Numerical Analyses and Experiments on the Characteristics of Ball-Type Constant-Velocity Joints*

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Ball-type constant-velocity joints are extremely important components of the front wheel drive systems of cars. However, the method to analyze the characteristics of this type of joint has not been established. The authors previously presented papers on the analysis based on statics, in that no frictional forces inside the joints were taken into account. Recently, the authors presented advanced analysis based on dynamics, in which the frictional forces were considered. In this paper, the numerical analyses of a ball fixed joint (BJ) and a double offset joint (DOJ) used for the front wheel drive of a car are carried out using the simultaneous equations induced in the former analysis and the simultaneous differential equations induced in the latter analysis. Contact forces acting inside DOJ are measured using piezoelectric sensors. The results of the numerical analyses based on dynamics show better coincidence with the measured value than those based on statics.

Key Words: Machine Element, Universal Joints, Constant-Velocity Joints, Ball-Type, Numerical Analysis, Statics, Dynamics, Characteristics, Contact Force, Friction

1. Introduction

Ball-type constant-velocity joints are used extensively in the drive shafts of machines. This component is extremely important in the drive system of automobiles. Analyzing the characteristics of this component with respect to the forces acting inside it, the secondary couples, rotational speed variation, etc., is of great value for optimum design and application.

The authors have previously carried out analysis of this component based on statics for the relative displacement between the parts and the internal forces without taking frictional force into account. Watanabe and coworkers and Ichikawa and Watanabe presented analyses on the kinematic characteristics of this component.

In these previous studies, it can be said that the prerequisites for analyses do not fully reflect the operating conditions of actual universal joints.

In order to simulate the actual operating conditions of universal joints more accurately, the authors recently carried out analysis based on dynamics taking the frictional force acting on the contact area of their parts into consideration. The analysis resulted in simultaneous ordinary differential equations relating to the motion of each part of the components.

In this paper, both the simultaneous ordinary differential equations and the simultaneous nonlinear equations presented in Ref. (1) are applied to the ball-type constant-velocity joints used practically for the front wheel drive shafts of automobiles. Numerical analyses are carried out, and a comparative study of the results obtained is performed. Furthermore, the adequacy of the analysis is verified by experiments.

2. Constant Velocity Universal Joint Adopted for Analyses and Experiments, and Symbols Used

Numerical analyses were carried out for Ball fixed joint (BJ) and Double offset joint (DOJ). Experiments were carried out for DOJ. Figure 1 shows DOJ.

Table 1 shows the conditions of the numerical analyses and the experiments. It was assumed that both BJ and DOJ are free from manufacturing errors such as pitch er-
ror of the track. As for the clearance between the ball and the inner/outer race track, and between the cage spherical surface and the inner/outer race, the median of the range caused by the dimensional tolerance of each part was used. Pre-loading was provided between the ball and the cage window guide-plane.

In the analyses(1),(7) a right-hand orthogonal coordinate system was set in which origin O coincides with the intersection point of the inner race axis and outer race axis, the \( xy \)-plane includes both axes, and the \( yz \)-plane coincides with the homo-kinetic plane (see Appendix). In the numerical analyses and experiments shown hereafter, the inner race axis is placed in the region of \( x > 0 \) and \( y < 0 \) on the \( xy \)-plane, the outer race axis is placed in the region of \( x < 0 \) and \( y < 0 \), the reference state of phase angle \( \phi \) of the outer race track is such that ball \( B1 \) is in the region of \( y > 0 \) on the \( y \)-axis, and the positive direction is assumed as the counterclockwise direction observing from inner race axis side. The vector of torsional moments \( T1 \) and \( T2 \) exerted on the inner race axis and outer race axis should be in the \( x > 0 \) and \( x < 0 \) directions, respectively. Such a case in which the rotational direction of the axis agrees with the positive direction of the phase angle is referred to as “Inner race drive”, and the case that the rotational direction is opposite, is referred to as “Outer race drive”.

Key symbols, other than the above, used in this paper are summarized below.

- \( 2\theta \) : Operating angle (deg)
- \( \sigma \) : Coefficient of friction
- \( PN \) : Contact force between ball and track when \( 2\theta = 0 \) (see Appendix)
- \( P1, P3 \) : Contact force between ball and inner/outer race
- \( P2, P4 \) : Contact force between ball and inner/outer race track where torque is carried
- \( P5 \) : Contact force between ball and inner/outer race track where torque is not carried
- \( P6 \) : Force in \( x > 0 \) direction acting from ball on cage window
- \( F2 \) : Force acting from cage pillar spherical surface (see Fig. 1) on outer race inner surface
- \( M2XY \) : \( xy \)-plane component of secondary couple induced to outer race axis
- \( M2Z \) : Component in \( z \)-axis direction of the same

3. Results of Numerical Analyses

Figure 2 shows the results of analysis of BJ based on statics. \( P3 \) and \( P4 \) are shown here. \( P1 \) and \( P2 \) are theoretically \( P1 = P3 \) and \( P2 = P4 \). Figure 3 shows the angle formed by the common tangential planes at the contact track where torque is carried.
In the case of BJ, however, since the inner sphere and the outer sphere are concentric, the moment balance on the cage is governed by $P5\sim P6$. In BJ, a peak of $P5\sim P6$ is generated by $P4$ and $P2$ even in the valley region of $P3$ and $P1$ at $\phi = 60\sim120^\circ$ under the condition that $2\theta$ is large.

Figure 6 shows the results of the analysis of BJ based on dynamics. It is noted that $P1$ and $P3$, and $P2$ and $P4$ are also nearly equal, although $P1$ and $P2$ are not shown here. Figure 7 shows the results of the analysis of DOJ based on dynamics. In this case also, $P1$ and $P3$ are nearly equal. $P2$ and $P4$ are not shown here since these are small although not zero.

The trend of the peaks and valleys of $P3$ and $P5\sim P6$ is similar to that of the analysis based on statics for both BJ and DOJ. However, it is noted that the phase angles where they are generated shift in the direction of rotation of the shaft (to the right for inner race drive, and to the left for outer race drive in the figures). The amount of the shift is more marked if $\sigma$ is large or $2\theta$ is small. In the case of DOJ, the reason for these phenomena is considered as follows; due to a frictional force acting from the outer race track on the ball, the ball is pushed towards the wedge angle narrowing side against the axial component of the frictional force. Therefore the shift of the peak is significantly due to the narrow wedge angle. In the case of BJ, however, since the inner sphere and the outer sphere are concentric, the moment balance on the cage is governed by $P5\sim P6$. In BJ, a peak of $P5\sim P6$ is generated by $P4$ and $P2$ even in the valley region of $P3$ and $P1$ at $\phi = 60\sim120^\circ$ under the condition that $2\theta$ is large.

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Although the rotational speed condition used for the numerical analyses mentioned above is 200 rpm, no significant difference is caused in the results even if any rotational speed in the range of 10 – 1000 rpm is used. Experiments shown below were carried out at 10 rpm due to limitations of the measurement method.

4. Experiments

4.1 Measurement of track load and outer race cylindrical surface load

4.1.1 Measurement object Forces acting from the ball of DOJ 82 on the outer race track and forces acting from the cage pillar sphere on the outer race inner cylindrical surface were measured. Clearances between the ball and the inner race track/outer race track/cage guide-plane, and between the cage and the inner race/outer race were set to the same conditions as used for numerical analysis. Lubricants used were commercially available types A and B grease for constant velocity universal joints. The value of coefficient of friction $\sigma$ measured using the Savan-type friction and abrasion testing machine$^{(10)}$ are approximately 0.05 for A and approximately 0.1 for B.

4.1.2 Measurement method As shown in Fig.10, an outer race equipped with two piezoelectric sensors for the measurement of three components of a force was produced. One sensor is for track load measurement and the other is for contact force measurement between the cage and outer race. The former is installed so that the normal direction of the sensor agrees with the direction of contact angle between the track and the ball, and the latter is installed in the direction perpendicular to the axis of the inside cylindrical surface of the outer race. The rigidity of the sensor is 300 kgf/µm in the normal direction, and 100 kgf/µm in the shearing direction. The rigidity in the normal direction is sufficiently high compared with that of the elastic contact between the ball and track. The track surface and the inside cylindrical surface were machined by grinding to be formed into one piece after the piezoelectric sensors were assembled. The sensor cables were wound around the shaft of the inner race, and measurements were taken in the range in which rotation was possible. Rotational speed was set at 10 rpm. Figure 11 shows the data acquisition system.

4.1.3 Results of measurements Figure 12 shows the results of the measurements of $P_3$. These correspond to Fig. 7 (a) through (d) which show the results of the analyses based on dynamics. The comparison of the measurement and analysis results reveals that they agree fairly well under all conditions considered.

Figure 13 shows the results of the measurements of $F_{2J}$. Compared with those in Fig. 8, it appears that although the tendencies are basically similar, the degree of
coincidence is inferior to that of $P_3$.

The forces in the outer race axis direction detected by two piezoelectric sensors were summed six times with phase angle correction, and an induced thrust force with a cyclic of 60 degree was obtained. Figure 14 shows the 6th-order thrust force components with regard to shaft rotation obtained by Fourier analysis. In Fig. 14, the results of analysis based on dynamics are also shown. Experimental values obtained by two experiments exhibit considerable scattering and do not exhibit good coincidence with analytical values.

$F_{2J}$ or induced thrust force is a resultant force of the forces acting at several locations and the contribution of the frictional force is significant. It is considered necessary that in order to improve the accuracy of simulation of analysis, processing of the frictional force should be improved.
Fig. 8  Analysis of DOJ based on dynamics (contact force — from one cage pillar to outer race)

(a)  $F_{2J} \sigma = 0.05$ Inner race drive
(b)  $F_{2J} \sigma = 0.05$ Outer race drive
(c)  $F_{2J} \sigma = 0.1$ Inner race drive
(d)  $F_{2J} \sigma = 0.1$ Outer race drive

Fig. 9  Angular displacement of inner race against outer race in DOJ

(a)  $\phi = 0.05$ Inner race drive
(b)  $\phi = 0.05$ Outer race drive
(c)  $\phi = 0.1$ Inner race drive
(d)  $\phi = 0.1$ Outer race drive
(e)  Static analysis

Fig. 10  Outer race equipped with piezoelectric sensors

Fig. 11  Data acquisition system

4.2 Measurement of secondary couple

The secondary couple induced at the outer race axis of DOJ is $M_{2XY} = (T_2) \tan \theta$ and $M_{2Z} = 0$ according to the analysis based on statics\(^{(1)}\). However, it is known that with actual universal joints, the average of $M_{2XY}$ is affected by torque loss\(^{(9,11)}\). The secondary couple (average) induced at the outer race axis of DOJ 75 was measured according to the method described in Ref. (11). The results of the measurement are shown in Fig. 15. For comparison purposes, dynamic analysis of DOJ 75 was also performed. Since the analytical value varies with a cyclic of 60 degrees, the average value was adopted and plotted in the
same figure. It is noted from the figure that measurement and analytical values agree fairly well.

5. Conclusions

Using the simultaneous nonlinear equations derived by statics analysis\(^1\) and the simultaneous ordinary differ-
The differential equations derived by dynamics analysis\(^7\), numerical analyses were successfully performed for BJ and DOJ of ball-type constant-velocity joints. For DOJ, forces acting on the inside and secondary couple were measured. As a result, the following findings were obtained:

(1) Internal forces obtained by static analysis without taking frictional forces into account have a phase shift compared with experimental values. However, their magnitude and distribution exhibit similar tendencies. Static analysis appears to aid in the understanding of fundamental characteristics.

(2) Track loads and secondary couples obtained by dynamic analysis taking frictional forces into account showed fairly good agreement with experimental values irrespective of the direction of driving and magnitude of the coefficient of friction of the lubricants. Dynamic analysis taking frictional forces into account appears to be effective as the means to determine the characteristics of the universal joint in operation.

(3) Compared with a case of track loads, normal force acting from the cage on the outer race cylindrical surface and rotational 6th-order components of a thrust force induced at the outer race obtained by dynamic analysis do not agree well with experimental values. This appears to be because these are the resultant values of the forces acting at several locations and the contribution of the frictional force is significant.

The authors thank Mr. K. Kawashima of NTN Corporation for his contribution to the development of the measurement method of internal forces of DOJ.

6. Appendixes

6.1 Coordinate system

Coordinate systems used in the analyses\(^{(4),(7)}\) are shown here again.
6.2 Contact force $PN$

Contact force $PN$ is a normal load generated by loading torque $T$ in the contact area between the ball and the track with operating angle $2\theta = 0$. This is expressed by the following equations (see Fig. 17 for BJ):

$$
DOJ: \quad PN = \frac{T}{n \cdot R \cdot \sin \alpha} \quad (1)
$$
$$
BJ: \quad PN = \frac{T}{n \cdot \sqrt{R^2 - F^2} \cdot \sin \alpha} \quad (2)
$$

in which $T$: Torque
$n$: Number of balls
$R$: Diameter of pitch circle
$\alpha$: Contact angle (design value)
$F$: Amount of offset

6.3 Inner/outer race relative displacement angle $\Omega$

The prerequisite for analysis$^{(7)}$ states “The outer race rotates at constant velocity”. In contrast, the inner race rotates with an advance or delay as much as $\Omega$ with regard to the outer race due to the clearance and elastic deformation inside the constant velocity universal joint. Figure 18 shows this state diagrammatically. Advance or delay is determined by the direction of torque. In the current analysis, it is assumed that if a torque acts in the directions of $a_T1$ and $a_T2$, then $\Omega$ is positive. In the meantime, $\Omega$ is handled as an unknown quantity from the analysis standpoint$^{(7)}$.

References