p_o}{\partial r} + \frac{1}{r^2} \frac{\partial \psi_j^e}{\partial \theta} \frac{\partial p_o}{\partial \theta} \right) + \frac{B\Omega r \cos \theta}{2} p_o \frac{\partial \psi_j^e}{\partial \theta} - i\omega p_o r \cos \theta \psi_j^e \right] d\Omega_e \quad (28)$$

where $\dot{m}_{xn}$ and $\dot{m}_{yn}$ are the first-order mass flow rates outward Γ_e .

5. Seal Performance Characteristics

The finite element Eqs. (11), (17), (21) and (22) are assembled for the whole seal domain. The zeroth- and first-order pressure fields are computed by solving the global finite element equations⁽¹³⁾. Seal opening force (F_z) and restoring moments ($M_{\varphi x}$, $M_{\varphi y}$) are given by the following expression.

$$\begin{Bmatrix} F_z \\ M_{\varphi x} \\ M_{\varphi y} \end{Bmatrix} = \iint_{\Omega} \begin{Bmatrix} (p_o - p_{\text{ref}}) \\ -(p_o - p_{\text{ref}}) r \sin \theta \\ (p_o - p_{\text{ref}}) r \cos \theta \end{Bmatrix} r d\theta dr \quad (29)$$

where p_{ref} represents the reference pressure defined as the lowest pressure of the seal inner and outer pressures. The dynamic force and moment coefficients are given by

$$K_{\sigma\beta} + i\omega C_{\sigma\beta} = - \iint_{\Omega} h_{\sigma} p_{\beta} r d\theta dr, \quad (\sigma, \beta = Z, \varphi_x, \varphi_y) \quad (30)$$

where $h_z = 1$, $h_{\varphi x} = -r \sin \theta$, and $h_{\varphi y} = r \cos \theta$. Equation (30) is rewritten in matrix form as

$$\begin{bmatrix} K_{zz} & K_{z\varphi x} & K_{z\varphi y} \\ K_{\varphi xz} & K_{\varphi x\varphi x} & K_{\varphi x\varphi y} \\ K_{\varphi yz} & K_{\varphi y\varphi x} & K_{\varphi y\varphi y} \end{bmatrix} + i\omega \begin{bmatrix} C_{zz} & C_{z\varphi x} & C_{z\varphi y} \\ C_{\varphi xz} & C_{\varphi x\varphi x} & C_{\varphi x\varphi y} \\ C_{\varphi yz} & C_{\varphi y\varphi x} & C_{\varphi y\varphi y} \end{bmatrix} = \iint_{\Omega} \begin{Bmatrix} -1 \\ r \sin \theta \\ -r \cos \theta \end{Bmatrix} \{p_z \ p_{\varphi x} \ p_{\varphi y}\} r d\theta dr \quad (31)$$

6. Numerical Results

This section is divided into two parts: 6.1. Validation and 6.2. Performance Characteristics of SGGFSs. An example of mechanical gas face seal is selected to evaluate the accuracy of the finite element procedure implemented for SGGFSs. Results computed by this procedure are compared with predictions available in the literature for aligned gas seals. Then, the influence of the static face misalignment on gas

grooved face seals is shown through some performance characteristic curves.

6.1 Validation

Predictions obtained by the high-order FEM procedure⁽¹³⁾ for axial stiffness and damping coefficients of aligned gas face seals are compared with those obtained by the incremental Galerkin FEM scheme implemented by Zirkelback and San Andrés⁽¹⁸⁾. The seal parameters are given in Table 2 with $c=6\mu\text{m}$. Figures 3 and 4 present the dimensionless axial stiffness and damping coefficients for a SGGFS versus the frequency number ($\sigma=2\Lambda(\omega/\Omega)$), respectively, at three speed numbers ($\Lambda=\frac{6\mu\Omega}{\dot{p}_{\text{ref}}}\cdot\left(\frac{r_o}{c}\right)^2$), computed by both the incremental FEM (dashed line) and the high-order FEM (solid line) schemes. Both schemes render practically the same results for low speed number problems. To avoid

Table 2 SGGFS parameters for validation

$r_i = 0.062\text{ m}$	(stationary grooves)	$c = 6\mu\text{m}$
$r_{gi} = 0.069\text{ m}$	$\Omega = 4774.7\text{ rpm}$	$c_g = 6\mu\text{m}$
$r_o = 0.0805\text{ m}$	$\rho = 1.09\text{ kg/m}^3$	$\mu = 18\times 10^{-6}\text{ Pa.s}$
$\beta = 161.4^\circ$		$p_i = 0.1\text{ MPa}$
$N_g = 12\text{ grooves}$		$p_o = 0.3\text{ MPa}$
$\alpha_g = 0.567$		$\Lambda\text{ varies from }5\text{ to }100$

Fig. 3 Comparative results for dimensionless axial stiffness versus frequency number in a SGGFS

Fig. 4 Comparative results for dimensionless axial damping versus frequency number in a SGGFS

numerical indeterminacy, the procedure uses a very small frequency number ($\sigma=0.001$) to evaluate the static stiffness and damping coefficients at $\omega=0$.

400 finite elements are used to model a ridge-groove pair in both procedures. The predictions by the incremental FEM scheme are computed using five steps and step increment of 0.05Λ . The normalization for stiffness and damping coefficients is shown in Table 3.

6.2 Performance characteristics of SGGFSs

The influence of the static misalignment on the performance characteristics of spirally grooved gas face seals is analyzed at moderate and high speed numbers (Λ). The seal baseline geometry for this analysis is given in Table 4, and represents a high speed inward pumping gas seal employed in nitrogen pumps.

The seal performance characteristics are evaluated with respect to the static misalignment about the axis X . In this study, the fixed seal face (stationary face) is slightly misaligned and the flexibly mounted face is perfectly aligned. Coning effects are not taken into account in the computations. The misalignment angle is given in degrees. The minimum and maximum values of seal outer clearances are about $0.9\mu\text{m}$ and $4.1\mu\text{m}$ (for $\varphi_x=0.001^\circ$), respectively. Table 5 shows the normalization of the seal opening force ($\bar{F}_z$), leakage flow rate ($\bar{Q}$), stiffness coefficients ($\bar{K}_{ij}$) and damping coefficients ($\bar{C}_{ij}$) employed in this work. R_g represents the gas constant.

Figure 5 shows the dimensionless seal opening force ($\bar{F}_z$) and inward flow rate ($\bar{Q}$) versus misalignment about the axis $X(\varphi_x)$ at moderate (dashed line) and high speed numbers (solid line). $\bar{Q}$ and $\bar{F}_z$ increase with increasing φ_x . The misalignment generates regions of converging and diverging clearances in

Table 3 Normalization of force coefficients for numerical validation

Normalized Stiffness	Normalized Damping
$\bar{K}_{zz} = K_{zz} / (\pi p_{ref} (r_o^2 - r_i^2) / c)$	$\bar{C}_{zz} = C_{zz} / (\pi p_{ref} (r_o^2 - r_i^2) / c \Omega)$

Table 4 Geometric and operating parameters for a SGGFS

Stationary grooves		
$r_i = 0.07112\text{ m}$	$\beta = 160^\circ$	$p_{in} = 0.101\text{ MPa}$
$r_{gi} = 0.076454\text{ m}$	$\alpha_g = 0.5$	$p_{out} = 0.505\text{ MPa}$
$r_o = 0.0889\text{ m}$	$N_g = 12\text{ grooves}$	$\Lambda = 300\text{ or }1253$
$c = c_g = 2.54\mu\text{m}$	$\mu = 10.963\times 10^{-6}\text{ Pa.s}$	$\Omega = 3600\text{ or }15000\text{ rpm}$
Mesh: 1320 elements		
(132 circumferential x 10 radial elements)		

Table 5 Normalization parameters for seal performance characteristics

$\bar{F}_z = F_z / F^*$	$F^* = p_{av} \pi (r_o^2 - r_i^2)$	$p_{av} = (p_o + p_i) / 2$
$\bar{Q} = Q / Q^*$	$Q^* = \frac{\pi p_{av} (c + c_g)^3 p^*}{6 \mu \ln(r_o / r_i)}$	$p_{av} = \frac{p_o + p_i}{2 R_g T}$
where $p^* = (p_i - p_o)$		
Axial stiffness :	$\bar{K}_{zz} = K_{zz} / K_{zz}^*$	$K_{zz}^* = F^* / c$
Angular stiffness :	$\bar{K}_{ij} = K_{ij} / K_{xx}^*$; $i, j = \varphi_x, \varphi_y$	$K_{xx}^* = \frac{F^* r_o^2}{c}$
Cross-Coupled Stiffness :	$\bar{K}_{zi} = K_{zi} / K_{xx}^*$; $i = \varphi_x, \varphi_y$	$K_{xx}^* = F^* r_o / c$
Axial damping :	$\bar{C}_{zz} = C_{zz} / C_{zz}^*$	$C_{zz}^* = K_{zz}^* / \Omega$
Angular damping :	$\bar{C}_{ij} = C_{ij} / C_{xx}^*$; $i, j = \varphi_x, \varphi_y$	$C_{xx}^* = C_{zz}^* r_o^2$
Cross-Coupled Damping :	$\bar{C}_{zi} = C_{zi} / C_{xx}^*$; $i = \varphi_x, \varphi_y$	$C_{xx}^* = F^* r_o / c \Omega$
	$\bar{C}_{iz} = C_{iz} / C_{xx}^*$

Fig. 5 Dimensionless opening force and flow rate versus misalignment angle in a SGGFS

the circumferential direction. The hydrodynamic pressure increases in the region of converging clearances.

The static ($\omega=0$) dimensionless direct stiffness and damping coefficients versus the misalignment angle about axis X are shown, respectively, in Figs. 6 and 7 for two values of speed number: $\Lambda=300$ (dashed line) and $\Lambda=1253$ (solid line). The direct stiffness coefficients increase and the damping coefficients generally decrease as φ_x increases for moderate and high speed numbers. The axial damping becomes negative approximately at $\varphi_x=0.008^\circ$ for $\Lambda=300$ and at $\varphi_x=0.007^\circ$ for $\Lambda=1253$. $\bar{C}_{\varphi_x\varphi_x}$ becomes negative for small misalignment angle φ_x at high Λ , but it always positive at moderate Λ . The sign change of the direct damping coefficients can indicate that the gas lubricated seal may become susceptible to unstable motions operating at high speeds⁽¹⁹⁾. However, the diametral tilt of the seal surface generates increased hydrodynamic direct stiffness coefficients (see Fig. 6).

Fig. 6 Dimensionless direct static stiffness coefficients versus misalignment angle in a SGGFS

Fig. 7 Dimensionless direct static damping coefficients versus misalignment angle in a SGGFS

That is, the hydrodynamic action of the thin gas film is to counterbalance the angular motions of the seal face.

Figures 8 and 9 depict the cross-coupled stiffness and damping coefficients, respectively, versus the static misalignment angle about the axis X at $\Lambda=1253$. As expected, the cross-coupled coefficients associated with the coupling between the axial and tilt motions ($\bar{K}_{z\varphi_x}$, $\bar{K}_{\varphi_x z}$, $\bar{C}_{z\varphi_x}$) vary significantly with increasing misalignment angle about axis X . For high speed numbers, these cross-coupled stiffness coefficients ($\bar{K}_{z\varphi_x}$, $\bar{K}_{\varphi_x z}$) have a trend to become more negative as φ_x increases, indicating that there may be a potential to transverse angular motions at high speeds. On the other hand, the cross-coupled damping coefficient ($\bar{C}_{z\varphi_x}$) may have a stabilizing effect on the seal transverse angular motion at high speed numbers, since it increases as φ_x increases.

The SGGFS analyzed is assumed to be perfectly balanced in this study. Angular misalignment between seal faces is a common feature in flexibly mounted gas face seals. The analysis shows that the hydrodynamic action of the thin gas film is to counterbalance any angular motion of the seal face. However, a potential to unstable angular motions may be present at high speeds. Moreover, the seal elastic

Fig. 8 Dimensionless cross-coupled static stiffness coefficients versus misalignment angle in a SGGFS ($\Lambda=1.253$)

Fig. 9 Dimensionless cross-coupled static damping coefficients versus misalignment angle in a SGGFS ($\Lambda=1.253$)

support mechanism is designed to act as self-aligning element working against any angular motion. The effects of the seal elastic support mechanism and damping associated with the secondary seal are not taken into account in the analysis.

The Reynolds number ($Re = (\rho \cdot \Omega \cdot r_o \cdot c) / \mu$) reaches the value of 362.5 at maximum speed, which indicates that the flow is basically laminar. Fluid inertia effects are expected to have an secondary effect in the computation of steady-state and dynamic force performance characteristics of grooved face seals lubricated with a low viscosity, low mass density fluid⁽²⁰⁾ even at very high operating speeds.

7. Conclusions

This work presents a finite element analysis of misaligned spiral groove gas face seals (SGGFSs) operating at high speeds. An efficient and accurate finite element procedure specially devised for high speed aligned SGGFSs⁽¹³⁾ is extended to include the seal face static misalignment into the analysis. Some steady-state and dynamic performance characteristics are predicted for spirally grooved gas seals operating under stringent conditions accounting for the angular misalignment between seal faces. Curves of

seal opening force, flow rate and dynamic force coefficients show the influence of the static misalignment angle on the behavior of SGGFSs operating at high speeds.

The finite element predictions indicate that the static seal misalignment results in increased seal opening force and direct stiffness coefficient, offering increasing resistance to rubbing contact between the seal mating plates. From the computational analysis carried out in this work, it can be concluded that the hydrodynamic action of the thin gas film is to counter-balance any angular motion of the seal face. However, at high speeds, large misalignment angles can lead to negative direct damping coefficients. This damping coefficient pattern shows that the seal may become more susceptible to unstable angular motions as the operating speed increases. This finite element study of the misalignment effects on the behavior of SGGFSs does not take into account the seal elastic support mechanism and damping associated with the secondary seal.

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