Combined Feedback Design of Active Noise Control and Face Velocity Control Based on a Novel Secondary-Path Model of Speaker-Duct Systems*

Jong-Yih LIN** and Ching-Wen LIAO**

Active noise control (ANC)-based systems provide good low-frequency noise attenuation by destructive interference between the secondary signal from an acoustic actuator (speaker) and the primary noise of the acoustic field (duct). A novel secondary-path model that described by two cascaded transfer functions and two disturbances for speaker-duct systems is first developed in this study. One disturbance is imposed on face velocity of actuator speaker that is usually ignored in ANC systems while the other is on sound pressure at a given location in the duct. The transfer functions are identified with measurements from a developed velocity sensor and a microphone, respectively. A combined design of ANC and face velocity control (FVC) based on the identified secondary-path model is further proposed to set up an adaptive feedback ANC/FVC controller with a modified filtered-X recursive least square (FXRLS) algorithm to update controller coefficients to reduce noise levels near microphone location. Results of experiments show that the adaptive feedback ANC/FVC controller has a noise reduction performance of $\sim 27 – 46$ dB for noise at a frequency between 100 – 200 Hz, as compared with $\sim 10 – 35$ dB for that of the conventional feedback ANC design. Furthermore, the adaptive feedback controller reveals a good capability to reduce the noise level for time-varying noise of a frequency sweeping from 100 to 200 Hz. Our data demonstrate a substantial improvement in the low-frequency noise attenuation using the developed controller that includes commonly overlooked factors and support the feasibility of our proposed approach in practices.

Key Words: Speaker-Duct System, Secondary-Path Model, Active Noise Control, Face Velocity Control, Modified FXRLS Algorithm

1. Introduction

Active noise control (ANC)-based systems\(^1\) work by generating an anti-signal from the secondary source to attenuate the noise level from a primary source in an acoustic field. In practice, an acoustic actuator such as an audio speaker is usually used to produce the secondary source (control input) with a mass flow rate proportional to face velocity of the actuator speaker\(^2\) –\(^4\). This audio speaker takes a voltage command from an ANC controller to generate an adequate face velocity to provide low-frequency noise reduction. However, an effect of sound pressure on the actuator speaker face velocity is ignored in the current ANC systems for most of the cases; the occurrence of a strong sound pressure on the actuator speaker face velocity may significantly obliterate ANC performance. In an attempt to reduce the effect of sound pressure on face velocity of actuator speaker, various face velocity sensors\(^5\) –\(^8\) are proposed and motional feedback\(^9\),\(^10\) technique is developed to control actuator speakers. Nonetheless, this feedback technique may increase response for higher order modes of face velocity of actuator speaker, hence causing a potential instability problem in ANC applications.

To improve noise reduction performance, a new approach of ANC design is developed here to take account of the influence of this factor in a speaker-duct ANC system. We first develop a novel secondary-path model based on two disturbances and two measured outputs as compared with only one measured output from error microphone and one disturbance at the microphone output in
the conventional model. An adaptive feedback controller based on the derived model is subsequently designed for the ANC system. This adaptive feedback controller includes two sub-controllers: face-velocity-control (FVC) controller and ANC controller, and is therefore referred to as ANC/FVC controller in this study. Performance of the ANC/FVC controller will be presented in experiments.

2. A New Secondary-Path Model of Speaker-Duct Systems for ANC

Consider an ANC-based system as shown in Fig. 1 where a noise speaker is placed at one end of an acoustic duct to produce a noise source into the duct while a dual-voice-coils (DVC) speaker, an acoustic actuator, is applied within the domain to generate a secondary source in order to reduce noise level in the duct. The noise and actuator speakers generate mass flow rates as $m_n(t)$ and $m_d(t)$, respectively, to the duct. The mass flow rates are proportional to their speaker-face velocities $v_n(t)$ and $v_d(t)$. Let $\hat{v}_n(s)$ denotes the Laplace transform of $v_n(t)$, etc. Then the mass flow rates can be described as

$$\hat{m}_n(s) = \rho S_n \hat{v}_n(s)$$  \hspace{1cm} (1)  \\
$$\hat{m}_d(s) = \rho S_d \hat{v}_d(s)$$  \hspace{1cm} (2)

where $\rho$ is density of the medium and $S_n$ and $S_d$ are speaker-face areas of noise and DVC speakers driving the mass flows. The speaker-face velocities are affected by input voltages of $e_n(t)$ and $e_d(t)$ of the noise and DVC speakers as well as pressure loadings of $p_n(t)$ and $p_d(t)$ on the speaker faces\(^{(7,8)}\). Thus, they can be depicted as

$$\hat{v}_n(s) = G_1(s)\hat{e}_n(s) + G_2(s)\hat{p}_n(s)$$  \hspace{1cm} (3)  \\
$$\hat{v}_d(s) = G_3(s)\hat{e}_d(s) + G_4(s)\hat{p}_d(s)$$  \hspace{1cm} (4)

where $G_1(s)$, $G_2(s)$, $G_3(s)$ and $G_4(s)$ represent associated transfer function models of the speakers. Two power amplifiers are applied with a primary noise source $r(t)$ and a secondary source (control input) $e(t)$ to generate the inputs voltages to the noise and DVC speakers, respectively, and can be described as

$$\hat{e}_n(s) = G_{\text{amp-A}}(s)\hat{r}(s)$$  \hspace{1cm} (5)  \\
$$\hat{e}_d(s) = G_{\text{amp-B}}(s)\hat{e}_n(s)$$  \hspace{1cm} (6)

where $G_{\text{amp-A}}(s)$ and $G_{\text{amp-B}}(s)$ denote transfer function models of the amplifiers. In addition, given the pressure loadings $p_n(t)$ and $p_d(t)$ arise from the sound pressure of the acoustic duct at locations of the noise speaker and DVC actuator speaker, respectively, we have

$$\hat{p}_n(s) = G_5(s)\hat{m}_n(s) + G_6(s)\hat{v}_n(s)$$  \hspace{1cm} (7)  \\
$$\hat{p}_d(s) = G_7(s)\hat{m}_d(s) + G_8(s)\hat{v}_d(s)$$  \hspace{1cm} (8)

where $G_5(s)$, $G_6(s)$, $G_7(s)$ and $G_8(s)$ represent associated transfer function models of the acoustic duct\(^{(2)-(4)}\). Furthermore, we have sound pressure at the error microphone location as

$$\hat{p}_{\text{em}}(s) = G_9(s)\hat{m}_n(s) + G_{10}(s)\hat{v}_d(s)$$  \hspace{1cm} (9)

where $G_9(s)$ and $G_{10}(s)$ are associated transfer function models. A block diagram of the speaker-duct system based on the transfer function models given in Eqs. (1)-(9) can then be given as Fig. 2, showing coupling dynamics among the primary source, secondary source and acoustic duct.

For the purpose of control design, the above representation of speaker-duct system is to be simplified. Substituting Eqs. (7) and (8) into Eqs. (3) and (4) and using Eqs. (1) and (2) leads to

$$\begin{align*}
1 - \rho S_n G_2(s)G_5(s) &- \rho S_d G_2(s)G_6(s) \\
- \rho S_n G_4(s)G_7(s) &- \rho S_d G_4(s)G_8(s) \\
\frac{\hat{v}_n(s)}{\hat{e}_n(s)} &- \frac{\hat{v}_d(s)}{\hat{e}_d(s)} \\
\frac{\hat{v}_d(s)}{\hat{e}_d(s)} &- \frac{\hat{v}_{\text{em}}(s)}{\hat{e}_{\text{em}}(s)}
\end{align*}$$  \hspace{1cm} (10)

Solving the above equation, we have

$$\begin{align*}
\hat{v}_n(s) &= G_{11}(s)\hat{e}_n(s) + G_{12}(s)\hat{e}_n(s) \\
\hat{v}_d(s) &= G_{13}(s)\hat{e}_d(s) + G_{14}(s)\hat{e}_d(s)
\end{align*}$$  \hspace{1cm} (11, 12)

where

$$\begin{align*}
G_{\text{den}} &= 1 - \rho [S_n G_2(s)G_5(s) + S_d G_2(s)G_6(s)] + \rho^2 S_n S_d [G_2(s)G_5(s) + S_d G_2(s)G_5(s)] \\
&- G_4(s)G_7(s)G_6(s)G_8(s)
\end{align*}$$  \hspace{1cm} (13)

Fig. 1 Speaker-duct system

Fig. 2 Block diagram of the speaker-duct system
Define

\[ G_{11}(s) = \frac{1 - \rho S_n G_4(s) G_8(s)}{G_{den}} \]  \hspace{1cm} (14) \\
\[ G_{12}(s) = \frac{\rho S_n G_2(s) G_6(s) G_3(s)}{G_{den}} \]  \hspace{1cm} (15) \\
\[ G_{13}(s) = \frac{\rho S_n G_2(s) G_7(s) G_1(s)}{G_{den}} \]  \hspace{1cm} (16) \\
\[ G_{14}(s) = \frac{1 - \rho S_n G_2(s) G_5(s) G_3(s)}{G_{den}} \]  \hspace{1cm} (17)

Substituting Eqs. (5) and (6) into Eq. (12), we have

\[ \hat{e}_d(s) = \hat{d}_4(s) + G_{P21}(s) \hat{e}_c(s) \]  \hspace{1cm} (20)

where

\[ \hat{d}_4(s) = G_{P12}(s) \hat{r}(s) \]  \hspace{1cm} (21)

Substituting Eqs. (1) and (2) into Eq. (9) leads to

\[ \hat{p}_m(s) = \rho S_n G_0(s) \hat{p}_m(s) + \rho S_n G_{10}(s) \hat{e}_c(s) \]  \hspace{1cm} (22)

Furthermore, substituting Eq. (11) into Eq. (22) and using Eqs. (5) and (6) leads to

\[ \hat{p}_m(s) = \rho S_n G_0(s) G_{11}(s) G_{amp-A}(s) \hat{r}(s) + \rho S_n G_0(s) G_{12}(s) G_{amp-A}(s) \hat{r}(s) + \rho S_n G_{10}(s) \hat{e}_c(s) \]  \hspace{1cm} (23)

Define

\[ G_{P11}(s) = \rho S_n G_0(s) \]  \hspace{1cm} (24)

Manipulating Eq. (23) by subtracting and adding a term of \( \rho S_n G_0(s) G_{12}(s) G_{13}^{-1}(s) G_{13}(s) G_{amp-A}(s) \hat{r}(s) \) to the right hand side of this equation, we have

\[ \hat{p}_m(s) = G_{P11}(s) \hat{r}(s) + \rho S_n G_0(s) G_{12}(s) G_{13}^{-1}(s) \hat{d}_4(s) + G_{P21}(s) \hat{e}_c(s) \]  \hspace{1cm} (25)

Define

\[ G_{P22}(s) = \rho S_n G_0(s) G_{12}(s) G_{13}^{-1}(s) + S_n G_{10}(s) \]  \hspace{1cm} (26)

By use of Eq. (20), Eq. (24) can be rewritten as

\[ \hat{p}_m(s) = \hat{d}_p(s) + G_{P22}(s) \hat{e}_c(s) \]  \hspace{1cm} (27)

where

\[ \hat{d}_p(s) = G_{P11}(s) \hat{r}(s) \]  \hspace{1cm} (28)

 Consequently, a simplified representation of the speaker-duct system can be obtained based on Eqs. (20), (21), (27) and (28) where focus is on two output variables of \( \hat{e}_c(s) \) and \( \hat{p}_m(s) \). A face-velocity sensor and a microphone are applied to measure these two outputs, respectively, as shown in Fig. 1 and can be described as

\[ \hat{e}_a(s) = G_{mic}(s) \hat{p}_m(s) \]  \hspace{1cm} (29) \\
\[ \hat{e}_u(s) = G_{mic}(s) \hat{p}_m(s) \]  \hspace{1cm} (30)

3. Feedback ANC/FVC Design

In this section, a combined design of ANC and FVC is developed based on the derived discrete-time
secondary-path model given in Eqs. (39) and (40) as shown in Fig. 4. Let control input $e_c(n)$ be represented as

$$e_c(n) = e_{ANC}(n) + e_{VC}(n)$$ (41)

where $e_{ANC}(n)$ and $e_{VC}(n)$ are two components of the control input to be designed. Substituting Eq. (40) into Eq. (39) and using Eq. (41), we have

$$e_m(n) = \tilde{d}_p(n) + \bar{h}_{P2}(n) + e_{ANC}(n) + \tilde{h}_{P2}(n) \ast [\tilde{d}_p(n) + \bar{h}_{P2}(n) + e_{VC}(n)]$$ (42)

where $\tilde{h}_{P2}(n)$ is the impulse response of $\tilde{H}_{P2}(z)$ defined as $\tilde{H}_{P2}(z) = \tilde{h}_{P2}(z)\bar{H}_{P21}(z)$ (43)

Let a desired output be given as $e_{m,\text{desired}}(n) = 0$ and define an error $e(n)$ as

$$e(n) = e_{m,\text{desired}}(n) - e_m(n) = -e_m(n)$$ (44)

By use of Eq. (42), this error can be obtained as

$$e(n) = e_{ANC}(n) + \tilde{h}_{P2}(n) + e_{VC}(n)$$ (45)

where

$$e_{ANC}(n) = -\tilde{d}_p(n) - \bar{h}_{P2}(n) + e_{ANC}(n)$$ (46)

$$e_{VC}(n) = -\tilde{d}_p(n) - \bar{h}_{P2}(n) + e_{VC}(n)$$ (47)

According to Eq. (45), component errors $e_{ANC}(n)$ and $e_{VC}(n)$ result in total error $e(n)$. If both the component errors can be made zeros, the total error will consequently be zero. Therefore in below subsections, we consider to design an ANC controller $W_{ANC}(z)$ to generate $e_{ANC}(n)$ to make $e_{ANC}(n) = 0$ and to design a FVC controller $W_{VC}(z)$ to generate $e_{VC}(n)$ to achieve $e_{VC}(n) = 0$ based on Eqs. (46) and (47), respectively.

### 3.1 ANC design

An ANC controller $W_{ANC}(z)$ is designed based on Eq. (46) and is shown in the bottom dashed box in Fig. 4. This controller is chosen to be a finite impulse filter (FIR) of order $\Theta_{ANC}$, i.e., $W_{ANC}(z) = w_{ANC0} + w_{ANC1}z^{-1} + \cdots + w_{ANC(\Theta_{ANC}-1)}z^{-(\Theta_{ANC}-1)}$. A reference input $\tilde{d}_p(n)$ is used for the controller to produce $e_{ANC}(n)$. The reference input will be derived as an estimate of $\tilde{d}_p(n)$ later. Define vectors

$$w_{ANC}^T(n) = [w_{ANC0}(n) \ w_{ANC1}(n) \ \cdots \ w_{ANC(\Theta_{ANC}-1)}(n)]$$ (48)

$$\tilde{d}_p(n) = [\tilde{d}_p(n) \ \tilde{d}_p(n-1) \ \cdots \ \tilde{d}_p(n-\Theta_{ANC}+1)]$$ (49)

Then the controller can be described as

$$e_{ANC}(n) = w_{ANC}^T(n)\tilde{d}_p(n)$$ (50)

Substituting the above equation into Eq. (46) leads to

$$e_{ANC}(n) = -\tilde{d}_p(n) - \bar{h}_{P2}(n) + [w_{ANC}^T(n)\tilde{d}_p(n)]$$ (51)

assuming that $w_{ANC}(n)$ varies slowly, where the filtered reference signal vector is

$$\tilde{d}_p(n) = \bar{h}_{P2}(n) + \tilde{d}_p(n)$$ (52)

Define a cost function as

$$\zeta_{ANC}(n) = \sum_{i=1}^n \lambda^{n-i}e_{ANC}^2(i)$$ (53)

where $0 < \lambda < 1$ is a forgetting factor. In addition, define a $\Theta_{ANC} \times \Theta_{ANC}$ covariance matrix as $Q_{ANC}(n-1)$ to have

$$q_{ANC}^i(n) = \lambda^{n-i}Q_{ANC}(n-1)\tilde{d}_p(n)$$ (54)

Then define

$$k_{ANC}(n) = \frac{q_{ANC}^i(n)}{\tilde{d}_p^T(n)q_{ANC}^i(n)+1}$$ (55)

According to the filtered-X recursive-least-squares (FXRLS) algorithm(4), the optimum weight vector at time $n$ that minimizes $\zeta_{ANC}(n)$ recursively can be obtained as

$$w_{ANC}(n+1) = w_{ANC}(n) + k_{ANC}(n)e_{ANC}(n)$$ (56)

along with

$$Q_{ANC}(n) = \lambda^{n}Q_{ANC}(n-1) - k_{ANC}(n)q_{ANC}^T(n)$$ (57)

The above FXRLS algorithm is based on the assumption that $w_{ANC}(n)$ varies slowly. This assumption may be violated in a real application and, therefore, convergence to the optimum weight vector may fail. Furthermore, determinant of $Q_{ANC}^n$ may approach to infinite as $n$ approaches to infinite if persistently exciting order(10) of $\tilde{d}_p(n)$ is smaller than $\Theta_{ANC}$. To avoid the numerical difficulty and secure the convergence, we modify the change of the weight vector in Eq. (56) with a convergence factor $\nu_1$ as

$$w_{ANC}(n+1) = w_{ANC}(n) + \nu_1 k_{ANC}(n)e_{ANC}(n)$$ (58)

where $0 < \nu_1 \leq 1$ and modify Eq. (57) with a resetting mechanism as
In addition, define a algorithm is shown in the bottom dashed box in Fig. 4. Modified FXRLS algorithm as approximation of \( \overline{\text{ANC}} \)

Substituting the above equation into Eq. (47) leads to

\[
\text{Then the controller can be described as}
\]

\[
\text{where} \quad \tilde{h}(n) \text{ is the impulse response of } H_p(z). \text{ The ANC controller } W_{\text{ANC}}(z) \text{ with the modified FXRLS adaptive algorithm is shown in the bottom dashed box in Fig. 4.}
\]

3.2 FVC Design

A FVC controller \( W_{\text{VC}}(z) \) is designed based on Eq. (47) and is shown in the middle dashed box in Fig. 4. This controller is chosen to be a FIR of order \( \Theta_{\text{VC}} \), i.e., \( W_{\text{VC}}(z) = w_{\text{VC}0} + w_{\text{VC}1}z^{-1} + \cdots + w_{\text{VC}v_0}(z^{-1})^{-v_0}. \) A reference input \( d(n) \) is used for the controller to produce \( e_{\text{VC}}(n) \). The reference input will be derived as an estimate of \( d(n) \) later. Similar to the previous approach, define vectors

\[
\begin{align*}
\mathbf{w}_{\text{VC}}(n) &= [w_{\text{VC}0}(n) \quad w_{\text{VC}1}(n) \quad \cdots \quad w_{\text{VC}v_0}(n)] \\
\tilde{d}_{a}(n) &= [\tilde{d}_a(n) \quad \tilde{d}_a(n-1) \quad \cdots \quad \tilde{d}_a(n-\Theta_{\text{VC}}+1)]
\end{align*}
\]

Then the controller can be described as

\[
e_{\text{VC}}(n) = \mathbf{w}_{\text{VC}}(n)\tilde{d}_{a}(n)
\]

Substituting the above equation into Eq. (47) leads to

\[
\text{assuming that } w_{\text{VC}}(n) \text{ varies slowly, where the filtered reference signal vector is}
\]

\[
\tilde{d}_{a}(n) = \tilde{h}(n) + \tilde{d}_{a}(n)
\]

Define a cost function as

\[
\zeta_{\text{VC}}(n) = \sum_{i=1}^{n} \lambda^{n-i} e_{\text{VC}}^2(i)
\]

In addition, define a \( \Theta_{\text{VC}} \times \Theta_{\text{VC}} \) covariance matrix as \( Q_{\text{VC}}(n-1) \) to have

\[
\begin{align*}
q'_{\text{VC}}(n) &= \lambda^{-1} Q_{\text{VC}}(n-1) - k'_{\text{VC}}(n)q'_{\text{VC}}(n) \\
& \text{if } Q_{\text{VC}}(n-1) < \alpha_1 \\
Q_{\text{VC}}(n) &= I_{\text{ANC}} + \Theta_{\text{VC}}
\end{align*}
\]

where \( I_{\text{ANC}} \) is an identity matrix and \( \alpha_1 \) is a large positive number, i.e., \( \alpha_1 \gg 1 \). This algorithm is referred to as a modified FXRLS algorithm in this study.

In practical applications, \( H_p(z) \) is unknown and must be estimated for application of Eq. (52). Assume that approximation of \( H_p(z) \) can be obtained as \( \tilde{H}_p(z) \), specifically, \( \tilde{H}_p(z) \approx H_p(z) \). Then the modified FXRLS algorithm can be expressed as Eqs. (50)–(55), (58) and (59), except with

\[
\tilde{d}_{a}(n) = \tilde{h}(n) + \tilde{d}_{a}(n)
\]

where \( \tilde{h}(n) \) is the impulse response of \( H_p(z) \). The ANC controller \( W_{\text{ANC}}(z) \) with the modified FXRLS adaptive algorithm is shown in the bottom dashed box in Fig. 4.

3.3 Feedback ANC/FVC Controller

Combine the ANC controller with FVC controller together as a controller to generate a control signal \( e_{\text{VC}}(n) = e_{\text{ANC}}(n) + e_{\text{VC}}(n) \). This controller is referred to as an ANC/FVC controller and requires two error signals \( e_{\text{ANC}}(n) \) and \( e_{\text{VC}}(n) \) and two reference signals \( \tilde{d}_{a}(n) \) and \( \tilde{d}_{a}(n) \) for applications. The error signals can be obtained as follows. Substituting Eq. (41) into Eq. (40) leads to

\[
\tilde{d}_{a}(n) = e_{\text{VC}}(n) - \tilde{h}(n) + e_{\text{ANC}}(n) + e_{\text{VC}}(n)
\]

Then substituting the above equation into Eq. (47), we have

\[
e_{\text{VC}}(n) = -e_{\text{VC}}(n) + \tilde{h}(n) + e_{\text{ANC}}(n)
\]

Also equaling Eq. (44) to Eq. (45), we have

\[
e_{\text{ANC}}(n) = -e_{\text{ANC}}(n) - \tilde{h}(n) + e_{\text{VC}}(n)
\]

By use of the above two equations, \( e_{\text{ANC}}(n) \) and \( e_{\text{VC}}(n) \) can be computed based on the sensor outputs \( e_{\text{a}}(n) \) and \( e_{\text{m}}(n) \) and the secondary-path models \( H_p(z) \) and \( H_{p2}(z) \). In practical applications, approximations of \( H_p(z) \) and \( H_{p2}(z) \) are acquired as \( \tilde{H}_p(z) \) and \( \tilde{H}_{p2}(z) \), respectively. \( \tilde{H}_{p2}(z) \) and \( \tilde{H}_{p2}(z) \) are acquired as \( \tilde{H}_p(z) \) and \( \tilde{H}_{p2}(z) \) specifically. Therefore, the error signals can be approximated as

\[
e_{\text{VC}}(n) = -e_{\text{VC}}(n) + \tilde{h}(n) + e_{\text{ANC}}(n)
\]

\[
e_{\text{ANC}}(n) = -e_{\text{ANC}}(n) - \tilde{h}(n) + e_{\text{VC}}(n)
\]

As to the reference signals, they can be obtained by use of the available \( e_{\text{ANC}}(n) \) and \( e_{\text{VC}}(n) \) according to Eqs. (46) and (47). In practices, the reference signals that are approximate to \( \tilde{d}_{a}(n) \) and \( \tilde{d}_{a}(n) \) can be computed as

\[
\tilde{d}_{a}(n) = -e_{\text{ANC}}(n) - \tilde{h}(n) + e_{\text{ANC}}(n)
\]

\[
\tilde{d}_{a}(n) = -e_{\text{VC}}(n) - \tilde{h}(n) + e_{\text{VC}}(n)
\]

Since no independent signal source is included in the reference signals here for the ANC/FVC controller, this controller is thus referred to as a feedback ANC/FVC controller. The feedback ANC/FVC controller is shown in


JSME International Journal
Fig. 4 and utilizes sensor signals $e_a(n)$ and $e_m(n)$ to generate a control signal $e_c(n)$ to attenuate disturbance effects of $\bar{d}_p(n)$ and $\bar{d}_s(n)$ to achieve $e_{\text{ANC}}(n) = 0$ and $e_{\text{VFC}}(n) = 0$. As a result, $e_c(n)$ can follow a control-command signal of $e_{\text{com}}(n) = b_{P21}(n) * e_{\text{ANC}}(n)$ and $e_m(n)$ can reduce.

3.4 Reduction to the conventional ANC design

By removal of $e_m(n)$, the new secondary-path model given as the top dashed box in Fig. 4 can be reduced to a conventional secondary-path model $H_{P21}(z)$ in Fig. 5. This conventional secondary-path model shows only one disturbance $\bar{d}_p(n)$ on one measured output $e_m(n)$. This disturbance can be derived by lumping $\bar{d}_p(n)$ and $\bar{d}_s(n)$ together as

$$ \bar{d}_p(n) = \bar{d}_p(n) + h_{P22}(n) * \bar{d}_s(n) $$

Thus the measured output can be expressed as

$$ e_m(n) = \bar{d}_p(n) + h_{P21}(n) * e_{\text{ANC}}(n) $$

Based on this model, a conventional feedback ANC controller $W_{\text{ANC}}(z)$ is designed as shown in Fig. 5 where a reference signal $\bar{d}_p(n)$ for the controller is given as

$$ \bar{d}_p(n) = e_{\text{com}}(n) - h_{P21}(n) * e_{\text{ANC}}(n) $$

$W_{\text{ANC}}(z)$ is chosen to be of the same form as given in section 3.1. Similarly, a modified FXRLS adaptive algorithm is used to update the controller coefficients as described in Fig. 5. The conventional ANC design can be regarded as a reduction of the developed ANC/FVC design.

4. Experiments

The speaker-duct system depicted in Fig. 1 was applied in experiments to obtain frequency responses of $\hat{G}_{P21}(s)$ and $\hat{G}_{P22}(s)$, approximations of $G_{P21}(s)$ and $G_{P22}(s)$. A function generator was used to produce a sinusoidal signal $e(t)$ of frequency at $\omega = 40\, \text{Hz}$ to excite the DVC actuator speaker. Two voltage followers were applied to measure primary coil voltage $e_1(t)$ and secondary coil voltage $e_2(t)$ of the DVC actuator speaker. A velocity sensor developed in Ref. (12) was adopted here with

$$ e_1(t) \text{ and } e_2(t) \text{ as sensor inputs. This sensor was implemented in a digital signal processor (DS1103) to compute face velocity of the actuator speaker as } e_1(t). \text{ In addition, an error microphone was used to measure sound pressure at its location as } e_m(t). \text{ At the steady state, signals } e_1(t), e_2(t) \text{ and } e_m(t) \text{ were simultaneously obtained for 1 second at } 0.001 \text{ second sampling rate to compute } \hat{G}_{P21}(j\omega) = \hat{e}_1(j\omega)/\hat{e}_2(j\omega) \text{ and } \hat{G}_{P22}(j\omega) = \hat{e}_m(j\omega)/\hat{e}_2(j\omega). \text{ Experiments were carried out from } \omega = 40 \text{ to } 300 \, \text{Hz at every } 5\, \text{Hz. Data of } \hat{G}_{P21}(j\omega) \text{ and } \hat{G}_{P22}(j\omega) \text{ were shown as "+" line in Fig. 6 and "+" line in Fig. 7, respectively. Transfer function models of } \hat{G}_{P21}(s) \text{ and } \hat{G}_{P22}(s) \text{ were then identified. Simulated frequency responses of the derived transfer functions were given as solid lines in Figs. 6 and 7, showing good agreement with experimental data. A bilinear transformation with sample time of 0.001 second was then used to transform } \hat{G}_{P21}(s) \text{ and } \hat{G}_{P22}(s) \text{ into their discrete-time equivalents } H_{P21}(z) \text{ and } H_{P22}(z) \text{ with poles/zeros and gain given in Tables 1 and 2. Subsequently, } H_{P22}(s) \text{ was obtained by multiplying } H_{P21}(z) \text{ with } H_{P22}(z) \text{ (data not shown).}
Table 1 \( \tilde{H}_{p_1}(z) \) with gain = \(-0.0310\)

<table>
<thead>
<tr>
<th>Zeros</th>
<th>Poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9898, 0.8852, 0.9935 + 0.0509j</td>
<td>0.9251, 0.8165, 0.9815 + 0.0202j</td>
</tr>
<tr>
<td>0.9865 + 0.0917j, 0.9826 + 0.1381j</td>
<td>0.9920 + 0.0567j, 0.9859 + 0.0953j</td>
</tr>
<tr>
<td>0.9764 + 0.1787j, 0.9578 + 0.2591j</td>
<td>0.9824 + 0.1411j, 0.9765 + 0.1842j</td>
</tr>
<tr>
<td>0.9193 + 0.3127j, 0.9096 + 0.3308j</td>
<td>0.9566 + 0.2658j, 0.9140 + 0.3123j</td>
</tr>
<tr>
<td>0.8239 + 0.3318j, 0.1254 + 0.2537j</td>
<td>0.9133 + 0.3335j, 0.8076 + 0.3561j</td>
</tr>
<tr>
<td>0.4810 + 0.2351j</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 \( \tilde{H}_{p_2}(z) \) with gain = \(-9.184 \times 10^{-7}\)

<table>
<thead>
<tr>
<th>Zeros</th>
<th>Poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (triple), 0.999, 0.567j</td>
<td>0.8699, 0.9931 + 0.0504j</td>
</tr>
<tr>
<td>0.9914 + 0.0757j, 0.8419 + 0.1056j</td>
<td>0.9915 + 0.0924j, 0.9867 + 0.1377j</td>
</tr>
<tr>
<td>0.9829 + 0.1543j, 0.9725 + 0.2255j</td>
<td>0.9788 + 0.1796j, 0.9595 + 0.2587j</td>
</tr>
<tr>
<td>0.9124 + 0.4032j, 0.8926 + 0.4370j</td>
<td>0.8329 + 0.2001j, 0.9340 + 0.3387j</td>
</tr>
<tr>
<td></td>
<td>0.8717 + 0.3903j, 0.8981 + 0.4327j</td>
</tr>
<tr>
<td></td>
<td>0.8769 + 0.4747j</td>
</tr>
</tbody>
</table>

![Fig. 8 Experimental configuration of ANC using DS1103](image)

A feedback ANC/FVC controller in Fig. 4 was then designed by use of the derived transfer functions \( \tilde{H}_{p_1}(z) \) and \( \tilde{H}_{p_2}(z) \) for experiments. Orders of ANC controller \( W_{\text{ANC}}(z) \) and FVC controller \( W_{\text{VC}}(z) \) were selected as \( \Theta_{\text{ANC}} = 6 \) and \( \Theta_{\text{VC}} = 6 \), respectively. Coefficients of \( W_{\text{ANC}}(z) \) and \( W_{\text{VC}}(z) \) were updated with the modified FXRLS algorithms of \( \lambda = 0.999, \nu_1 = 0.1, \nu_2 = 0.1, \alpha_1 = 50,000 \), and \( \alpha_2 = 50,000 \).

An experimental configuration was set up as shown in Fig. 8 to evaluate performance of this feedback ANC/FVC controller. A signal \( r(t) = \sin \omega t \) at frequency \( \omega \) was generated from the function generator. This signal was applied to produce disturbances \( d_p(t) \) and \( d_l(t) \) to the secondary path. The digital signal processor (DS1103) was used to implement the feedback ANC/FVC controller and the face velocity sensor.\(^{12}\) Face velocity of the DVC actuator speaker was obtained as \( e_v(t) \). An error microphone was used to measure sound pressure at its location denoted as \( e_m(t) \). At the steady state, signals \( r(t) \) and \( e_m(t) \) were simultaneously obtained for 1 second at sampling rate of 0.001 second to compute \( \hat{e}_m(j\omega) / \hat{r}(j\omega) \). Experiments were carried out from \( \omega = 100 \) to 200 Hz at every 5 Hz and data of magnitudes of \( \hat{e}_m(j\omega) / \hat{r}(j\omega) \) were shown as the “o” line in Fig. 9. There was \(-27 \sim -46 \) dB reduction of noise at frequency in the range of 100 – 200 Hz as compared to that of the uncontrolled one (“x” line in Fig. 9).

Measurements were also performed with a conventional feedback ANC controller in Fig. 4, which had \( W_{\text{ANC}}(z) \) of \( \Theta_{\text{ANC}} = 6 \) and the identified transfer function \( \tilde{H}_{p_2}(z) \). Coefficients of the conventional feedback ANC controller were updated with the modified FXRLS algorithm of \( \lambda = 0.999, \nu_1 = 0.1, \) and \( \alpha_1 = 50,000 \). The conventional feedback ANC controller was implemented in DS1103 for ANC experiments as for the developed ANC system (Fig. 8). Data of magnitude of \( \hat{e}_m(j\omega) / \hat{r}(j\omega) \) were shown as the “+” line in Fig. 9. There was \(-10 \sim -35 \) dB reduction of noise at frequency in the range of 100 – 200 Hz as compared to that of the uncontrolled one (“x” line in Fig. 9). As shown in Fig. 9, the conventional system nevertheless produced less noise reduction \((-5 \sim -29 \) dB) than did the feedback ANC/FVC controller.

Performance of the feedback ANC/FVC controller was further examined by time-varying noise as shown in Fig. 8. The function generator was used to generate a time-varying signal as \( r(t) = \sin \omega t \) where frequency \( \omega \) was changing from 100 to 200 Hz in 20 seconds. This sweeping sinusoidal signal was used to produce time-varying disturbances \( d_p(t) \) and \( d_l(t) \) to the secondary path. At the steady state, data of \( r(t) \) and \( e_m(t) \) were obtained for 20 seconds at sampling rate of 0.0001 second to compute \( \hat{e}_m(j\omega) / \hat{r}(j\omega) \). Data of \( e_m(t) \) were shown in Fig. 10. Similarly, experiments of ANC with (Fig. 12) and without the conventional feedback ANC controller (Fig. 12) were carried out respectively. As seen from Figs. 9 – 11, the feedback ANC/FVC controller displayed the best...
ANC controller; the dashed line, controlled with the feedback ANC/FVC controller, there was also an improved noise reduction performance for the feedback ANC/FVC controller with respect to time-varying noise in a wide frequency bandwidth of 100 Hz.

5. Conclusions

We have established a novel secondary-path model that includes the contribution from a frequently ignored disturbance imposed on face velocity output of actuator speaker apart from a commonly considered disturbance on sound pressure output in the ANC system for noise reduction design. Moreover, this model utilized two cascade transfer functions identified by a newly developed velocity sensor along with a microphone. We showed that an adaptive feedback ANC/FVC controller based on the novel secondary-path model displayed substantial improvement in the low-frequency attenuation as compared with the conventional one (∼5 – 29 dB improvement in the range of 100 – 200 Hz). Furthermore, the adaptive feedback ANC/FVC controller demonstrates a much robust capability to reduce the noise level for time-varying noise of a frequency sweeping from 100 to 200 Hz. Our data thus provide experimental evidence of the lost performance from the commonly overlooked factors in current ANC systems and support the feasibility of our proposed approach in practices.

Acknowledgments

This work was supported by the National Science Council of the Republic of China under the contract of NSC 93-2218-E-005-004.

References


(2) Hull, A.J., Radcliffe, C.J., Miklavic, M. and MacCluer,


