A Study of Adhesion Force Model for Wheel Slip Prevention Control

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In order to enhance the speed of wheel slip prevention control for railway vehicles, modeling of adhesion characteristics after saturation of adhesion force, which had hitherto not been performed with trains, was performed via a beam model. The behavior of adhesive force between the wheel and rail in a wet condition was examined with a brake performance test stand for an actual vehicle. Results of comparing the model and test values indicated that the beam model represented test values well.

Key Words:  Railway, Modeling, Observer, Adhesion, Wheel-Rail Contact, Wheel Slip Prevention, Disturbance Observer

1. Introduction

With a long train, the middle and rear vehicles are less affected by water on the rail surface than the lead vehicles, and a larger adhesion force can be provided\(^{(1),(2)}\). There was a problem with shorter trains, however, of an increased braking distance because the adhesion force could not be anticipated in the rear vehicles. To efficiently operate a train at high speeds regardless of the number of vehicles comprising it, a control system that can provide a braking force even with a shorter train must be developed.

In addition, enhanced speeds faster than the Shinkansen bullet train have been planned in Europe in recent years. Development of a Shinkansen intended for a maximum speed of 360 km/h has also been conducted in Japan as well\(^{(3),(4)}\). Thus, enhanced speed with adhesion force control in high-speed regions is also critical.

Modeling of controlled vehicles is important in achieving those demands, and modeling of the adhesion force with braking is critical. Confirmation of the effects of a designed control system beforehand via computer simulation has great significance to allow reduction of the costs required for that research and development.

There are numerous experimental formulas relating to adhesion between wheels of trains and rails for velocity characteristics with adhesion force as exemplified by the design formula for the Shinkansen. In addition are Carter's theoretical solution\(^{(5)}\) and the like for models of the relationship between slipping and adhesion force. However, the range in which this theoretical solution can be applied is in a range with a slip rate of 0 to 0.2%. Reports of experimental results with regard to adhesion force characteristics\(^{(6)-(8)}\) are mainly with a slip rate after saturation of the adhesion force. With regard to modeling via adhesion performance analysis for electric locomotives\(^{(9)}\) and conventional simulations of wheel slip prevention\(^{(10),(11)}\), discontinuous models ignoring characteristics prior to saturation of adhesion force were assumed\(^{(9),(10)}\) or an adhesion model was clearly not performed\(^{(11)}\) and research relating to modeling has been insufficient.

In addition, we can perform curve fitting for modeling of adhesion on the basis of experimental results, and obtain a model of adhesion-slip characteristics. Although besides the necessity of determining individual coefficients in each adhesive state, there is also the problem of the fact that the formula is physically meaningless.

There are various techniques to make a model of tire actuation and braking characteristics for automotive tires. But about the railway wheel, there is no complicated characteristic like the rubber tire. So we adopted the circular beam model\(^{(12)}\), which can express only longitudinal adhesion in this study. The conventional reports about adhesion were generally to use theory of CARTER for railway. Recently, the theory of KALKER\(^{(13)}\) was used about the study of pre-slipping phenomenon\(^{(14)}\) in order to compare with the experimental results in dry rail condition. But our study is about the adhesion force model in all slip range,

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so the theory of KALKER can not be adopted.

Therefore, modeling of adhesion force to provide a simulation required to confirm the control action of wheel slip prevention control beforehand was performed in this study using a beam model with physical meaning. We report the comparison with experimental results was performed and validity of the model.

2. Theory

Carter’s theoretical solution(5) cannot express adhesion characteristics theoretically in macro slip region (after adhesion force saturates). There are no research reports about modeling of adhesion in macro slip for railway research. Therefore, we adapted the beam model(12) railway adhesion characteristics. The beam model has been used for theoretical tire models to study dynamics and control of automobiles. This conceptual model is shown in Fig. 1.

The signature $\mu$ is the tangential force coefficient (traction coefficient), which is the tangential force acting between wheel and rail, divided by the wheel load at braking. $\mu_{\text{max}}$ is coefficient of adhesion, that is, the maximum value of the tangential force coefficient. We show the contact surface form(15) of wheel and rail. As the contact surface of the tire for automobiles is approximated a rectangle in the beam model, we also approximated the contact surface of the wheel and rail as a rectangle as shown in Fig. 2.

If we defined the contact pressure between wheel and rail as the Eq. (1) of secondary function, a tangential force distribution becomes as shown in Fig. 3. In the figure, $p$ is the contact pressure between wheel and rail, $\mu_d$ is dynamic friction coefficient in slipping area, $f_s$ is the tangential force in adhesive area (locking area, called in railway research), $x$ is the displacement from tip of contact surface, and $l_h$ is the start point of slipping in contact surface of wheel and rail.

$$p = \frac{3}{2} \frac{F_z}{w} \left( \frac{l_2}{2} - \left( x - \frac{l_2}{2} \right)^2 \right)$$  \hspace{1cm} (1)

Here,

- $p$ : contact pressure between wheel and rail
- $l$ : contact length
- $w$ : contact width
- $F_z$ : wheel load
- $x$ : displacement from tip of contact surface

According to Ref. (12), dynamic friction coefficient was defined using the average velocity of the slipping area. Therefore, dynamic friction coefficient in slipping area is defined as the following equation using $\mu_{\text{max}}$ and slip ratio.

$$\mu_d = \mu_{\text{max}} - \frac{a \cdot s \cdot v}{(l - l_h)}$$  \hspace{1cm} (2)

With,

- $\mu_d$ : dynamic friction coefficient in slipping area
- $a$ : constant to determine dynamic friction coefficient in slipping area
- $v$ : vehicle velocity
- $v_w$ : wheel velocity
- $s$ : slip ratio

$$l_h = \left( 1 - \frac{K_s s}{3\mu_{\text{max}} F_z} \right)$$  \hspace{1cm} (3)

With,

- $l_h$ : start point of slipping in contact surface of wheel and rail

The wheel load is equal to the integrated value of contact pressure between wheel and rail on the whole contact surface. Therefore, tangential adhesion force between wheel and rail is expressed the Eq. (4)
\[ F_x = C_x s \left( 1 - \frac{K_x s}{3 F_z \mu_{\text{max}}} \right)^2 \left( \frac{1}{2} \right) \]

We used the parameters as shown in Table 1 for a wheel of railway. The theoretical results through the beam model with those parameters are shown in Fig. 4. In addition, we used the parameters of the contact length and the contact width according to Ref. (15). Because the constant “\(a\)” is determined so as to agree with the experimental results, in case of the automobile research, we also determined “\(a\)” as 0.001 3 from the following adhesion experimental results at 140 km/h.

3. Adhesion Test

3.1 Disturbance observer\(^{16),(17)}\)

It is very difficult to measure adhesion force directly on running vehicles. As we will explain in section 3.2, it is also impossible to measure adhesion force directly on the brake performance test stand. Therefore, we estimated adhesion force by using a disturbance observer\(^ {16),(17)}\)

The relationship between wheel velocity and the brake torque is given by the Eq. (6).

\[ J \dot{v_w} = \mu M g r_w - T_b \]

Here,
- \(J\): Inertia of wheel set
- \(v_w\): wheel velocity
- \(r_w\): wheel diameter
- \(\mu\): tangential force coefficient (traction coefficient)
- \(M\): train mass
- \(T_b\): brake torque
- \(g\): gravity acceleration

To define “\(\mu M g r_w = T_x\)” and re-write the last equation using \(T_x\), the following equation is obtained.

\[ T_x = T_b + J \frac{\dot{v_w}}{r_w} \]

\[ (7) \]

To execute LAPLACE transformation, we obtained the Eq. (8). But the Eq. (8) contains differentiation term. Therefore, we adapted primary order low-pass filter to the Eq. (8), and executed the equivalent conversion to obtain the Eq. (9). The block diagram of this disturbance observer is shown in Fig. 5.

\[ T_x(s) = T_b(s) + s J \frac{V_w(s)}{r_w} \]

\[ (8) \]

\[ \hat{T}_x(s) = T_b(s) + \frac{J}{r_w \tau_0} \left( 1 - \frac{1}{\tau_0 s + 1} \right) V_w(s) \]

\[ (9) \]

Here, \(\tau_0\) is time constant of the disturbance observer.

We estimated the tangential force coefficient (traction coefficient) \(\hat{\mu}\) using next formula.

\[ \hat{\mu} = \frac{\hat{T}_x}{F_z} \]

3.2 Test equipment

For the adhesion test, we used the brake performance test stand in Railway Technical Research Institute. We show this test stand conceptually in Fig. 6, and the photograph in Fig. 7. The equipment consists of a main principal axle with a rail wheel, flywheels and a main motor, a
sub-axle with a wheel and a brake disk. After accelerating to a target velocity by main motor, the brake caliper acts brake force. The inertia of flywheels imitates the inertia of a vehicle when the train runs.

The test condition is shown in Table 2. And in this test, we sprayed water on a rail wheel to realize the rail wet condition. We measured brake torque, wheel load, wheel velocity and rail wheel velocity. This test unit can measure the real-time (dynamic) contact load between wheel and rail, which is used to calculate the tangential force coefficient (traction coefficient). The time constant of the disturbance observer is 400 ms.

4. Test Results

We show an example of adhesion test result in Fig. 8. We gave brake torque over maximum adhesion force at 4.5 second. As the result of braking, the macro sliding of the wheel occurred. At about 6.1 second, the slip ratio was over 50%, the controller stopped the brake action. Therefore, the wheel velocity recovered and the sliding diminished to zero by the adhesion force between wheel and rail (in this research, wheel and rail wheel). In this test, we performed to 50% to prevent to damage this test stand by wheel lock. The test results of slip – adhesion characteristics are shown in Fig. 9. We also calculated slip ratio using primary order low-pass filter equal to the disturbance observer. The \( \Delta \) shows the maximum values, \( \circ \) is mean values, and \( \times \) is minimum values. In Fig. 9, upper lines are

![Disturbance observer diagram](image)

Fig. 5 Disturbance observer

![Adhesion test unit diagram](image)

Fig. 6 Adhesion test unit of brake performance test stand (Conceptually)

![Photograph of adhesion test unit](image)

Fig. 7 The adhesion test unit of the brake performance test stand (Photograph)

<table>
<thead>
<tr>
<th>Test Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail wheel velocity [km/h]</td>
<td>30, 60, 100, 140</td>
</tr>
<tr>
<td>Slip rate [%]</td>
<td>0 ~ 50</td>
</tr>
<tr>
<td>Wheel contact load [kN]</td>
<td>34.5</td>
</tr>
<tr>
<td>Wheel inertia [kg · m²]</td>
<td>60.35</td>
</tr>
<tr>
<td>Rail wheel axle inertia [kg · m²]</td>
<td>699.8</td>
</tr>
<tr>
<td>Sprayed water [l/min]</td>
<td>1.0 ~ 3.0</td>
</tr>
</tbody>
</table>

Table 2 Test condition

![Experimental results graph](image)

Fig. 8 Experimental results
maximum theoretical value of the beam model with these test results of $\mu_{\text{max}}$. The middle lines are mean value, and the lower lines show minimum value. In these test results, adhesion force saturated at about 10% of slip ratio and the beam model simulates the feature of test results.

5. Discussion

According to Carter’s theoretical solution, we have known adhesion force saturates about slip rate 0.2%. But in this beam model and experiment result, adhesion force saturated at about 10% of slip ratio. The difference from Carter’s theoretical solution is considered to be as follows. Carter’s theory uses maximum traction coefficient (Coefficient of adhesion) only and argues about micro slip region. Moreover, Carter used $\mu_{\text{max}}$ as friction coefficient of slipping area in tangential force distribution as shown Fig. 3. But we used $\mu_d$ given by the Eq. (2) depending on slip velocity as coefficients of friction in the beam model. Therefore, we consider that the slip ratios at which the adhesion force saturates in case of the beam model and the experiment results are different from those by Carter’s theory.

We showed Fig. 10 the adhesion characteristics of velocity from these test results. The sign $\bullet$ shows the estimated coefficients of adhesion using the disturbance observer, and the real line is the experimental formula calculated from the test results. These test results at 100 km/h are high value as compared with the mean value of narrow gauge lines in Japan. Comparing to the mean values, they are about 156% of the values, and are considerably high. In order to make clear this reason, we measured the surface roughness value $R_z$ on wheel and rail wheel. The measured results of $R_z$ value is 6.4 $\mu$m and higher than those of in the Ref. (5). According to the Ref. (5) the higher the roughness is the higher the adhesion coefficient. Therefore, the high adhesion coefficients of the test are considered to result from high surface roughness.

6. Conclusion

In order to enhance the speed of wheel slip prevention control for railway vehicles, modeling of adhesion characteristics after saturation of adhesion force, which had hitherto not been performed with trains, was performed via a beam model. The behavior of adhesive force between the wheel and rail in a wet condition was examined with a brake performance test stand for an actual vehicle. Results of comparing the model and test values indicated that
the beam model represented test values well. The merit of using the beam model as examined in this research is that it can serve to confirm the effects of a control system via simulation since characteristics after saturation of the adhesion force can be expressed, which had not previously been indicated for trains.

Topics for the future will be comparison of control performance by the conventional control with the control system designed by robust control theory such as variable structure control system via simulations by using the adhesion model obtained. Moreover, we will design the control system to allow braking force even with shorter trains, and put that to practical use.

In addition, performing experiments controlling the roughness of the contact surface for the wheel and rail for actual vehicles and deepening understanding relating to adhesion will also be important in the future.

References