Acoustic Radiation Optimization Using the Particle Swarm Optimization Algorithm∗

Jin-Young JEON** and Masaaki OKUMA**

The present paper describes a fundamental study on structural bending design to reduce noise using a new evolutionary population-based heuristic algorithm called the particle swarm optimization algorithm (PSOA). The particle swarm optimization algorithm is a parallel evolutionary computation technique proposed by Kennedy and Eberhart in 1995. This algorithm is based on the social behavior models for bird flocking, fish schooling and other models investigated by zoologists. Optimal structural design problems to reduce noise are highly nonlinear, so that most conventional methods are difficult to apply. The present paper investigates the applicability of PSOA to such problems. Optimal bending design of a vibrating plate using PSOA is performed in order to minimize noise radiation. PSOA can be effectively applied to such nonlinear acoustic radiation optimization.

Key Words: Acoustics, Noise Reduction, Particle Swarm Optimization Algorithm, Finite Element Method, Rayleigh Integral Method, Sound Power Level, Modal Analysis, Genetic Algorithm, Noise

1. Introduction

Optimum structural design technology to reduce radiating noise is becoming an important subject of research in various industrial fields today. Previously, engineers were forced to design quiet structures by trial and error, a costly and time-consuming process. With the advent of inexpensive and powerful computers, however, the empirical methods of designing quiet mechanical components can be replaced with numerical modeling and optimization.

A number of studies have examined the minimization of sound power radiated from a vibrating structure using gradient-based recursive quadratic programming algorithms, such as the quasi-Newton method, sequential quadratic programming (SQP) method, genetic algorithms (GAs) and simulated annealing (SA), and an evolutionary algorithm that mimics natural phenomena. Lang and Dym(1) studied an optimal acoustic design for sandwich panels using a pattern search method and showed that the average transmission loss of a panel may be improved over a range of frequency and that the optimization is realized by shifting up the symmetric coincidence frequency of the panel beyond the frequency range of interest. Belegundu et al.(2) presented a general gradient-based optimization algorithm for minimizing the sound power radiated from baffled plates excited by single-frequency and broadband harmonic excitation. Their study was an attempt to determine the optimum thickness distribution of a plate in order to minimize the radiated acoustic power under excitation over a band of frequency. Akl et al.(3) investigated a comprehensive optimization of the design parameters of stiffened underwater shells using a rational multi-criteria optimization approach, whereby the shell vibration and associated sound radiation was totally minimized. In other words, the vibration, noise radiation, weight and cost of the stiffened shell were minimized. A two-step multi-criteria optimization approach was used to search the optimal design configurations of stiffened underwater shells: the number of stiffeners and their cross sectional dimensions. The adopted approach relies on the use of the Pareto and min-max optimum search procedures. Constans et al.(4) reported that sound power reduction can be accomplished by optimal placement and sizing of small point masses in the semi-cylindrical shell structure using a simulated annealing algorithm. The optimal small point masses can alter the critical mode shapes to quieter modes of vibration for sound power reduction. The simulated annealing technique is used to determine the location and/or...
mass of the point masses. In most of the abovementioned studies, the control parameters by which to optimally reduce the sound power radiated from vibrating structures are the thickness of plate and layers of composites, attachment of point masses and damping materials, and attachment of ribs and other types of stiffeners. The methods may effectively reduce sound power from structures, such as automobile bodies, gearbox casings, fuselage skins and engine nacelles. These approaches, however, may require additional weight and cost. Kaneda et al.\(^{(5)}\) reported that the reduction of sound power radiated from a plate in an infinite rigid baffle can be achieved effectively by the plate curvature design. This method uses a Genetic Algorithm, which is well known as a relatively efficient evolutionary optimization algorithm, together with the Finite Element Method (FEM) for analyzing vibration and the Rayleigh integral method for acoustic analysis.

The PSOA was originally proposed by Kennedy and Eberhart\(^{(6)}\), and other researchers have successfully applied PSOA to various optimization problems. For instance, Salman et al.\(^{(7)}\) presented an approach based on PSOA for the static task assignment problem in distributed or parallel computing systems. Fourie et al.\(^{(8)}\) reported an application of the PSOA to the optimal shape and size design with respect to static load. Furthermore, in geometry optimization of some simple truss structures, Fourie et al. reported that PSOA yielded better solutions than GAs. This algorithm has a big advantage in that it does not require explicitly mathematical expression of the relation between an objective function to be optimized and the design parameters. Neither derivative evaluations nor initial bracketing of optimal points is necessary. However, as opposed to well-established methods such as GA, SA and NN, PSOA is still in its infancy.

First, in selecting an optimization algorithm, one must decide whether a gradient or non-gradient method should be used. Gradient methods have the advantage of typically converging on an optimal solution quite rapidly if the objective function is well behaved. Belegundu et al.\(^{(9)}\) and Hambric\(^{(10)}\) have reported that non-gradient optimization methods are attractive compared to gradient methods for structural optimization with respect to acoustic response because of the presence of multiple local minima.

The purpose of the present study is to investigate the feasibility of PSOA for structural optimization in order to minimize sound power radiated from vibrating shell structures such as car floors. In the present paper, bending design of a vibrating plate is demonstrated. A new optimizer based on PSOA can optimally determine the degree of bending.

2. Theory and Modeling

In the present paper, structural bending optimization of a plate is performed in order to minimize noise radiation. The technical theory for the optimization is described in this section.

2.1 Structural analysis

The finite element method is a practical tool for linear structural dynamic analysis of arbitrary structures. In the present study, the vibration of a test plate is analyzed by the finite element method. Using four-node shell elements, a finite element model is developed for the basic test shell structure, as depicted in Fig. 1. The structure consists of three parts, namely an interior plate which behaves elastically, a rectangular frame that can be assumed to be rigid in the frequency range of interest, and four simple suspensions that support the entire shell plate. The equation of motion of the system can be expressed as

\[
[M]\{\ddot{X}_r\} + [K]\{X_r\} = [F_e] \tag{1}
\]

where \(M\) is the global mass matrix, \(K\) is the global stiffness matrix, \([F_e,F_r]\) is an external excitation force vector, \([X_r]\) is the nodal displacement vector, and the subscripts \(r\) and \(e\) represent the freedoms of the rigid frame and the elastic plate, respectively.

Since the motion of a rigid body can be expressed by the six-degree-of-freedom motion of a point on the body, the partial freedom \([X_e]\) of the response can be reduced to

\[
\begin{bmatrix}
X_e \\
X_r
\end{bmatrix} = \begin{bmatrix}
T & 0 \\
0 & I
\end{bmatrix} \begin{bmatrix}
X_g \\
X_r
\end{bmatrix} = \{X\} \tag{2}
\]

where \([T]\) is the coordinate transformation, \([I]\) is an identity matrix, \([X_g]\) is the vector of displacement of a point representing the motion of the rigid frame. Substituting Eq. (2) into Eq. (1) gives

\[
\begin{bmatrix}
0 & T \\
I & 0
\end{bmatrix} [M] \begin{bmatrix}
0 & T \\
I & 0
\end{bmatrix} \begin{bmatrix}
\dot{X}_g \\
\dot{X}_r
\end{bmatrix} + \begin{bmatrix}
0 & T \\
I & 0
\end{bmatrix} [K] \begin{bmatrix}
0 & T \\
I & 0
\end{bmatrix} \begin{bmatrix}
X_g \\
X_r
\end{bmatrix} = \begin{bmatrix}
0 & T \\
I & 0
\end{bmatrix} \{F_r\}. \tag{3}
\]

In this study, an external force is assumed to be applied by the vertical motion of four wheels in the system. Then, the external force vector in Eq. (3) can be expressed by

\[
[F_r] = [k] \delta - [X_r] \tag{4}
\]

\[
[F_e] = 0
\]
where \([k]\) and \([\delta]\) are the spring constant matrix of the spring suspensions and the displacement vector of the wheels (see Fig. 1), respectively.

Substitution of \([F_r]\) in Eq. (4) into Eq. (3) and rearrangement gives

\[
\begin{bmatrix}
M_{11a} & M_{12a} \\
M_{21a} & M_{22a}
\end{bmatrix}
\begin{bmatrix}
\ddot{X}_y \\
\ddot{X}_x
\end{bmatrix}
+
\begin{bmatrix}
K'_{11a} & K_{12a} \\
K_{21a} & K_{22a}
\end{bmatrix}
\begin{bmatrix}
X_y \\
X_x
\end{bmatrix}
=
\left[
\begin{bmatrix}T'_y\end{bmatrix}[k][\delta]
\right]
F_x
\]

where \([K'_{11a}] = [K_{11a}] + [T'_y][k][T].\)

Equation (5) is transformed into the modal domain using the following modal coordinate transformation:

\[
[X_a] = \begin{bmatrix}
X_y \\
X_x
\end{bmatrix} = [ \Phi_a ][\xi]
\]

where \([\Phi_a]\) is the eigenvector matrix of the system and \([\xi]\) is a modal coordinate vector. Next, we apply harmonic excitation. Substituting Eq. (6) into Eq. (5) and assuming proportional viscous damping, we obtain the uncoupled equations of motion in frequency domain as follows:

\[
\left(-\omega^2[m] + j\omega[c] + [k]\right)[\xi] = [f]
\]

\[
[\xi] = \left(-\omega^2[m] + j\omega[c] + [k]\right)^{-1}[f]
\]

where \([m], [c], [k]\) are the modal mass, the modal damping and the modal stiffness, respectively, \([f]\) is the load vector at the modal coordinate, \(j\) is \(\sqrt{-1}\), and \(\omega\) is the angular frequency.

The nodal displacements at each of the node points in the shell structure can be obtained by the superposition of the normal modes as follows:

\[
[u_j] = [X] = [\Phi][\xi]
\]

The normal displacement on the plate structure in the system is then used as the boundary condition for the Rayleigh integral in the acoustic analysis presented in the next section.

### 2.2 Acoustic analysis

In the present study, sound power radiated from the vibrating plate is approximately computed by the Rayleigh integral method. The plate structure is assumed to an infinite rigid baffle, as shown in Fig. 2. With reference to Fig. 2, the Rayleigh integral can be formulated as

\[
P_q = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \frac{j\rho c k e^{j(\alpha x + \beta y)}}{2\pi r} \cdot \left(\delta_1 + \delta_2 + \delta_3 + \cdots + \delta_{12}xy\right) dx dy
\]

where \(\alpha = ka\sin\theta\cos\phi\) and \(\beta = kb\sin\theta\sin\phi.\)

Equation (14), then, can be derived

\[
P_q = \frac{j\rho c k e^{j(\alpha x + \beta y)}}{2\pi r}
\]
\[ \{ \delta_1 A_0 B_0 + \delta_2 A_1 B_0 + \delta_3 A_0 B_1 + \cdots + \delta_{12} A_1 B_3 \} \]

where

\[ A_m = \int_{-\frac{a}{2}}^{\frac{a}{2}} x^n e^{i \omega t} dx, \quad B_m = \int_{-\frac{b}{2}}^{\frac{b}{2}} y^m e^{i \beta y} dy \]

\((m = 0 \sim 3)\).

In the present study, the evaluated parameter is the value of total radiated sound power. The radiated sound power \(W\) can be obtained by integrating the far-field acoustic intensity over a hemispherical surface centered on the plate. That is,

\[ W = \int_S I(r_s) dS \]  \hspace{1cm} \text{[W]} \hspace{1cm} \text{(15)}

where \(I(r_s)\) is the acoustic intensity and \(r_s\) is the position vector of the elemental surface \(dS\).

In discretized form for numerical computation, the radiated sound power \(W\) of Eq. (15) can be numerically represented by

\[ W = \frac{2 \pi r^2}{\rho c^2} \sum_{i=1}^{m} p^2_x(l) \]  \hspace{1cm} \text{[W]} \hspace{1cm} \text{(16)}

where \(p_x(l)\) is the sound pressure at the \(l\)th observation point, \(2 \pi r^2\) is the surface area of the hemisphere with radius \(r\), and \(m\) is the number of the observation point.

### 2.3 Particle swarm optimization

The particle swarm optimization (PSO), originally developed by Kennedy and Eberhart, was discovered through the simulation of a simplified social behavior of organisms such as bird flocking and fish schooling, and comprises a very simple concept that requires only primitive mathematical operators, and is computationally inexpensive in terms of both memory requirements and speed.

Let \(p\) be the size of the PSO population. For particle \(d\) \((d = 1, 2, \ldots, p)\), Kennedy and Eberhart initially proposed that, for simulating the social behavior of birds, the position \(x^d\) in PSO be updated as

\[ x^d_{k+1} = x^d_k + v^d_k \]  \hspace{1cm} \text{(17)}

and the velocity \(v^d\) be updated as

\[ v^d_k = \alpha v^d_k + c_1 r_1(p^d_k - x^d_k) + c_2 r_2(p^d_k - x^d_k) \]  \hspace{1cm} \text{(18)}

where the subscript \(k\) is a time increment counter, \(r_1\) and \(r_2\) are stochastic factors in the range \([0,1]\), \(c_1\) and \(c_2\) are constants, called acceleration coefficients, that are recommended by the developers and that control the maximum step size the particle can perform. \(p^d_k\) represents the best ever position (i.e. the best position of the \(k\)-th particle that minimizes the objective function until the \(k\)-th optimization process) of particle \(d\) at time \(k\), and \(p^d_k\) represents the global best ever position (i.e. the global best position among all particles that minimizes the objective function until the \(k\)-th optimization process) in the swarm until time \(k\).

Furthermore, by adding a new inertia weight \(\lambda\) into Eq. (18), a new modified formulation of PSO was proposed by Eberhart et al.\(^{(12)}\) as follows:

\[ v^d_{k+1} = \lambda v^d_k + c_1 r_1(p^d_k - x^d_k) + c_2 r_2(p^d_k - x^d_k) \]  \hspace{1cm} \text{(19)}

They proposed \(0.8 < \lambda < 1.4\) as the appropriate range of \(\lambda\). The inertia weight, which is a user-specified parameter, is used to control the impact of the previous historical values of the particle velocities on the current velocity. A larger inertia weight facilitates flying toward global exploration, searching a new area, while a smaller inertia weight tends to facilitate fine-tuning of the current search area. Appropriate selection of the inertia weight and acceleration coefficients can provide a balance between global and local exploration abilities, and thus, on average, the optimal solution can be found in fewer iterations. Equation (19) is employed in order to obtain the new velocity of the particle according to its previous velocity and the distance of its current position from both its own best historical position and its neighbors’ best position. Then, the particle \(d\) in PSO moves to a new position according to Eq. (17).

In the present study, the PSO algorithm is proposed for acoustic radiation optimization.

### 2.4 PSO algorithm for acoustic radiation optimization

In this section, we describe the outline of the implementation of PSOA for optimizing the sound power radiated from a vibrating structure.

As mentioned above, we employ the average radiated sound power over a frequency range of interest, which is calculated by evaluating the total radiated sound power \(W\) expressed by Eq. (16) at a closely spaced frequency increment count \(N_f\) between \(\omega_{\text{min}}\) and \(\omega_{\text{max}}\) as the objective function:

\[ H = \frac{1}{N_f} \sum_{i=1}^{N_f} W(\omega_i) \]  \hspace{1cm} \text{(20.a)}

Minimize \(H\)  \hspace{1cm} \text{(20.b)}

where \(\omega_i\) is an angular frequency. \(H\) is the average radiated sound power in the frequency range of interest.

The inequality boundary conditions for design variables are expressed as:

\[ z(j) \leq z(j) \leq \bar{z}(j) \hspace{1cm} (j = 1 \sim n) \]

where \(z(j)\) are the design variables with lower boundary \(\underline{z}(j)\) and upper boundary \(\bar{z}(j)\), \(j\) is the number of the design variable. In this case, the design variables used in the optimization process represent the heights of the bending line along the \(y\)-axis into the \(z\)-axis. The set of heights is considered as the position of each particle in the present optimization.

### 3. Two Examples

#### 3.1 Rectangular plate clamped at four sides

As the first example, we investigate the performance of PSOA for an acoustic radiation optimization problem.
Figure 3 shows a schematic view of a structure that is assumed as a clamped aluminum plate in an infinite baffle. The aluminum plate has dimensions of 0.45 × 0.4 m with thickness 0.001 m. The material properties of the aluminum are a Young’s modulus of 73 GPa, a Poisson’s ratio of 0.34, and density of 2690 kg/m³. The acoustic medium is air, having a density of 1.21 kg/m³ and a wave velocity of 343 m/s. The harmonic force with amplitude 1 N is applied at the center point of the plate over the frequency range of interest. As shown in Fig. 3, the plate is bent along nine parallel lines. The design variables of this optimization are the heights of the bending. Each height is independent. The feasible height is limited to within the range of −0.01 ~ +0.01 m.

As mentioned in section 2.4, we applied the objective function expressed by Eq. (20). That is, the average radiated sound power over 1 ~ 300 Hz in 1 Hz increments. Observation points are symmetrically employed at 13 points on the hemispherical surface 0.8 m away from the center of the plate (see Fig. 4).

Finite element analysis was used to obtain the first ten natural modes at 50.7, 95.2, 110.2, 147.9, 166.7, 202.6, 212.1, 234.1, 263.3, and 289.3 Hz, respectively. Of these, the first, the fifth, the sixth and the tenth modes produced noise, whereas the others were quiet due to volume velocity cancellation. Consequently, the optimization problem in this section comes down to the problem of minimizing the total radiated sound power generated by the well-radiating modes only (see Fig. 5). We then developed an optimization design procedure consisting of three steps: vibration analysis by FEM, acoustic radiation analysis by Rayleigh integral, and optimization computation by PSOA.

Figure 6 shows the radiated sound power level of the optimally bent plate using the optimization design procedure together with that of the original flat plate. The figure shows that the first natural frequency is shifted up 50.7 Hz to 408 Hz and that the sound power level is successfully reduced in the frequency range by the optimization. Figure 7 shows the schematics of the optimally bent plate by the optimization procedure. Figure 8 illustrates the drastic reduction of the objective function in early iterative stage by the proposed optimization method and the eventual determination of the optimal solution.
3.2 Plate model with four spring suspensions

In order to further verify the reliability of the method presented herein, bending design of a plate model with four spring suspensions, shown in Fig. 1, is assumed as a simple model of a vehicle and optimized to minimize its radiated sound power. The result is then compared to an optimum solution obtained by a GA.

The finite element mesh model consists of 165 nodes, 140 four-node shell elements, four spring elements and a rigid frame. The material properties, the size and the observation points for the plate are defined as in section 3.1. The model is excited by the displacement excitation through four wheels. The wheels connected to springs \( k_2 \) and \( k_3 \) are excited by displacement \( 0.01 e^{j\omega t} \). The wheels connected to springs \( k_1 \) and \( k_4 \) are excited by displacement \( 0.01 e^{j(\omega t - 0.0162)} \). The rotational velocity \( \omega \) changes from 0 rad/s to 600π rad/s, which is the frequency range of 0 Hz to 300 Hz. Thirteen design variables limited by lower and upper bounds to ±0.025 m, as shown in Fig. 11, are selected for the inside plate only, because the outside part of the objective structure is a rigid frame, as shown in Fig. 1.

In implementing the optimization process, the objective function is the average sound power radiated from the aluminum plate over a given frequency band 0–300 Hz and is calculated by means of the Rayleigh integral, where

\[
A\omega \approx \frac{1}{2\pi} \int_{0}^{300} P_\omega \, d\omega
\]

the acoustic medium is air having a density of 1.21 kg/m\(^3\) and a wave velocity of 343 m/s.

Finite element analysis revealed natural modes at 3.8, 6.6, 6.6, 70.2, 114.6, 132.4, 150.4, 177.9, 224.6, 236.3, and 243.2 Hz in the frequency range of interest. The plate suspended by four springs oscillates as a rigid plate in the first three natural modes and oscillates as an elastically deformed plate in other natural modes. According to the vibro-acoustic analysis reported in the present paper, the sound powers due to the fourth, sixth, eighth, and tenth modes are much larger than those due to the fifth, seventh, ninth, and eleventh modes in the frequency range (see Fig. 9). This means that the odd-odd modes, which are the 4th, 6th, 8th and 10th modes, radiate louder noise than even-odd and even-even modes, which are the 5th, 7th, 9th, and 11th modes.

In order to make ensure the effectiveness and viability of the optimal solution obtained by PSOA, the optimization based on PSOA is compared to that based on Steady State Genetic Algorithms (SSGA) with tournament selection, two-point crossover operator, a mutation operator and an elitism mechanism. Note that we employed the following values assigned to each of the parameters in both algorithms.

(A) PSOA
- Size of the swarm: 100
- Inertia weight \( \lambda_0 \): 0.98
- Acceleration coefficients \( (c_1 = c_2) \): 1.0
- Maximum Iteration number: 300

(B) SSGA
- Population: 100
- Crossover probability: 0.77
- Mutation probability: 0.0077
- Maximum number of generations: 300

Figure 9 shows a comparison of sound powers radiated from the optimized plate by the two methods. We
Fig. 10 Comparison of the convergence history by SSGA and PSOA

Fig. 11 Optimized geometry of the bent plate by SSGA and PSOA

can see in Fig. 9 that all large sound radiating modes have been moved successfully over the given frequency range by both methods. Figure 10 shows the average radiated sound power from the model at each iterative process for both optimization methods. We can see that the optimization by PSOA converged faster and was got better result than the optimization by SSGA. The optimized bending by both methods is illustrated in Fig. 11. These results demonstrate the validity and reliability of the optimization based on PSOA.

4. Conclusion

The present paper introduced a structural bending optimization method by which to minimize radiating noise based on a new evolutionary population-based heuristic algorithm called the particle swarm optimization algorithm (PSOA). The optimization procedure uses FEM for structural analysis, the Rayleigh integral method for acoustic analysis and the PSOA for optimization. The height of bending is adopted as a design variable. The objective function to be minimized is the average sound power radiated from an objective structure over a given frequency range. Two basic examples demonstrated the capability of PSOA for structural bending optimization in order to reduce noise. Also, in order to ensure the effectiveness and viability of the optimal solution obtained by PSOA, the optimized result based on PSOA was compared to that based on Steady State Genetic Algorithms (SSGA).

References


