Vibration Control Using a State Observer that Considers Disturbances of a Golf Swing Robot*

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In this paper, optimal control of a golf swing robot that is used to evaluate the performance of golf clubs is described. The robot has two joints, a rigid link and a flexible link that is a golf club. A mathematical model of the golf club is derived by Hamilton’s principle in consideration of bending and torsional stiffness and in consideration of eccentricity of the center of gravity of the club head on the shaft axis. A linear quadratic regulator (LQR) that considers the vibration of the club shaft is used to stop the robot during the follow-through. Since the robot moves fast and has strong non-linearity, an ordinary state observer for a linear system cannot accurately estimate the states of the system. A state observer that considers disturbances accurately estimates the state variables that cannot be measured. The results of numerical simulation are compared with experimental results obtained by using a swing robot.

Key Words: Robot, Motion Control, Optimal Control, Vibration Control, Disturbance Observer, Golf Swing, Measurement and Control

1. Introduction

There have been many studies on flexible manipulators(1) – (4), but few studies have focused on fast motion of flexible manipulators. Golf swing robots, which can be regarded as one type of fast motion manipulator with a flexible link, have been used to evaluate the performance of golf clubs and golf balls. However, since joints of a swing robot are connected by linkages such as gears and belts and relative motion is decided previously, users of the robot can adjust only its initial pose and drive speed. Furthermore, since each joint of most golf swing robots is braked down to stop forcibly after the downswing, golf clubs and swing robots are vibrated by the shock of braking, and this vibration reduces the degree of accuracy of the evaluation results. Many researchers have studied torque plans in order to accurately reproduce human golf swinging motions(5) – (8). Suzuki and Inooka(9) investigated a new golf-swing robot model in which flexibility of the golf club is taken into consideration, and they studied torque plans emulating the golfer’s skill. In those studies, wrist joint was considered as a passive joint containing only a brake mechanism that emulates the wrist cocking action of a golfer, and it was assumed that the swing motion is executed by using dynamic interference forces, i.e., centrifugal force, Coriolis’s force and gravity. Suzuki and Inooka(9) studied optimal feedforward torque plans utilizing shaft elasticity in the case in which gravity is ignored, and they have shown that the brake mechanism and the flexibility of a golf club play important roles in the golf swing. Because the robot proposed by them utilizes dynamic interference forces, we cannot linearize the dynamics of the robot by compensating these forces. However, there has not been sufficient study on methods to reduce the shock and vibration that occurs at completion of the swing.

In this study, optimal control is applied to a golf swing robot to suppress the shock and vibration due to braking after the downswing. The robot consists of two joints, a rigid link and a flexible link that is a golf club. The robot also has a hook mechanism to lock the wrist joint. The golf club is modeled as a flexible beam in consideration of bending and torsional stiffness and in consideration of eccentricity of the center of gravity of the club head on the shaft axis. Because of the boundary conditions of the club head, the bending in the swing plane and the torsional vibration of the club shaft are coupled. The natural frequencies and vibration modes are obtained by numerical and experimental modal analysis. The swing robot pulls up
the golf club (backswing) and then swings it in a swing plane. The backswing action is performed by a digital servo controller. Feedforward torque is applied only to the shoulder joint during the downswing. Control torque is applied to the shoulder and wrist after the club head hits a golf ball. A linear quadratic regulator (LQR) that takes into account the vibration of the club shaft is applied during the follow-through to stop the robot, and the degree of shock is decreased.

Since it is difficult to measure all of the state variables, a state observer is employed for the state feedback control system. An observer for estimating harmonic disturbance has been used in previous studies in which estimated disturbance was fed forward to the system to cancel actual disturbance\cite{10,11}. Good control performances were obtained in those studies, however, it is difficult to model the dynamics of the disturbance in this study because of Coriolis’s force, centrifugal force, gravity and constrained torque due to the mechanical hook. It is especially difficult to model the constrained torque due to the mechanical hook. Moreover, these disturbances cannot be modeled as sinusoidal ones. An ordinary state observer for a linear system cannot accurately estimate the states of the system because of dynamics not modeled or uncertainty due to those nonlinear forces. Although a nonlinear observer can estimate the state vector more accurately than a linear one can, a large amount of computing time and a large amount of memory space are required. To overcome these problems, we use a state observer that takes into account disturbances to improve the performance of the state feedback control. This observer cannot only accurately estimate the state of the system but can also estimate the disturbance using only a simple algorithm. Moreover, an exact model of the system, a model of the disturbance and an inverse model of the nominal system are not required.

Numerical simulations and experiments are carried out using a small swing robot. The abovementioned state observer is active through the swing action and improves the performance of the state estimation and the state feedback control.

2. Assumption and Modeling

2.1 Assumptions

The system of orthogonal axes $(x, y, z)$ is taken as shown in Fig. 1, and the robot swings a golf club in the swing plane $o – xy$. The angle between the direction of gravity vector $g$ and $x$-axis is $\alpha$ [rad]. The angle $\alpha$ depends on physique of the golfer and length of the golf club. We assume that the robot has two joints, two rigid links that are modeled on the arm of the golfer and the grip of the club. The robot also has a hook mechanism to lock the wrist joint\cite{9}. The golf club is modeled as a flexible beam in consideration of bending and torsional stiffness and in consideration of eccentricity of the center of gravity of the club head on the shaft axis. Figure 2 shows the coordinate system in the $xy$-plane. The local coordinates $\xi_1 - \xi_1, \eta_1 -$ $\xi_3, \eta_3$ and $\zeta_1 - \xi_4, \eta_4, \zeta_4$ are the coordinate fixed on the first link, the second link, and the tip mass as the club head. The $\xi_3 - \eta_3, \zeta_3$ is the local coordinate fixed on the second link, where $\xi_3$ is the fixed point of the second link and the flexible link. These coordinates are right hand coordinate systems. Angles $\theta_1$ and $\theta_2$ are the absolute rotating angles of the first joint and the second joint, respectively. The lengths of the first link and the second link are $L_1$ and $L_2$, respectively. The club shaft is assumed to be homogeneous and initially straight along the $\xi_2$-axis shown in Fig. 2. The flexible shaft is cantilevered at the rigid grip link, and the properties of the shaft are as follows: a uniform cross-sectional area $A_3$, diameter $d_3$, length $L_3$. Young’s modulus $E_3$, area moment of inertia $I_3$, shear modulus $G_3$, polar moment of inertia $I_{p3}$, mass per unit volume $\rho_3$ and eccentricity of the center of gravity of the club head $L_4$. Deflections of the flexible link along the $\eta_3$-axis and the $\zeta_3$-axis are $v_\eta$ and $v_\zeta$, respectively. Torsion angle of the flexible link is $\phi$. Position vectors $\eta_1, \eta_2, r_4$ and $r_5$ are the centers of gravity of the first link, second link, tip mass and element of flexible link at position $\xi$.
on the $\xi_3$-axis, respectively. The swing robot used in the experiment is shown in Fig. 3. A flexible beam made of ABS resin is used instead of a golf club due to restrictions of experimental equipment.

A mathematical model of the golf club is derived by Hamilton’s principle in consideration of bending and torsional stiffness and in consideration of eccentricity of the center of gravity of the club head on the shaft axis. The masses of the first link, the second link and the tip mass are $m_1$, $m_2$, and $m_4$, respectively. The moments of inertia of the first link and the second link are $J_1$ and $J_2$, respectively. The moments of inertia of the tip mass around the axes $\xi_4$, $\eta_4$ and $\xi_3$ are $J_{4\xi}$, $J_{4\eta}$ and $J_{4\xi}$, respectively. We define the following operators,

$$(\bullet)^{\prime} = \frac{\partial}{\partial t}(\bullet) \quad \text{and} \quad (\bullet)^{\prime\prime} = \frac{\partial}{\partial t}(\bullet)^{\prime}.$$ 

For example,

$$\ddot{v}_{\eta}(\xi,t) = \frac{\partial^4}{\partial t^2 \partial \xi^2}(v_{\eta}(\xi,t)).$$

Kinetic energies of the first link, second link and tip mass are expressed as

$$T_1 = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} J_1 \dot{\theta}_1^2, \quad T_2 = \frac{1}{2} m_2 \dot{r}_2^2 + \frac{1}{2} J_2 \dot{\theta}_2^2$$

$$\quad \text{and} \quad T_4 = \frac{1}{2} m_4 \dot{r}_4^2 + \frac{1}{2} J_4 \dot{\theta}_4^2$$

respectively, where $\Phi$ is a transformation matrix from the $\xi_3\eta_3\xi_2$ coordinate system to the coordinate system fixed on the tip mass and

$$\dot{\theta}_4 = [\dot{\varphi}(L_3,t) \ - \dot{\psi}_2(L_3,t) \ - \dot{\psi}_1(L_3,t)]^T$$

$$\quad \text{and} \quad J_4 = \text{diag}[J_{4\xi} \ J_{4\eta} \ J_{4\xi}].$$

Kinetic energy per unit length of the flexible link is described as

$$\mu_3 = \frac{1}{2} \rho_3 A_3 \dot{r}_4^2 + \frac{1}{2} \rho_3 I_3 \dot{\psi}_1^2(\xi,t).$$

$T$ is kinetic energy of the system defined by

$$T = T_1 + T_2 + \int_0^{L_3} \mu_3 d\xi + T_4.$$ 

Potential energies of the first link, second link and tip mass are expressed as

$$U_1 = -m_1 g^T r_1, \quad U_2 = -m_2 g^T r_2 \quad \text{and} \quad U_4 = -m_4 g^T r_4,$$

respectively, and potential energy per unit length of the flexible link is obtained as

$$u_3 = \frac{1}{2} E_3 I_3 \dot{\psi}_1^2(\xi,t) + \frac{1}{2} G_3 I_{3\psi} \dot{\psi}_1^2(\xi,t) - \rho_3 A_3 g^T r_3,$$

$$\quad \text{respectively}.$$ 

Potential energy of the system is given by

$$U = U_1 + U_2 + \int_0^{L_3} u_3 d\xi + U_4.$$ 

$L$ is the Lagrangian of the system defined by

$$L = T - U,$$

Hamilton’s principle is applied as

$$\int_{\tau_1}^{\tau_2} (\delta L + \delta W) dt = 0,$$

where

$$\delta W = \tau_1 \delta \theta_1 + \tau_2 (\delta \theta_2 - \delta \theta_1),$$

and $\tau_1$ and $\tau_2$ are the torques applied to the shoulder and wrist joints, respectively. Then we derive the equations of motion and the boundary conditions.

Ignoring the non-linear terms, the boundary conditions of the club head are obtained as

$$m_4 \ddot{v}_4(L_3,t) - \dot{\varphi}(L_3,t) m_4 L_4 - E_3 I_3 \ddot{\psi}_1(L_3,t) = 0,$$

$$J_{4\xi} \ddot{\psi}_1(L_3,t) + E_3 I_3 \ddot{\psi}_1(L_3,t) = 0$$

$$J_{4\xi} \ddot{\psi}_1(L_3,t) + E_3 I_3 \ddot{\psi}_1(L_3,t) = 0.$$ 

(9)

(10)

(11)

From these equations, we can know that the bending vibration in the swing plane and the torsional one are coupled to each other. Using the same time function $q(t) = \sin\omega t$, we assume the bending displacement $v_{\eta}$ and the torsional angle $\varphi$ to be

$$v_{\eta}(\xi,t) = q(t) \Gamma_\eta(\xi),$$

$$\varphi(\xi,t) = q(t) \Gamma_\varphi(\xi),$$

where $\Gamma_\eta$ and $\Gamma_\varphi$, respectively, are the mode shapes of bending and torsional angle, and $\omega$ is the circular frequency. We assume these mode shapes to be

$$\Gamma_\eta(\xi) = C_1 \left( \sin \frac{\lambda}{L_3} \xi - \sinh \frac{\lambda}{L_3} \xi \right),$$

$$C_2 \left( \cos \frac{\lambda}{L_3} \xi - \cosh \frac{\lambda}{L_3} \xi \right),$$

$$\Gamma_\varphi(\xi) = C_3 \sin \frac{\omega}{c} \xi,$$

(13.a)

(13.b)

where

$$\lambda^2 = \omega^2 L_3 A_3 \rho_3$$

$$c^2 = \frac{G_3}{\rho_3 E_3 L_3},$$

Substituting Eq. (12) into Eqs. (9)–(11), the frequency equation is derived as...
and then the natural frequencies for the coupling model are obtained as the eigenvalues of this matrix equation. Orthogonality relationships of modal functions are obtained as

\[
\rho I_p \int_0^{L} \Gamma_{\psi_i} \dot{\Gamma}_{\psi_j} d\xi + (J_{4\xi} + m_4 L_4^2) \Gamma_{\psi_i}(L_3) \dot{\Gamma}_{\psi_j}(L_3) + \rho A \int_0^{L} \Gamma_{\eta_i} \dot{\Gamma}_{\eta_j} d\xi + m_4 \Gamma_{\psi_i}(L_3) \dot{\Gamma}_{\eta_j}(L_3) + J_{4\xi} \Gamma_{\eta_i}(L_3) \dot{\Gamma}_{\psi_j}(L_3) - L_4 m_4 \Gamma_{\psi_i}(L_3) \dot{\Gamma}_{\eta_j}(L_3) - \Gamma_{\eta_j}(L_3) \dot{\Gamma}_{\psi_i}(L_3) = M_3 \delta_{ij},
\]

where \( \delta_{ij} \) is Kronecker’s delta. Figure 4 shows the changes in natural frequencies of the bending-torsion coupling model named ‘bending-torsion’ and the independent model, for which the bending-torsion coupling is neglected, named ‘bending’ or ‘torsion’. For example, we describe the \( i \)th natural frequency of the coupling model as ‘bending-torsion \( i \)th’. We use the parameters of the golf club shown in Tables 1 and 2 except for the eccentricity \( L_4 \). Natural frequencies of bending modes of the independent model do not change in Fig. 4. However, natural frequencies of the coupling model depend on the change in eccentricity of the center of gravity of the club head, \( L_4 \). Therefore we use the coupling model in this study. The natural frequencies and the vibration modes are obtained by numerical and experimental modal analyses. The bending displacement \( v_\eta \) and the torsional angle \( \varphi \) can be expressed as

\[
v_\eta(\xi, t) = \sum_{i=1}^{\infty} q_i(t) \Gamma_{\eta,i}(\xi), \quad \varphi(\xi, t) = \sum_{i=1}^{\infty} q_i(t) \Gamma_{\psi,i}(\xi),
\]

An approximate system of finite dimensional equations is obtained using a finite number of modes in Eq. (16). First two modes are sufficient to describe the dynamics adequately within the frequency bandwidth of interest in this paper.

Applying orthogonality relationships of modal functions for the equations of motion, the non-linear equations of motion of the finite dimensional model are given by

\[
J(x) \ddot{x} + D \dot{x} + Kx + h(x, \dot{x}) + g(x) - pu = 0,
\]

where

\[
x = [\theta_1 \quad \psi \quad q_1 \quad q_2]^T, \quad \psi = \theta_2 - \theta_1, \\
u = [\tau_1 \quad \tau_2]^T, \quad p = [I_{2 \times 2} \quad 0_{2 \times 2}]^T,
\]

and \( J, D \) and \( K \) are inertia, damping and rigidity matrices, and \( h, g \) and \( u \) are nonlinear force, gravity and input vectors, respectively. The wrist joint has a mechanical hook that constrains the relative angle of the wrist joint in \( \psi \geq -\pi/2 \) [rad] and the hook can achieve wrist cock action in the downswing \( ^{(5)} \). The nonlinear force vector \( h \) consists of centrifugal force, Coriolis’s force and constrained force of the hook.
2.2 Controller design

In this study, optimal control is applied to the swing robot to suppress the shock and vibration due to braking after the downswing. Equation (17) can be rewritten as

\[
\ddot{x} = -J^{-1}(x)Dx - J^{-1}(x)Kx + J^{-1}(x)pu + J^{-1}(x)[-h(x, \dot{x}) - g(x)].
\] (18.a)

Inertia matrix \(J(x)\) depends on \(\psi, q_1\) and \(q_2\). We set the terminal posture of the follow-through action as \(x = [\theta_1 \ \psi \ q_1 \ q_2]^T = 0\), then linearize Eq. (18.a) around \(x = 0\). Equation (18.a) can be rewritten as Eq. (18.b) by using the inertia matrix equation at the terminal posture, \(J_n = J(0)\).

\[
\ddot{x} = -J_n^{-1}D\dot{x} - J_n^{-1}Kx + J_n^{-1}pu + J_n^{-1}[d_j + d_k + d_h],
\] (18.b)

where

\[
d_j = (J_nJ_n^{-1}(x) - I)(-D\dot{x} - Kx + pu),
\]

\[
d_k = -J_nJ_n^{-1}(x)g(x)
\]

and

\[
d_h = -J_nJ_n^{-1}(x)h(x, \dot{x}).
\]

By introducing the equivalent disturbance vector \(d\) as substitution for the vector \((d_j + d_k + d_h)\), we can obtain the linear state variable equation from Eq. (18.b) as,

\[
\ddot{X} = A_nX + B_nu + E_n\dot{d},
\] (19)

where

\[
X = [x^T \ \dot{x}^T]^T, \quad A_n = \begin{bmatrix} 0 & I \\ -J_n^{-1}K & -J_n^{-1}D \end{bmatrix},
\]

\[
B_n = \begin{bmatrix} 0 \\ J_n^{-1}p \end{bmatrix} \quad \text{and} \quad E_n = \begin{bmatrix} 0 \\ J_n^{-1} \end{bmatrix}.
\]

The output equation is

\[
Y = [\gamma \ \theta_1 \ \psi]^T = CX,
\] (20)

where

\[
C = \begin{bmatrix} 0_{1 \times 2} & r \Gamma_{\phi_2}(\xi_x) \\ I_{2 \times 2} & 0_{2 \times 2} \\ 0_{2 \times 2} & r \Gamma_{\varphi_2}(\xi_x) \end{bmatrix} \begin{bmatrix} 0_{3 \times 4} \\ \xi_x \end{bmatrix},
\]

\(r\) and \(\xi_x\) denote the radius of the club shaft and the position of strain gauges, respectively, and output \(\gamma\) is the shear strain due to the torsion of the flexible link. The discrete-time system of the Eq. (19) is given as

\[
X_{k+1} = A_nX_k + B_nu_k + E_n\dot{d}_k.
\] (21)

Control input is described as

\[
u_k = -FX_k,
\] (22)

where \(F\) is the LQR feedback gain matrix that minimizes the cost function

\[
J = \frac{1}{2} \sum_{k=1}^{\infty} [X_k^T QX_k + u_k^T R u_k].
\]

Since some state variables cannot be measured directly, estimation of those state variables is necessary for the perfect state feedback. In this study, the state variables except for \(\gamma, \theta_1\) and \(\psi\) in Eq. (20) are estimated by a state observer described in following subsection.

2.3 Observer design

Robot manipulators are greatly affected by Coriolis’s force and centrifugal force in fast motion. An ordinary state observer for a linear system cannot accurately estimate the states of the system because of the uncertainty due to those nonlinear forces.

An observer that estimates a harmonic disturbance has been used in previous studies in which estimated disturbance was fed forward to the system to cancel actual disturbance\(^{10,11}\). Good control performances were obtained in those studies. However, it is difficult to model the dynamics of disturbance in this study because of Coriolis’s force, centrifugal force, gravity and constrained torque due to the mechanical hook. Moreover, these disturbances cannot be modeled as sinusoidal ones.

Disturbances of the swing robot consist of non-linear and gravity terms that depend on the change of the angular velocities and the attitudes during the swing action. The period of the swing action is longer than several hundreds msec, and then the change of them is slow. Therefore the disturbances of the swing robot mainly consist of low frequency components. The basic idea of the observer proposed in this paper is a type 1 digital servo system that tracks the outputs of the actual system as

\[
\dot{\hat{X}}_{k+1} = A_n\hat{X}_k + E_n\dot{d}_k + B_nu_k,
\] (23.a)

\[
\dot{\hat{Y}}_k = CX_k,
\] (23.b)

\[
e_k = Y_k - \dot{\hat{Y}}_k
\] (23.c)

\[
\dot{d}_k = F_e \sum_{i=1}^{k} e_i + F_\chi (\hat{X}_k - \hat{X}_0) + \dot{d}_0.
\] (23.d)

where \(F_e\) and \(F_\chi\) are the feedback gain matrices for the observer, and \(Y_k\) is the output vector of the actual system. We define the symbol, \(\hat{\cdot}\), as the estimated variable of \(\cdot\). If the initial conditions of the actual system and observer system are the same, the behavior of the estimated states must be similar to the behavior of the actual states. Thus, we can obtain the estimated equivalent disturbances by integration of the errors of outputs. It is, however, difficult to prove convergence of errors of the states except for both outputs of the observer and the actual system when the initial conditions of the observer are different from the actual initial conditions. In this paper, the dynamics of the equivalent disturbance vector \(d_k\) is assumed to be

\[
\eta_{k+1} = \Gamma \eta_k,
\] (24)

\[
d_k = H\eta_k,
\]

where

\[
\Gamma = H = I_{4 \times 4},
\]

in order to express the property of the disturbance of the swing robot and construct an integrator of the estimation error as Eq. (23.d). Equations (21) and (24) can be combined into the state variable equation as
where

\[ X_{k+1}^i = \tilde{A}X_k^i + Bu_k, \]

\[ Y_k^i = \tilde{C}X_k^i, \] (25)

and

\[ \tilde{C} = [ C \ 0 ]. \]

Then we simply construct a full-order observer

\[ \hat{X}_{k+1} = \tilde{A}\hat{X}_k + Bu_k + F_\omega(Y_k^i - \tilde{C}\hat{X}_k^i) \] (26)

of the nominal system described as Eq. (25) to estimate the state vector

\[ \hat{X}_k = \begin{bmatrix} \hat{x}_k \\ \hat{\eta}_k \end{bmatrix}. \]

Then the estimated equivalent disturbance vector \( w_k \) can be obtained as

\[ w_k = \hat{d}_k = H\hat{\eta}_k \]

in addition to the correct estimation of states. Now, Eq. (26) can be rewritten as

\[ \hat{X}_{k+1} = A_{\omega}\hat{X}_k + B_\omega u_k + E_\omega w_k + F_{\omega 1}\epsilon_k, \]

\[ w_{k+1} = w_k + F_{\omega 2}\epsilon_k, \] (27)

where

\[ \epsilon_k = Y_k^i - \tilde{C}\hat{X}_k^i \]

and

\[ F_\omega = \begin{bmatrix} F_{\omega 1} \\ F_{\omega 2} \end{bmatrix}. \]

Then \( w_k \) can be solved as

\[ w_k = F_{\omega 2}\sum_{i=1}^{k}\epsilon_{i-1} + w_0. \] (28)

A block diagram of this observer is shown in Fig. 5. The sensor noise vector is described as \( n \). The structure of this observer is similar to that of a steady-state disturbance observer. It can be seen from Eq. (26) that the estimation errors of all states converge to zero even if the initial conditions of the observer and actual systems are different. Because Eq. (28) has a structure similar to Eq. (23.d) and the first term of Eq. (28) expresses an integrator, this observer is a combined system of a type 1 digital servo system tracking the actual system and a full-order observer. The joint angle of the wrist joint is constant \((-\pi/2[\text{rad}])\) while the mechanical hook is working. The reference signal (the output of the actual system) for the estimated angle of the wrist joint of the observer is constant value \((-\pi/2[\text{rad}])\). Because this observer has a structure of the type 1 digital servo system, the tracking error (the estimation error) for the constant reference signal becomes zero in steady state. If the estimation error, \( e \), converge to zero, \( e \to 0 \), we can obtain the following relation, \( \hat{d} \to \hat{d} \), because of the equation obtained by subtracting Eq. (27) from Eq. (21). This mechanism works mainly in the low frequency region. In the other hand, the part of the full-order observer works mainly in the high frequency region to converge the estimation error to zero. Then the high frequency component of the disturbance hardly appears in \( \hat{d} \). Moreover, this observer has the property of a low-pass filter for sensors. This property has some advantages, because measurement noise is usually large in a high frequency region. Therefore, this observer is not greatly affected by sensor noise and can accurately estimate the states in a low frequency region, and \( F_\omega \) decides its bandwidth. In this paper, we use the LQR to decide \( F_\omega \). Now Eq. (22) is rewritten as

\[ u_k = -F\hat{X}_k \] (29)

and then the estimated state \( \hat{X}_k \) is used to obtain the feedback input to the system.

3. Simulation and Experiment

The swing robot has two links and two AC servomotors to drive its joints. The shoulder joint consists of a 100 W AC servomotor and a ball reducer whose reduction ratio is 1 : 10. The wrist joint consists of a 50 W AC servomotor and it drives the wrist joint directly. The rotating angle of each joint is measured with an incremental encoder on each motor axis whose resolution is 2000 counts per revolution. The wrist joint has a mechanical hook that constrains the relative angle of the wrist joint in \( \psi \geq -\pi/2[\text{rad}] \), and the hook can achieve wrist cock action in the downsing \( \psi = -\pi/2[\text{rad}] \). A flexible solid beam made of ABS resin is cantilevered on the second link. Tables 1 – 4 show parameters of the swing robot. In these tables, each parameter \( \beta \) describes the position of the center of grav-

\[ X_{k+1}^i = \tilde{A}X_k^i + Bu_k, \]

\[ Y_k^i = \tilde{C}X_k^i, \] (25)

\[ X_k^i = \begin{bmatrix} X_k \\ \eta_k \end{bmatrix}, \]

\[ \tilde{A} = \begin{bmatrix} A_d & E_dH \\ 0 & I \end{bmatrix}, \]

\[ \tilde{B} = \begin{bmatrix} B_d \\ 0 \end{bmatrix} \]

\[ \tilde{C} = \begin{bmatrix} C \\ 0 \end{bmatrix}. \]
ity. Figure 6 shows the experimental system. Shear strain due to torsional deformation of the beam is measured with four strain gauges, where bending and elongation are canceled by a bridge circuit. A digital signal processor (DSP) is installed on a ISA slot of a PC/AT host computer. The controller is implemented on the DSP and its sampling rate is 1 kHz.

The swing motion procedure is described as follows. First, the robot tracks a reference signal \( r(t) \) in the back-swing action by the type 1 digital servo-system. The signal is generated by the function

\[
r(t) = (r_e - r_s) \left( t - T_e \right) \left( T_e - T_s \right) - \frac{1}{2\pi} \sin \left( 2\pi \frac{t - T_s}{T_e - T_s} \right) + r_s,
\]

which has an S-type profile as shown in Fig. 7. In this paper, \( r_s = 0 \) [rad], \( r_e = -\pi \) [rad], \( T_s = 0 \) [sec] and \( T_e = 4.5 \) [sec] are used for the reference signal of the shoulder joint, and \( r_s = 0 \) [rad], \( r_e = -\pi/2 \) [rad], \( T_s = 0 \) [sec] and \( T_e = 4.5 \) [sec] are used for the reference signal of the wrist joint. Next, the triangular feedforward torque shown in Fig. 8 is added to only the shoulder joint to make the robot swing down. In this paper, \( \tau_{\text{max}} = 4.0\,\text{[N\cdotm]} \), \( T_s = 9.0\,\text{[sec]} \), \( \Delta T_{\text{max}} = 0.600\,\text{[sec]} \) and \( \Delta T_e = 0.620\,\text{[sec]} \), where \( \Delta T_{\text{max}} = T_{\text{max}} - T_s \) and \( \Delta T_e = T_e - T_s \). Finally, the robot is stopped by the LQR of Eq. (29) in follow-through.

Suitable weight matrices \( Q, R \) and feedback gain of the observer \( F_o \) are decided by the simulation results in this paper.

We obtained simulation results by the 4th-order Runge-Kutta method in which control inputs are changed at only each sampling period. Quantization errors of the output of the system have been taken into account for accuracy of the results. Figure 9 shows the displacements of the club head when the swing robot is stopped by the mechanical brake or the LQR. When the robot is stopped by the mechanical brake, the amplitude of the vibration of the club shaft is large. However, the vibration is suppressed when the robot is stopped by the LQR considering the
flexibility of the club shaft. Figure 10 shows a comparison of the results of numerical simulations of the actual and the estimated angular velocities of the wrist joint. Actual results are obtained by numerical integration of the nonlinear Eq. (17). The robot is stopped by the LQR with the minimal-order observer in consideration of the torque due to the gravity. The constrained force of the hook mechanism, the torque due to the gravity and the moment of inertia of the arm, which change non-linearly due to the angle of the joint, cause modeling errors between the nonlinear real model and the observer for the linear one. Because of those errors, the observer cannot estimate the correct states, and the performance of state feedback control becomes worse.

An observer that considers disturbances, however, can estimate the states more accurately. Figures 11 – 13 show simulation results obtained by using this observer. Experimental results are shown in Figs. 14 and 15. In Figs. 11 – 15, weight matrices of LQR are decided as

\[ Q = \text{diag} \{ 1 \times 10^3, 1 \times 10^3, 1 \times 10^3, 1 \times 10^3, 1 \times 10^3, 1 \times 10^3 \} \]

and weight matrices to decide feedback gain of the observer, \( F_o \), are decided as

\[ Q_o = \text{diag} \{ 1 \times 10^{-1}, 1 \times 10^{-1}, 1 \times 10^{-3}, 1 \times 10^{-3}, 1 \times 10^{-1}, 1 \times 10^{-3}, 1 \times 10^{-3}, 1 \times 10^{-3}, 1 \times 10^{-3}, 1 \times 10^{2}, 1 \times 10^{-6}, 1 \times 10^{-6} \} \]
Fig. 14 Experimental results of the estimated angular velocity of the wrist joint

\[ R_o = \text{diag} \begin{bmatrix} 10^3 & 1 & 10 \end{bmatrix} \]

The upper parts of Figs. 11 – 13 show results through the swing motion, and the lower parts of the figures show the enlarged results in the region around the downswing. This observer accurately estimates the states in spite of above-mentioned modeling errors. Figure 11 shows better estimation performance for angular velocities in comparison with the results shown in Fig. 10. Figure 12 shows the estimated state variable \( q_2 \) obtained by Eq. (27) in comparison with actual results obtained by Eq. (17). The actual state variable \( q_2 \) includes a low-frequency component due to the effect of gravity. The present observer eliminates the low-frequency component and shows good estimation performance of the vibration component. The equivalent disturbance torques estimated by this observer are shown in Fig. 13. This observer estimates not only the torque due to gravity but also the constrained torques due to the mechanical hook during the downswing. Experimental results shown in Figs. 14 and 15 agree with simulation results shown in Figs. 11 and 12.

4. Conclusions

In this study, optimal control was applied to a golf swing robot to suppress the shock and vibration due to braking after the downswing. The golf club was modeled as a flexible beam in consideration of bending and torsional stiffness and in consideration of eccentricity of the center of gravity of the club head on the shaft axis. An LQR that considers the vibration of the club shaft was applied during the follow-through to stop the robot and reduce the degree of shock.

An ordinary state observer for a linear system did not accurately estimate the states of the system because of dynamics not modeled or uncertainty due to those nonlinear forces. A state observer that considers disturbances was introduced to improve the performance of state feedback control.

Numerical simulations and experiments were carried out using a small swing robot. The state observer that considers disturbances estimated the states accurately in spite of non-linear torques due to gravity, the mechanical hook, Coriolis’ force and centrifugal force. The feedback gains for the states estimated by them were decided more easily than the gains for the states estimated by an ordinary observer, and the control performance to stop the robot and to suppress the vibration of the flexible link was improved by using the present state observer.

References


