Vibration Isolation by a Variable Stiffness and Damping System*

Yanqing LIU**, Hiroshi MATSUHISA**, Hideo UTSUNO** and Jeong Gyu PARK**

Most passive vibration isolation systems are composed of springs and dampers. Although it is possible to improve the isolation performance by active vibration control, the complexity, power requirements and cost of such a system have restricted its use. A vibration isolation system with variable damping is practical and has good performance in the high frequency region, but it was found not to improve the responses in the low frequency region. On the base of a damping on-off control method, a stiffness on-off control method and a combination of damping and stiffness on-off control method were proposed. Comparison of the responses among the proposed methods and the conventional methods showed that the damping and stiffness on-off control method had the best isolation properties in the whole frequency region. A new system with controllable dampers of two Voigt elements in series was used to achieve the proposed idea.

Key Words: Variable Stiffness, Voigt Element, Controllable Damper, Semi-Active Control

1. Introduction

Active vibration control systems require actuators and lots of power. On the other hand, semi-active vibration control systems expend small energy because only the system parameters, such as damping and stiffness, are altered. The idea of providing variable damping in vibration system has been studied by many researchers. Karnopp proposed a semi-active electro-hydraulic damper with on-off skyhook control method(1), (2). It was established that mass deceleration was accomplished by the damper when the sign of absolute velocity of the mass is the same as that of the relative velocity. Rakheja discussed vibration isolation with a variable sized orifice damper. He proposed the on-off control method with the damper on when the relative displacement and velocity had opposite signs(3). However, it is difficult to reduce the vibration in the region below the natural frequency with a variable damper(4), (5).

Variable stiffness systems have been discussed in several papers. Walsh and Lamancusa investigated control of rotating imbalance prior to reaching the constant rotational velocity(6). An adjustable stiffness of the compound leaf spring was used to provide a variable stiffness element. The spring stiffness was changed depending on the rotational velocity. In another study, Kobori proposed an active variable stiffness system to suppress the building response to earthquakes(7). His system was to achieve a non-stationary and non-resonant state during earthquakes. He proposed the objective function to minimize the root mean squared values of the system responses. An optimal control law governing the simultaneous control of damping and spring stiffness was studied by Youn and Hac(5). Since it was more difficult to change the stiffness than the damping in vehicle suspension system, a change of stiffness occurred only when the required control force could not be generated by variable damping alone. They used the air spring in three distinct rates: soft, medium and firm. The suspension system with variable damping and stiffness showed significant improvements in controlling vehicle vibration when it was compared to a semi-active suspension with a variable damper and fixed stiffness. They proposed the adaptive control method to minimize the performance index by the active control system with constraint conditions. Several researchers have proposed control methods which use the frequency spectrum of the response to design the variation of spring stiffness(8). The responses of the vibration system were improved by the above control methods, but their hard structures and control algorithms are complicated to design.

In this paper, on the base of the damping on-off skyhook control method, a new stiffness and damping on-off control method was proposed. It was found to be more controllable and realizable than the previous control methods.
2. Variable Stiffness Control Method and Simulation Results

2.1 Variable stiffness control method

Figure 1 shows one degree-of-freedom (1 DOF) vibration model subjected to a base displacement \(x_0\) and a force \(F\). In a case of vehicle suspension, \(x_0\) is the road bumpiness and \(F\) comes from the vibration of engine. When the model describes a building vibration isolation system, \(x_0\) is the earthquake and \(F\) is caused by the wind. The parameters \(k^*\) and \(c^*\) are variable spring stiffness and damping coefficient, respectively. The variable \(x\) is the displacement of mass \(m\).

The most popular method for variable damping is the on-off skyhook control \(2\), which is described as

\[
f_d = \begin{cases} 
-c(x-x_0) & \text{if } x(x-x_0) > 0 \\
0 & \text{if } x(x-x_0) < 0 
\end{cases}
\]

where \(f_d\) is the damping force, the damping coefficient \(c^*\) is equal to \(c\) in the on-state and zero in the off-state, \(x\) and \(x_0\) are the velocity of \(m\) and the base, respectively. The damper exerts a force tending to reduce the velocity of the mass. By analogy, the spring force on-off control scheme is proposed as

\[
f_s = \begin{cases} 
-k(x-x_0) & \text{if } x(x-x_0) > 0 \\
0 & \text{if } x(x-x_0) < 0 
\end{cases}
\]

where \(f_s\) is the spring force, the stiffness \(k^*\) is equal to \(k\) in the on-state and zero in the off-state, and \(x-x_0\) is the relative displacement of the spring. The on-off spring force is controlled by the sign of \(x(x-x_0)\). When \(x-x_0\) is positive, the spring is extended and can generate a downward force for the mass. When \(x-x_0\) is positive, the mass moves upwards. Thus, the spring can exert a force to reduce \(x\) (named on-state). When \(x-x_0\) is negative, the mass moves downwards. In this case, because the passive spring cannot generate an upward force with the positive value of \(x-x_0\), the best spring can do is to supply no force (named off-state). When \(x-x_0\) is negative, the control logic is described similarly as the above process. The spring force in “on” or “off” state is shown in Table 1. Here ‘+’ and ‘-’ express that the signs of variables are positive and negative, respectively.

In this paper, three types of semi-active control schemes are discussed. Type 1: only the damping force is on-off controlled with fixed spring stiffness as shown in Eq. (1) (named “D on-off”). Type 2: only the spring force is on-off controlled with fixed damping, as shown in Eq. (2) (named “S on-off”). Type 3: both the damping and stiffness are on-off controlled (named “D+S on-off”). The damping and stiffness of the “D+S on-off” are changed in accordance with the on-off control schemes expressed in Eqs. (1) and (2), respectively. The Type 1 system is the conventional damping on-off control system. The Type 2 and Type 3 systems are proposed in this paper.

2.2 The responses to a sinusoidal input

In the system, the sinusoidal inputs are described as

\[
x_0 = X_0 \sin(\omega t),
\]

\[
F = F_0 \sin(\omega t)
\]

where \(\omega\) is the input frequency, \(t\) is the time, \(X_0\) and \(F_0\) are the amplitudes of \(x_0\) and \(F\). Figure 2 shows the frequency responses of the system of four control methods, where \(X_0 = 0.01\) m, \(X_a = F_0/k, F_0 = 0.04\pi^2 \) N, \(k = 4\pi^2\) N/m, \(m = 1\) kg, the natural frequency \(\omega_p = 2\pi \text{ rad/s} (f_p = 1\) Hz), the damping \(c = \pi \text{Ns/m}\) and the damping ratio \(\zeta = 0.5\). The calculation in time domain is carried out until the response to the input signal reaches a steady state. Because the base does not move in the force input system (\(x_0 = 0\)), the term \(x(x-x_0)\) is always positive and the damping force of on-off damper is the same as that of the passive damper and the system with D+S on-off control is the same as the S on-off control system.

Figure 2 (a) shows that the D on-off system has a smaller response in the high frequency region \((\omega/\omega_p \gg 1)\) than the passive system, and the S on-off has smaller response in the resonance region \((\omega/\omega_p \approx 1)\). The D+S on-off system has a smaller response in the resonance and high frequency regions than the passive system. When the system is subjected to a force, the system with S on-off control method has better characteristics than the passive system at the resonance frequency region shown in

![Fig. 1 Model of 1 DOF vibration system](image)

![Fig. 2 Frequency responses of 1 DOF system with sinusoidal input](image)

### Table 1 Signs of variables

<table>
<thead>
<tr>
<th>(x-x_0)</th>
<th>+</th>
<th>+</th>
<th>-</th>
<th>-</th>
</tr>
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<tbody>
<tr>
<td>(\dot{x})</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(f_s)</td>
<td>on</td>
<td>off</td>
<td>on</td>
<td>off</td>
</tr>
</tbody>
</table>


Fig. 2 (b).

Figures 3 – 5 show a set of typical time responses of the system. The base motion $x_0(t)$ is sinusoidal with the same amplitude, and the frequency is equal to 0.5, 1.0 and 4 times the natural frequency ($f_n = 1$ Hz). In the time responses, the acceleration of mass is not drawn because its shape is equal to the summation of the damper and spring force $f_d + f_s$.

In the resonance input case ($f = f_n$), as shown in Fig. 3, the amplitude $x$ of the S on-off system is a little smaller than that of the D on-off control system. The amplitude of $x$ of the D+S on-off control system is about $1/10$ of the D on-off control. The reasons can be derived from two aspects. Firstly, the amplitude of the total force ($f_d + f_s$) of the D+S on-off control is smaller than that of the D on-off control. Secondly, the total force acts on the mass (in on-state) during the attenuation portion of the vibration cycle, but it is zero (in off-state) when it accelerates the mass. This is shown by $\dot{x}$ and $f_d + f_s$ in Fig. 3. In the low frequency input case as shown in Fig. 4, the response of D+S on-off is about half of D on-off. In the high frequency input case shown in Fig. 5, the response of S on-off is larger than that of D on-off but the response of D+S on-off is remarkably smaller than that of D on-off.

Figure 6 shows the steady state responses of the system with the force input. The force $F$ is sinusoidal with the frequency of 0.25, 1.0 and 4 times of the natural frequency. In the resonance input case ($f = f_n$), the amplitudes of $x$ and $\dot{x}$ of the S on-off system are a little smaller than those of the passive system, but they are almost same in the high frequency input case ($f = 4f_n$). In the low frequency input...
put case \((f = 0.25f_n)\), the responses of the system with \(S\) on-off control are worse than that of passive system. This may be caused by the shock force at switching time.

The conclusion obtained from the above analysis is that the \(D+S\) on-off control system has good isolating characteristics.

### 2.3 The responses to random input

This control is supposed to be adopted to a car suspension. The response of the system with random input gives a fairly good indication of the behavior. There is research on the random input which has a power spectral density (PSD) approximating the roadway unevenness\((9)\).

The input \(x_0(t)\) can be generated by passing white noise through a filter
\[
\dot{x}_0 + n_0 v x_0 = w(t)
\]
where \(v\) is the vehicle velocity, \(n_0\) is a measure datum of the spatial frequency, \(w(t)\) is a white noise function with intensity \(2\pi n_0 v\), and \(\sigma\) is a roughness constant which is dependent on the quality of the road. In this study, following values are used, \(f_n = 1.0\) Hz, \(n_0 = 1/2\pi\) cycle/m, \(v = 20\) m/s (72 km/h), \(\sigma^2 = 64 \times 10^{-6}\) m\(^3\)/cycle (corresponding to the C class of the road surface\((10)\)) and \(w(t)\) is generated by the Matlab Simulink with a cutoff frequency of 15 Hz. The simulation is run for 100 sec with the sampling time of 0.005 sec. The PSD of \(w(t)\) is flat from 0.1 Hz to 10 Hz, as shown in Fig. 7 (a). The PSD of \(x_0(t)\) shown in Fig. 7 (b) decreases with the increasing frequency. The transmissibility of \(X/X_0\) and the power spectral density (PSD) of \(\ddot{x}\) are plotted for frequency ratios \(\omega/\omega_n\) of 0.1 up to 10 in Fig. 8.

According to Fig. 8 (a), the vibration system with \(D+S\) on-off control method has better performance than the other control methods in the whole frequency region. In the acceleration response shown in Fig. 8 (b), the response of the \(D+S\) on-off control is better than those of other controls in both the resonance and the high frequency regions, but it is larger than that of the \(D\) on-off control in very low frequency region.

The root mean square (RMS) values of the displacement and acceleration are calculated with three kinds of random inputs. Tables 2 and 3 show the RMS values of the system with the input of road surface type B (good), C (average) and D (bad)\((10)\). According to the RMS values, the accelerations of the \(D\) on-off control are 1.5 times of those of the \(D+S\) on-off control. The displacements of the \(D+S\) control are much smaller than those of other control methods.

### 3. Proposed Model of the Variable Damping and Stiffness System

The vibration system with \(D+S\) on-off control method has an excellent performance. In general, the structure of variable stiffness is very complicated and its response has a delay time. On the other hand, the variable damping can be easily made by a fluid damper with a variable orifice or MR (Magnetorheological) damper. Therefore, a new structure combining a variable damping and a variable stiffness is proposed. In this system, the stiffness is varied by the damper. Figure 9 (a) shows the concept of the variable damping and stiffness system. There are two Voigt elements which are named Unit 1 and Unit 2. Each of the units is composed of a controllable damper and a fixed spring. The stiffness \((k_1\) and \(k_2)\) of the two springs are not changed. However, the stiffness of the whole system can be varied by changing the damping coefficient \(c_1(t)\) or \(c_2(t)\). In other words, one of the two controllable dampers can be used to realize the variable stiffness of the sys-

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**Table 2 RMS values of acceleration**

<table>
<thead>
<tr>
<th>Road class</th>
<th>RMS values of acceleration (m/s(^2))</th>
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</thead>
<tbody>
<tr>
<td>Passive</td>
<td>D on-off</td>
</tr>
<tr>
<td>B (Good)</td>
<td>0.5901</td>
</tr>
<tr>
<td>C (Average)</td>
<td>0.9924</td>
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<tr>
<td>D (Poor)</td>
<td>1.6091</td>
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</tbody>
</table>

**Table 3 RMS values of displacement**

<table>
<thead>
<tr>
<th>Road class</th>
<th>RMS values of displacement (m × 10(^{-5}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive</td>
<td>D on-off</td>
</tr>
<tr>
<td>B (Good)</td>
<td>4.2</td>
</tr>
<tr>
<td>C (Average)</td>
<td>10.4</td>
</tr>
<tr>
<td>D (Poor)</td>
<td>17.5</td>
</tr>
</tbody>
</table>
tem. And the other damper provides the variable damping for the system. In this way, a vibration system combined damping and stiffness on-off control (D+S on-off control) is achieved. Here \( x_0, x \) and \( m \) are the same as shown in \( \text{Fig. 1} \), and \( x_m \) is the displacement of the point between the two control units. This system with variable damping and stiffness is described by the equivalent model shown in \( \text{Fig. 9} \). Here \( c' \) and \( k' \) are the equivalent damping and stiffness, respectively.

The equations of motion for the variable damping and stiffness system shown in \( \text{Fig. 9} \) (a) are

\[
\begin{align*}
\dot{m}\ddot{x} &= -k_2(x - x_m) - c_2(t)(\dot{x} - \dot{x}_m), \\
    & \text{(6)} \\
    \dot{k}_2(x - x_m) + c_2(t)(\dot{x} - \dot{x}_m) &= k_1(x_m - x_0) + c_1(t)(\dot{x}_m - \dot{x}_0).
   & \text{(7)}
\end{align*}
\]

In this system, the following parameters are introduced

\[
\begin{align*}
\omega_{n1} &= \sqrt{k_1/m}, \quad \omega_{n2} = \sqrt{k_2/m}, \quad \alpha = \sqrt{k_2/k_1}, \\
\varphi_1 &= c_1(t)/2 \sqrt{mk_1}, \quad \varphi_2 = c_2(t)/2 \sqrt{mk_2},
   & \text{(8)}
\end{align*}
\]

where \( \omega_{n1} \) and \( \omega_{n2} \) (\( f_{n1} \) and \( f_{n2} \)) are the natural frequencies of the system with only one of the control units, \( \varphi_1 \) and \( \varphi_2 \) are the damping ratios of the two controllable dampers, and \( \alpha \) is the ratio of spring stiffness of the two units. From Eqs. (6) and (7), \( k' \) and \( c' \) are given as

\[
\begin{align*}
k' &= \frac{1 + \alpha^2 + 4\alpha^2(\varphi_1^2 + \varphi_2^2)}{(1 + \alpha^2)^2 + 4(\varphi_1 + \alpha\varphi_2)^2} k_2, \\
c' &= \frac{\varphi_2^2 + \alpha^2 \varphi_1 + 4\alpha(\varphi_1 + \alpha\varphi_2)^2}{(1 + \alpha^2)^2 + 4(\varphi_1 + \alpha\varphi_2)^2} 2 \sqrt{mk_2},
   & \text{(9)} \text{ and } (10)
\end{align*}
\]

where \( r \) is the frequency ratio \( \omega/\omega_{n1} \). When \( \varphi_2(t) \approx 1, \) Unit 2 does not move and the total stiffness \( k' \) becomes \( k_1 \). When \( \varphi_2/\varphi_1 = \alpha, k' \) is \( k_1 k_2/(k_1 + k_2) \). This is the minimum value of \( k' \). Therefore \( k' \) can be changed from \( k_1 k_2/(k_1 + k_2) \) to \( k_1 \) by \( \varphi_2 \). In order to have a wide variable range of \( k', k_1 \) must be large and \( k_2 \) must be small. In this study \( k_2/k_1 = 1/3 \) is adopted, and \( k' \) can be changed from 0.25\( k_1 \) to \( k_1 \). Let \( m = 1 \) kg, \( k_1 = 4\pi^2 N/m, r = 1 \). Figure 10 shows \( k' \) and \( c' \) \( (c' = c'/2 \sqrt{mk}) \) as functions of \( \varphi_2 \) with the fixed \( \varphi_1 = 0.15 \) and 0.5, \( k' \) varies from \( \pi^2 \) to \( 4\pi^2 N/m \).

In this system, appropriate adjustment of damping is also needed. The total damping ratio \( \zeta' \) can be controlled by \( \varphi_1 \). Figure 11 shows \( k' \) and \( c' \) as functions of \( \varphi_1 \) with the fixed \( \varphi_2 = 0.28 \) and 5.0. With the value of \( \varphi_1 \) changing from 0.15 to 0.5, when \( \varphi_2 = 5.0 \), \( c' \) varies from 0.22 to 0.56, when \( \varphi_2 = 0.28 \), \( c' \) varies from 0.19 to 0.25. Therefore in any value of \( \varphi_2, c' \) can be changed more or less by \( \varphi_1 \).

In the numerical calculation of the D+S on-off control of the proposed model, damper 1 has a damping ratio \( \varphi_1(t) = 0.15 \) in the off-state and 0.5 in the on-state to give the variable damping, and damper 2 has a damping ratio \( \varphi_2(t) = 0.28 \) in the off-state and 5 in the on-state to give the variable stiffness. In the D on-off control, only damper 1 is on-off controlled while damper 2 has a constant damping ratio \( \varphi_2(t) = 5 \). In the S on-off control, only damper 2 is on-off controlled and damper 1 has a constant damping ratio \( \varphi_1(t) = 0.5 \). For the passive case, \( \varphi_1(t) = 0.5, \varphi_2(t) = 5 \). Figure 12 shows the frequency responses of the proposed system with random input. The results are similar to those of the 1 DOF model as shown in Fig. 8. The RMS values of the displacement and acceleration are shown in Tables 4.
Table 4    RMS values of acceleration

<table>
<thead>
<tr>
<th>Road class</th>
<th>RMS values of acceleration (m/s²)</th>
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<tbody>
<tr>
<td></td>
<td>Passive</td>
</tr>
<tr>
<td>B (Good)</td>
<td>0.5005</td>
</tr>
<tr>
<td>C (Average)</td>
<td>0.8418</td>
</tr>
<tr>
<td>D (Poor)</td>
<td>1.4158</td>
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</table>

Table 5    RMS values of displacement

<table>
<thead>
<tr>
<th>Road class</th>
<th>RMS values of displacement (m × 10⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Passive</td>
</tr>
<tr>
<td>B (Good)</td>
<td>5.9</td>
</tr>
<tr>
<td>C (Average)</td>
<td>9.9</td>
</tr>
<tr>
<td>D (Poor)</td>
<td>16.7</td>
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</table>

and 5 respectively. The accelerations of the D+S on-off control are about 70% of those of the D and S on-off control. The displacements of the D+S control are smaller than those of other control methods.

4. Conclusions

On the base of a damping on-off control method, the stiffness on-off control method and the combination of damping and stiffness on-off control method were proposed. The responses of the one degree of freedom vibration system with the proposed control methods were compared with the conventional control methods in frequency and time domains with sinusoidal and random input. In the base input case, the system with D+S on-off control had the best isolating performance in the whole frequency region. In the force input case, the S on-off control had better performance than the passive system in the resonance and the high frequency regions. That was the advantage of the proposed control method for vibration isolation systems.

A new structure of the variable damping and stiffness system was proposed by using controllable dampers in series of two Voigt elements. The stiffness of the whole system was changed by one damper, and another damper provided the variable damping for the system. The simulating results showed that the proposed variable damping and stiffness system had a good performance.

References