Trends and Solutions in CAD/CG*

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The past and current trends in CAD/CG are discussed. An overview of the approach used in our ongoing project is given. Our final goal is primarily focused on developing a shape modeling system for solving problems of surface generation and enhancement, which includes polygon generation from unorganized points, shape smoothing, simplification, and improvement of mesh quality parameters of 3D polygonal sets.

Key Words: Radial Basis Function, Finite Element Interpolation, Surface Generation and Enhancement, Scattered Data

1. Introduction

A challenging goal in computer graphics (CG) and computer aided design (CAD) is to provide powerful technique for modeling the shape of an object.

Boundary representation or polygons can define quite large a variety of surfaces. Yamaguchi has noticed⁽¹⁾ that Euclidean processing provides the basis for CAD, nevertheless, there will always be a limit to the level of reliability in systems.

Polynomial functions were chosen according to the properties that were considered best, tangency, curvature that could be considered as sums of Bernstein's functions. Thus CAD systems based on a parametric representation have been widely used in the traditional process of body engineering in the late 1960s. However, in order to obtain a satisfactory design result, the surface must be totally expressed with numbers⁽²⁾. The basic idea is rather transparent but many problems remained to be solved: choosing adequate functions, blending curves and surface patches. There is also another problem concerning to the graphic user interface.

Solid modeling⁽³⁾ emerged in the late 1970s as a technology in mechanical CAD, with the first industrial systems appearing in 1980–81, provide good support for graphics and mass properties calculations. The major problems appear to be the following:

- Computational and representational intensity. Current systems can handle only small problems effectively.
- A little progress in incorporating of “sculptured surfaces.”

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- Weak human-computer interfaces.

Increasingly developing volume graphics provides tools for visualization and manipulation of voxel data represented by scalar values in the nodes of a regular 3D grid. Its perspective is to replace “polygon pushers” by rendering hardware based on a volume buffer. In addition to rendering, volume graphics provides few operations on volumetric objects. They are Boolean operations and metamorphosis in discrete space. However, “very little has yet been done on the topic of volume modelling”⁽⁴⁾. There are such problems here as pointed by Kaufman et al.⁽⁵⁾:

- Sculpturing in discrete space.
- Feature mapping and warping.
- Morphing and changing of the model.
- Intermixing volumetric and analytically defined geometric objects.

One can observe these topics being intensively investigated in geometric modelling. Above mentioned operations and many others like blending, sweeping, hypertexturing have found quite general solutions for geometric solids represented by continuous real functions as $f(x, y, z) \geq 0$. We believe that a CAD system should be based on a combination of various representations, for instance, the use of volume representation (a combination of continuous real functions and voxel based models)⁽⁶⁾ for solids provides effective solutions to the domain, application support, and some system problems.

The most noticeable trend in geometry modeling and CG is point-sampled geometry that has been researched intensively recently. A problem of reconstructing a surface from point clouds has been solved with interpolated implicit function^{(7)–(10)}, moving least square surface⁽¹¹⁾, level set method⁽¹²⁾ (see also Refs. (13)–(15)). Rendering point-sampled surface is proved to be efficient as shown in Refs. (16)–(18)).

Reverse engineering problems and the pursuit of the reality in CG as well as the rapid improvement of computation performance expands use of more and more elaborate geometric models with rich optical attributes. It is a big issue to acquire such models as quickly and cost-effectively as possible, however problems of editing or modification of such models is still a challenging task in geometry modeling. Recently introduced multiresolution modeling paradigm where a complex surface is decomposed into a low-resolution part and additional displacement information from the base for increasing resolution before retouching partly solves the problems. See, for instance, paper of Eck et al. in Ref. (19), where multiresolution analysis based on subdivision connectivity is extended to arbitrary mesh and geometric details are expressed as wavelets coefficients. Spectral analysis of point-sampled surfaces is proposed by Pauly et al. in Ref. (20). Their approach enables surfaces to be retouched in spectrum domain by modifying Fourier coefficients of the surface. Pauly et al.⁽²¹⁾ proposed multiresolution modeling of point-sampled surface. Their idea is to decompose a complex surface into a smooth base surface and a displacement distribution that is measured along the normal direction from the base. Deformation operators are applied to the base surface efficiently and a surface is reconstructed from the deformed base and the displacement distribution with original geometric features preserved well.

In our approach we exploit radial basis functions (RBF) for various applications as it will be shown later. Interpolation of point clouds can be used for getting low-resolution surface and filling holes on the input surfaces. The problem is now under consideration, see our preliminary result in Fig. 8.

Indeed, when we use only generic properties such as position of a point of the object that is deformed the problem of constructing smooth surface satisfying certain constraints can be formulated as a mapping function from $\mathbf{R}^3$ to $\mathbf{R}^3$. Such a space-mapping technique based, for instance, on RBFs is a powerful tool, which offers simple and quite general control of simulated shapes. Our previous approach to the surface retouching based on this idea. Kojekin et al. (see Ref. (22) and references therein) proposes an automatic retouching method of a selected area. The method produces smooth and visually pleasing results, however, suffers from failure in retouching huge hole areas. An application of polygon stitching before retouching reduces the problem, nevertheless, polygon stitching is not practical to apply when the sizes of holes are big enough. It is because the computational complexity of the problem is $O(N^5)$, where N is the number of vertices of a hole. Implementation of dynamic programming makes it possible to decrease computational complexity, nevertheless, a manual sculpting is necessary for fine-tuning of surfaces.

A model of extended space mapping was proposed in Ref. (23) and incorporates geometric space mappings and function mappings from $\mathbf{R}^3$ to $\mathbf{R}$. Constructive solid geometry (CSG) is usually used in many CAD applications. Traditionally, CSG modeling uses simple geometric objects for a base model, which can be further manipulated by implementing a certain collection of operations such as set-theoretic operations, blending, or offsetting. The operations mentioned above and many others have found quite general descriptions or solutions for geometric solids represented as points (x, y, z) in space satisfying $f(x, y, z) \geq 0$ for a continuous function f . RBFs offer a mechanism to obtain extrapolated points of a surface for various parts of a reconstructed object that can be used as “CSG components” to design a volume model.

Questions of surface reconstruction are related to the problem of surface smoothing. Recently, in CG and CAGD communities, attention has been paid to mesh smoothing. In Ref. (24) a signal processing approach to mesh smoothing was proposed. Laplacian smoothing is considered in Ref. (25) as time integration of the heat equation on an irregular mesh and a curvature flow approach is used to remove undesirable noise. The algorithm described there is very fast, but over-smoothing can be observed in Fig. 5 of that reference. Subdivision schemes⁽²⁶⁾ can be thought of as an alternative approach to the problem. Methods exploiting anisotropic diffusion techniques became rather popular recently (See, for instance, Ref. (27) and references therein). The technique of Tasdizen et al. is based on filtering normals of the surface, rather than processing the positions of points on a mesh. The method is applied for smoothing complex, noisy surfaces, while preserving (and enhancing) sharp, geometric features, and applies to any shape that can be modeled as an isosurface. A fast mesh-smoothing algorithm based on multi-resolution techniques, in combination with constrained minimization of the discrete energy functional, was proposed in Ref. (28). Using mesh hierarchies, where components of the geometric shape at each level of detail are characterized by a fairing, solves the problem. It allows interactive response times for moderately complex models, up to about 5 K triangles. Nevertheless, off-line preprocessing (an incremental mesh decimation algorithm) is applied. Many authors have considered very carefully questions of geometric distortion during smoothing and problems of precise shaping. See, for instance, the recent papers of Zhang and Fiume⁽²⁹⁾ and Yagou et al.⁽³⁰⁾, where algorithms to preserve the sharp features of a mesh were presented, and also the article by Belyaev et al.⁽³¹⁾ and references therein. These techniques provide excellent feature-preserving behavior, but a number of issues regarding their application remain open, and require more thorough consideration. For general example, techniques which strongly preserve features can also

produce additional artifacts. A shortcoming of the methods discussed above is that they do not provide resampling of data sets.

Approaches using methods of interpolation of scattered data based on minimum-energy properties have recently received increased attention for geometrical design problems, computer animation, and medical applications. While providing good restoration qualities, such methods sometimes produce excessive smoothing. Uniting this concept with simultaneous smoothing has become an important problem in CG and CAGD because 3D range data often suffers from noise (See recent work of Carr et al.⁽³²⁾). In Ref. (32), scattered range data are smoothed by fitting a RBF to the data and convolving with a smoothing kernel (low-pass filtering). Proposed method exhibits very good smoothing features. However, the technique is computationally very expensive. In spite of that, the algorithm allows for producing data resampling. We seek a fast method for surface denosing that does not exploit repositioning of (x,y) local coordinates but guarantees a valid mesh after processing that discussed in section 2.3.

Mesh improvement is an almost obligatory step to obtain a valid finite element mesh. In this case, the requirements on automatic mesh generator can be weakened. Because of their low computational cost, Laplacian and smart Laplacian smoothing are the most commonly used techniques to simultaneously improve quality of meshes and smooth 3D data. But both methods may produce badly shaped or even inverted elements. In Ref. (33) an effective method, called an angle-based approach, with low computational cost was proposed. The quality of a mesh optimized with this method is much better than after Laplacian and smart Laplacian smoothing, and the likelihood of obtaining inverted elements (with negative area) is reduced. But in Ref. (34) it was shown that this is true mostly for meshes with regular connectivity. When the mesh contains strongly distorted elements, an angle-based approach also may fail. Our algorithms improving triangular, quadrilateral and hexahedral meshes based on a statistical approach. Each free vertex is geometrically repositioned according to information at the incident vertices and the vertex's new position may improve quality of the mesh according to some quality parameters such as an aspect ratio.

Collaborative research with Tokyo Institute of Technology attacks the computational intensity problem and provides a computational environment for the work overviewed here. Our work is primarily focused on developing a shape modeling system for solving the problems of surface generation and enhancement, which includes polygon generation from unorganized points, shape smoothing, simplification, and improvement of mesh quality parameters of 3D polygonal sets.

2. Applications

2.1 Volume visualization and modeling

Volume visualization is one of the most interesting and fast-growing areas in computer graphics today. In terms of its impact on modern computer graphics, it is sometimes compared to "raster graphics revolution" that happened in the early 1980s. Today's volume visualization system creating high-quality images from scalar and vector multidimensional data are actually using the model that was classified as spatial occupancy enumeration more than 15 years ago⁽³⁵⁾. Most volume visualization techniques are based on one of about five foundation algorithms discussed in Ref. (36).

Some graphics workstations recently arrived in the market can perform high-speed volume rendering that could be considered as a step toward replacing "polygon pushers" with "voxel pushers".

By comparing raster graphics with its "next of kin" volume visualization, we would not avoid identifying some problem that is common for both of them. This problem is a limited set of operations that can be directly applied to pixels and voxels. To perform some complex geometric operation, one has to raise the level of abstraction and consider other geometric models that then can be visualized with the raster/voxel model.

Volume visualization can be thought as mapping 3D objects onto 2D images. The scope of modelling is quite different. It starts with some 3D objects, manipulates them and results in new 3D objects. A generated object model can be used further in different ways:

- Visualization.
- Integral properties calculations (volume, center of inertia, and so on).
- Rapid prototyping.
- Manufacturing.
- Application problems solving.

There is a trend in volume graphics to distinguish between modelling and visualization. We believe that the spatial occupancy enumeration requires unification with another model that would imply more sophisticated operations and ability to use voxel model together with another solid model. This higher level geometric abstraction would allow one to use all abilities of solid modelling with the advantages of volume visualization.

Such fruitful alliance could be achieved if we combine together spatial occupancy enumeration and the function representation of solid objects. Our aim is to get some real function $f(x,y,z,vol) \geq 0$ where vol is a voxel data parameter and x, y, z are Cartesian coordinates. This function has positive values for the internal points, equal to zero for the boundary points and less than zero for the points outside the solid.

Figure 1 presents an example of intermixing volumet-

Fig. 1 Volumetric head and hair unified with a pillar

ric and analytically defined geometric objects to form a solid object with curved surfaces. In this example we use volume data of a voxel head with simulated hair in synthetic carving on a pillar.

2.2 Surface reconstruction

In spite of a flurry of activity in the field of scattered data reconstruction and interpolation, this matter remains a difficult and computationally expensive problem. A vast amount of literature is devoted to the subject of scattered data interpolation methods and their applications, see, for instance Refs. (37)–(42). However, the required computational work is proportional to the number of grid nodes and the number of scattered data points. Special methods to reduce processing time were developed for thin plate splines and discussed in Refs. (43) and (44), see also recent publications^{(45), (46)}.

Methods exploiting RBFs can be divided into three groups. The first group is “naïve” methods, which are restricted to small problems, but they work quite well in applications, dealing with shape transformation. The second group is fast methods for fitting and evaluating RBFs⁽⁴⁵⁾. The third is compactly supported RBFs^{(47), (48)}. Let us notice here about recent outstanding work of Ohtake et al.⁽⁴⁹⁾ where compactly supported radial basis functions (CSRBS) are used as blending functions. The significance of this work is that the method proposed enables obtaining high quality reconstruction results and handle realistic amounts of data.

Here, we shall give a short account of the shape transformation method used in the applications considered in this paper. To interpolate the overall displacement, we use a volume spline based on Green’s function, for more references, see Ref. (50). This is well known problem — to find an interpolation spline function $u \in W_2^m(\Omega)$, where $W_2^m(\Omega)$ is the space of functions whose derivatives of order $\leq m$ are square-integrable over $\Omega \subset \mathbf{R}^n$, such that the following two conditions are satisfied: (1) $u(p_i) = h_i$, $i = 1, 2, \dots, N$, and

(2) u minimizes the bending energy, if the space transformation is seen as an elastic deformation. For an arbitrary three-dimensional area Ω , the solution of the problem is well-known: the volume spline $f(P)$ having values h_i at N points P_i is the function

$$f(P) = \sum_{j=1}^N \gamma_j \phi(|P - P_j|) + p(P), \quad (1)$$

where $p = v_0 + v_1x + v_2y + v_3z$ is a degree one polynomial. To solve for the weights γ_j we have to satisfy the constraints h_i by substituting the right part of Eq. (1), which gives

$$h_i = \sum_{j=1}^N \gamma_j \phi(|P_i - P_j|) + p(P_i). \quad (2)$$

Solving for the weights γ_j and v_0, v_1, v_2, v_3 it follows that in the most common case there is a doubly bordered matrix T , which consists of three blocks, square sub-matrices A and D of size $N \times N$ and 4×4 respectively, and B , which is not necessarily square and has the size $N \times 4$.

Since the RBF $\phi(r)$ is not compactly supported, the corresponding system of linear algebraic equations is not sparse or bounded. Storing the lower triangle matrix requires $O(N^2)$ real numbers, and the computational complexity of a matrix factorization is $O(N^3)$. Thus, the amount of computation becomes significant, even for a moderate number of points. Wendland in Ref. (51) constructed a new class of positive definite and compactly supported radial functions for 1D, 3D and 5D spaces of the form $\phi(r) = \psi(r)$, $0 \leq r \leq 1$; $0, r > 1$, where $\psi(r)$ is a univariate polynomial whose radius of support is equal to 1. A space-mapping in $\mathbf{R}^n$ defines a relationship between each pair of points in the original and deformed objects. Nevertheless, heights h_i are not necessarily arbitrary points of Euclidean space E^n . Our approach presents an attractive possibility of using function mapping for controlling local deformations by placing arbitrary control points inside or outside an initial implicitly defined object G , and they are assumed to belong to the surface of a modified object G^m . Thus, the control points define the deformation of G resulting in G^m .

The application shown in Fig. 2 (an example of reconstruction of CSG object) demonstrates applicability of an extremely simple approach proposed in Ref. (42) that allows us to attain rather acceptable results. In our software implementation, we employ a standard approach for creating a binary tree from an initial point data set with an additional required parametric value K , which denotes the maximum number of points in a leaf. Such a tree allows to provide an efficient sorting of scattered data⁽⁴⁸⁾ that leads to obtaining a band diagonal sub-matrix A ; after that Cholesky decomposing and block Gaussian solution are applied.

2.3 Hole filling

Point sets obtained from computer vision techniques are often non-uniform and even contain large missing ar-

eas of points. Another source of such a data, for instance, is partly destroyed natural objects, for instance, such as teeth that need a treatment.

Three approaches to reconstruct missing parts have been dominant in the CAD area: the first one works with 3D polygonal models to stitch damaged or incorrectly calculated nodes of 3D geometric objects, the second one is an approach dealing with fitting of the data generated according to some geometrical features such as curvature, and the third one is actually based on a well-founded mathematically set-level approach. Partial differential equations are widely used to model a surface subject to certain constraints.

Another way is to apply space mapping technique based on the use of RBFs, for more references, see Ref. (52). In the application shown in Fig. 3, the optimization task is to find a functionally transformed occlusal surface (an inlay part) of a model tooth that best matches the remaining occlusal surface of a treated tooth. Actually, this approach can be called “cloning”. We suppose that RBFs are suitable for sufficiently moderate 3D data sets. Nevertheless, they possess many features that make them very attractive for CAD applications dealing with modification of geometric objects under certain constraints,

Fig. 2 “Seashell” CSRBFs surface reconstruction where sphere as a “carrier” function is used. Number of points: 917. Processing time: 0.56 sec (includes surface extraction time: 0.41 sec)

see Ref. (53), where occlusal surface modeling for restorations, based on jaw articulation simulation was used as shown in Fig. 4. Figure 4 (a) shows the complexity of the surface of teeth, which are the subject of correction. Requirements of avoiding of interpenetrations with the opponent teeth and preserving main topological features of the occlusal surface of teeth are imposed on design of the occlusal surface of restoration. Arbitrarily placed points (Fig. 4(b)) induced by the distance distribution are used to control 3D occlusal surface deformations by RBFs.

In Ref. (22) an approach to hole filling of polygonal data was proposed. The algorithm includes holes extraction step, polygon stitching, and a hole surface improvement based on space-mapping technique. For sufficiently small holes, polygon stitching based on the use of dynamical programming demonstrates quite good results as it can be seen in Fig. 5.

To illustrate the applicability of the space-mapping technique (third step of our algorithm) to the surface-retouching problem, we first show an example of image inpainting in Fig. 6.

The principal contribution of the approach is a surface-retouching algorithm based on a local approximation of missing data. Example in Fig. 7 demonstrates the applicability of the approach for a rather complicated geometric object with 16 holes and the result of completely automatic reconstruction of the missing parts of the object. Let us note that regions where one-to-one mapping

Fig. 4 (a) Surface of teeth. (b) Distance maps of collision removal by applying RBF mapping function

Fig. 3 (a) Model tooth. (b) Tooth to be treated. (c) Approximation area (extended, pre-set boundary). (d) Treated tooth after application of genetic optimization. (e) Treated tooth after final refinement based on RBFs. (f) Resulting approximated CSG object

Fig. 5 Example of surface retouching of a real polygonal model. (a) "Stoned" model (courtesy of R. Scopigno and M. Callieri of Institute CNUCE). Model size — 88 478 points. Red lines show hole areas. (b) Model after surface retouching

Fig. 6 Example illustrating image inpainting approach. "Wool", one additional sloping scratch was added to the test image from Ref. (54). Processing time: 0.1 sec

Fig. 7 Solution based on CRRBFs. (1) Original model "Port6" (19 467 polygons) contains 16 holes. (2) Prediction of the surface inside the holes area — triangulation (left) and linear subdivision of the triangulated surface (right). (3) Result of the extrapolation for the holes area. Total processing time 7.1 sec

or neighborhoods that are not homeomorphic to a disc can be observed in this example.

Figure 8 is an example of hole filing based on the multiresolution paradigm.

Fig. 8 An example of hole filing of the "Sangiovanino" model (courtesy of R. Scopigno and M. Callieri of Institute CNUCE). New sample points are generated on the hole arias.

Fig. 9 (a) The original noisy sphere "Epcot" model, (770 vertices, 1 536 polygons), (b) smoothed model after 5 iterations based on 11-point interpolation. Processing time: 0.6 sec

Fig. 10 (a) The "Monk" model, whose surface was constructed from range data (courtesy of A. Belyaev and Y. Ohtake of Max-Planck-Institut für Informatik). Model size: 29 795 points. (b) Approximation of "Monk" model

2.4 Surface smoothing

The approach discussed above shows the obvious relationship between the surface-retouching problems and shape smoothing. Figure 9 presents an example of compactly supported RBF smoothing of polygonal surfaces. The shape-smoothing algorithm exhibits, in practice, good features and volume-preserving properties.

Figure 10 shows an implementation of a finite element method (FEM) for smoothing of polygonal surfaces.

The problem of constructing interpolating functions by FEM for 2D space Ω is stated as follows: We are given data points $P_i(x, y)$, $i = 1, 2, \dots, N$, are scattered, in the sense that there are no assumptions about the disposition of the independent data. Data set r is associated with the points. We must construct a smooth function $\sigma_h(x, y)$ which takes on the value r_i at points $P_i(x, y) \in \text{space } \Omega$, if possible, or satisfies the condition:

$$\|A\sigma_h - r\|^2 = \min,$$

where A is some linearly bounded operator. Along with this condition, the function $\sigma_h(x, y)$ has minimum energy of all functions that interpolate values r_i . This conforms to the following minimum condition, which defines operator T :

$$\int_{\Omega} [\sigma_{xx}^2 + \sigma_{yy}^2] d\Omega = \min.$$

The algorithm attains overall smoothness and preserves most fine features of the object. Volume characteristics are preserved without special treatment. For instance, the difference between the volume of the original “Monk” model and its approximation is 0.1%.

2.5 Real time facial animations

3D geometric modeling systems based on shape deformations have been pursued by many researchers and take mainly advantage of the simple idea that tangible geometry of deformations can be defined by the user assigned starting and destination points. Probably this approach firstly was implemented in the papers of Wolberg⁽⁵⁵⁾ and Beier and Neely⁽⁵⁶⁾ for 2D warping.

For real-time applications computing of the transformations is the most critical part in the sense of time optimizations. Figure 11 shows an example of facial animations by compactly supported RBFs. For every point, which is inside the radius of support distance is calculated one time and after that space transformations are calculated in accordance to a phase parameter (value varies from 0 to 1) that defines total deformation⁽⁵⁷⁾. The deformation process is regarded as taking place step by step so that the transition from a known state to a new state takes

place with small increments in deformations. That is, intermediate transformations for every step of animation are generated according to the phase parameter

2.6 Hatching

Recently, sampled point clouds have received much attention in the CG community for visualization purposes (see Refs.(11) and (58) and references therein), where a moving least-square method (MLS)⁽⁵⁹⁾ is used. Approximation of a single point is based on using a radial weight function, such as a Gaussian with a fixed parameter, reflecting anticipated spacing between neighboring points. Nevertheless, results of surface reconstruction depend on the selected parameter, and the computational cost of the MLS can be high.

Figure 12 shows drawing non-photo realistic images of volume models. The turtle graphics approach to imitate painting operations is implemented in 3D space. The “head” object is defined by sets of scattered range data. From a geometrical point of view the problem considered can be stated in terms of allowable transformations or as

Fig. 12 An example of hatching fractal-like objects with different levels of details on a point-sampled object represented by 1 487 scattered 3D coordinates

Fig. 11 An example of facial animations (7 024 polygons, deformation defined by 32 vectors, radius of support is equal to 0.2): (a) — original model, (b) — “smile”, (c) — “upset”, (d) — “kiss”. Processing time: 107 fps

a one-to-one mapping of a portion of a surface S onto a portion of a surface S^* . Of particular practical interest are mappings that preserve certain geometric properties such as the length of every arc (in our case the length of line segments on the plane, for instance) on a surface and angle-preserving mappings, which preserve the angle between every pair of intersecting curves on a surface.

2.7 Mesh generation

Surface reconstruction methods can be broadly classified into two main categories: Delaune-based methods and implicit surface approaches. Compared with the Delaune-based methods, the implicit surface approach approximates rather than interpolates sample point. Also, the implicit surface approach is based on the idea of global approximation of scattered data. From our point of view, methods based on the idea of local reconstruction are promising in CAD and CG applications dealing with huge amounts of scattered data.

The partition of unity method (PUM) for the construction of interpolation and approximation was pioneered by Shepard⁽⁶⁰⁾ and was later extended by Franke and Nielson⁽³⁷⁾. In recent years, it has received much attention due to the works of Melenk and Babuska⁽⁶¹⁾ and Krysl et al.⁽⁶²⁾

Shepard's approximation on a set of scattered points $\mathbf{x}$ of domain Ω is as follows:

$$u_h(\mathbf{x}) = \sum_{I=1}^N \omega_I(\mathbf{x}) u_I,$$

where u_I are the nodal parameters, and $\omega_I(\mathbf{x})$ are the basis functions of compact support. They are constructed from weight functions $W_I(\mathbf{x})$ by means of the formula

$$\omega_I(\mathbf{x}) = W_I(\mathbf{x}) / \sum_{k=1}^N W_k(\mathbf{x}).$$

The CSRBF is used as a weight function

$$W_I(\mathbf{x}) = \begin{cases} (1-r)^4(4r+1), & 0 \leq r \leq 1 \\ 0, & r > 1 \end{cases},$$

where $r = \|\mathbf{x} - \mathbf{x}_I\|$ is the Euclidean distance between an interpolated point and an input point, and N is the number of points in a predefined area. Other choices of the weight function are also acceptable; however, theoretical proofs can be given to show that, to achieve extrapolation efficiency, weight functions, with small third derivatives should be used.

A general cover construction algorithm or partition of the domain Ω into overlapping rectangular patches ω_I to cover the complete domain has to be used. Let us note that our main premise is to take account of a surface variation σ (an analog of surface curvature) that might be useful for correct choice of a radius of support (r -sphere) for reconstruction taking into consideration the orientations of local surface elements. In our work⁽⁶³⁾, we have investigated rather simple scheme taking account of the local geometry of a surface.

Figure 13 shows the results of reconstruction using the approach discussed above.

Figure 14 shows an implementation of the PUM for generation of polygonal surfaces for point sets represented by elevation data. We demonstrate the applicability of the approach to data homeomorphic to a disc; nevertheless, since a closed object can be partitioned into a collection of bordered patches homeomorphic to a disc, this is no serious restriction, as it was mentioned by Horman and Greiner in Ref. (64).

2.8 Polygon simplification

Surface remeshing has become very important today for CAD and CG. This question is also very important for technologies related to engineering applications. Simplification of a geometric mesh involves constructing a mesh element which is optimized to improve the element's shape quality. Recently, a tremendous number of very sophisticated algorithms have been invented to obtain a simplified model. One exceedingly good overview⁽⁶⁵⁾ presents a problem statement and a survey of polygonal

Fig. 13 Implementation of the partition of unity for generation of polygons from scattered data of the fragment of Mount Bandai: (a) Curvature analysis. In the blue area, the surface variation $\sigma > 0.3$. (b) Result of reconstruction (ray tracing). Number of scattered points: 10 000, processing time: 0.941 sec, number of vertices after reconstruction: 90 000. (c) Fragment of the mesh as a wire-frame with color attributes according to calculated heights

Fig. 14 Surface reconstruction of a technical data set. (a) Cloud of points (4 100 scattered points are used). (b) Simplified mesh shaded (processing time: 0.1 sec). (c) Fragment of the initial mesh, 31 234 triangles. (d) Combined mesh modification (polygon reduction and statistical improvement of the mesh), 12 132 triangles

simplification methods and approaches.

We present here in more details a surface simplification method which uses predictor-corrector steps for predicting candidate collapsing points with consequent correcting them by the use of a statistical approach for triangles enhancement. The predictor step is based on the idea of selecting candidate points according to a bending energy.

We show that since the simplification method is sufficiently efficient for up to 90 percent of reduction, there is no need for user-tuned parameters and the approach allows for obtaining a realistic time response (few seconds on AMD Athlon 1 000 MHz) for sufficiently complex models (70 K triangles).

Finding the optimal decimation sequence is a complex problem. The traditional strategy is to find a solution that is close to optimal; this is a greedy strategy, which involves finding the best choice among all candidates. Our simplification algorithm is sufficiently simple and is based on an iterative procedure for performing simplification operations. In each iteration step, candidate points for an edge collapse are defined according to a local decimation cost of points belonging to a shaped polygon. We call such a polygon a star. After all candidates have been selected, we produce a contraction step by choosing an optimal point.

A specific error metric is employed. We propose using the bending energy $\mathbf{h}'\mathbf{A}^{-1}\mathbf{h}$ as an error/quality cost to select candidates for an edge collapse. The approach is based on the use of displacements of N control points as the difference between the initial and final geometric forms. The central point of a star polygon is considered as a point that can slide to the neighboring points. The selection of candidate points is made according to the bending energy. We exploit a simple idea that the more smoothly we transform a central point, the fewer residuals there will be between an initial mesh and the subsequent mesh. In this step, we form a list of points to be contracted; this list contains a number of candidate points. In the contraction step we eliminate processing of points that can be contracted twice or more.

Vertex placement is produced in two steps. In the first step we generate a position on the line connecting two vertices of an edge to be contracted. In fact, the optimal point is generated at the next step. The main underlying assumption of the algorithm is that a local mesh refinement automatically results in improved global mesh quality, bearing in mind the distribution of a limited set of polygons in the entire model.

Statistics (see Ref. (66)) on the values of the mesh quality criteria parameters of the neighbors of each vertex of the triangle mesh in order to predict the most likely value are exploited. This provides some latitude in the choice of point placement allowing softer “transformations” of polygons to be produced. Figure 15 shows examples of polygon simplification of the “Horse” model.

Figure 16 (c) demonstrates a good visual appearance of the simplified model that is verified by luminosity histograms.

Notice that in all the examples in this paper the processing time is shown for our test configuration AMD Athlon 1 000 MHz, 128 MB RAM, Microsoft® Windows 2000, ATI Radeon 8500 LE.

3. Conclusion

RBFs as a tool mainly used in our ongoing project seem ready-made for many applications in shape modeling and CG, even for interactive 3D modification and sculpting. We have to state that according to our experiments with various applications of RBFs for surface modifications, for instance teeth reconstruction and optical design, we have a good alliance of geometric modeling and optimization techniques to determine the reconstructed surface and assure overall smoothness.

There is no single restoration and simplification method that provides the best results for every surface in the sense of quality and processing time. Experimental results indicate that the algorithms discussed in this paper provide rather good results and look promising for implementation in CAD and computer-aided engineering applications.

Let us note here that, for example shown in

Fig. 15 (a) Fragment of the “Horse” model; (b) Fragment of the mesh after statistical processing; (c) Combined mesh modification: polygon reduction (40% of the original number of triangles) and statistical improvement

Fig. 16 (a) Original “Horse” model (96 966 triangles) and a luminosity histogram; (b) Simplified model produced accordingly to the use of the bending energy (10% of the original number of triangles) and the luminosity histogram; (c) Simplified model produced accordingly to the use of the bending energy and statistical approach (9% of the original number of triangles) and the luminosity histogram; (d) Simplified model produced accordingly to the use of the bending energy and statistical approach (3% of the original number of triangles) and the luminosity histogram

Fig. 14(d), the volume was well preserved, with a difference between the initial mesh and the processed one of about 0.65%.

One shortcoming of the approach discussed in section 2.5 is that in some areas (almost vertical) of a surface the triangular vertices may be spaced far apart. Selecting local data sets in the reconstruction algorithm according to the surface curvature is still an interesting research topic waiting for a solution. Our most urgent problem is to extend the algorithm given in section 2.5 to provide adaptive remeshing (enriching) according to local features of a surface geometry.

Algorithm proposed for hole filling based on polygonal stitching and space mapping techniques produces smooth and visually pleasant results, however, it is assumed that any hole does not have islands. In practice, holes filling problem becomes user dependent whether

holes could be filled or not. Our current task is to find an approach to combine automatic defect detection and repairing algorithms.

The research discussed suggests future directions such as a development of a method based on an implementation of quasi-statistical modeling for improvement of quadrilateral and hexahedral meshes.

Our case studies hold promise to enable a new paradigm of computer assisted geometry applications.

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