Dynamic Response of a Spinning Timoshenko Beam with General Boundary Conditions under a Moving Skew Force Using Global Assumed Mode Method

T.N. SHIAU**, E.C. CHEN***, K.H. HUANG** and W.C. HSU****

In this study the global assumed mode method (GAMM) is used to analyze the dynamic behavior of a spinning Timoshenko beam subjected to a moving skew force with general boundary conditions. The moving skew force is usually caused by the frictional effect or the weight of the components. Considering three general geometric boundaries, i.e. hinged-hinged, clamped-clamped, and clamped-hinged, the system equations of motion are derived by the Lagrangian approach combining with the GAMM. The transient response of the system due to a moving skew force is evaluated by the Runge-Kutta method. The numerical results show that the lateral deflections due to the skew force for the hinged-hinged boundary case are smaller than those in the case of a clamped-clamped or clamped-hinged boundary. And the axial deflections due to the skew force are larger in the case of a hinged-hinged boundary.

Key Words: GAMM, Moving Skew Force, Rotating Shaft, Ball Screw

1. Introduction

The dynamic analysis of an elastic structure under moving loads has been a topic of interest for over a century. Interest in this problem originated in civil engineering for the design of the railroad bridges and highway structures. Recently, the problem of moving loads on a rotating shaft has also arisen in many modern machining operations such as high-speed feed drive systems and high-speed drilling.

Katz et al.(1) presented a study of the transient response of a rotating shaft subjected to a load moving with constant velocity. The modal analysis and an integral transformation method were employed to obtain the transient response. Katz(2) also studied the dynamic response of a rotating shaft subjected to an axially moving and rotating load. Zu and Han(3) obtained, for the first time, closed-form solutions for the free vibration analysis of spinning Timoshenko beams for all the classical boundary conditions, namely, simply-supported or hinged-hinged, hinged-free, free-free, fixed-free, fixed-fixed and fixed-hinged. In addition, under the effect of a moving load the dynamic responses of a spinning Timoshenko beam with general boundary conditions was also analyzed by Zu and Han(4). Lee(5) employed the assumed mode method and the Timoshenko beam theory to analyze the dynamic response of a rotating shaft subjected to an axial force and a moving load. The dynamic behavior of an Euler beam with multiple point constraints traversed by a moving concentrated mass was also analyzed by Lee(6). Abu-Hilal et al.(7) even studied the dynamic behavior of a uniform elastic Bernoulli beam with general boundary conditions when the beam is subjected to a moving harmonic force. Abu-Hilal(8) also studied transverse vibrations of elastic homogeneous isotropic beams with general boundary conditions due to a moving random force.

On the part of dynamic analysis, the global assumed mode method (GAMM) introduced by Shiau and Hwang(9),(10) was first employed in a rotor-bearing system to determine its critical speeds and the unbalance response. They even demonstrated the efficiency and the accuracy by using the GAMM as compared with the finite element method. It is noted that some researchers call the GAMM a different name: generalized polynomial expansion method (GPEM).
Most researchers had used the modal analysis or the assumed mode method to investigate the dynamic behavior of rotating beams subjected to moving lateral forces. From the theoretical analysis of the above methods, we find that the GAMM can not only construct simpler system equations of motion, but also can be employed to analyze different system models combining distinct geometric boundaries. Therefore, this present work employs the GAMM to study the dynamic behavior of a rotating shaft subjected to a moving skew force, which includes an axial driving force and a lateral force. Several distinct geometric boundary conditions are also taken into account in this study.

2. Equations Formulation

A Timoshenko beam of length \( l \), spinning along its longitudinal axis at a constant speed \( (\Omega) \) is shown in Fig. 1. A set of inertial coordinates \((OXYZ)\) with the origin \( O \) on the middle of the beam, are adopted. In addition to this inertial frame, a rotating coordinate \((O\overline{XY}Z)\) is also defined. It is noted that the inertial coordinate \( OX \) coincides with the rotating coordinate \( Ox \) in the undeformed configuration. The skew force is assumed to be of constant magnitude and a constant angle \( \theta \) inclined along axis \( OY \) and it moves along axis \( OX \) in plane \( X-Y \) with a constant axial velocity \( v \). The Timoshenko beam theory is used to model the rotating shaft.

The system is considered to have three degrees of freedom for translation \((U, V, W)\) and two degrees of freedom for rotation \((B, \Gamma)\), as shown in Fig. 1. They are functions of the time and the axial position as shown below

\[
\begin{align*}
U &= U(x,t) \\
V &= V_b(x,t) + V_l(x,t) \\
W &= W_b(x,t) + W_l(x,t) \\
B &= B(x,t) = -\frac{dW_b}{dx} \\
\Gamma &= \Gamma(x,t) = \frac{dV_b}{dx}
\end{align*}
\]

(1)

where the translations \((V, W)\) consist of the contribution due to bending effects \((V_b, W_b)\) and transverse shear effects \((V_l, W_l)\). Based on the GAMM(9), the deflections \(U, V, W, B\) and \(\Gamma\) can be expressed as simple polynomial forms as follows.

\[
\begin{align*}
U &= \sum_{i=1}^{N_u} a_i(t)x^{i-1} \\
&= a_1(t)x + a_2(t)x^2 + \cdots + a_{N_u}(t)x^{N_u-1} \\
V &= \sum_{i=1}^{N_v} b_i(t)x^{i-1} \\
&= b_1(t)x + b_2(t)x^2 + \cdots + b_{N_v}(t)x^{N_v-1} \\
W &= \sum_{i=1}^{N_w} c_i(t)x^{i-1} \\
&= c_1(t)x + c_2(t)x^2 + \cdots + c_{N_w}(t)x^{N_w-1}
\end{align*}
\]

where \(a_i, b_i, c_i, d_i\) and \(e_i\) are time-dependent generalized coordinates, \(x\) is the axial position on the shaft, and \(N_p\) is the number of terms of the polynomial. It should be noticed that the deflections in Eq. (2) have not satisfied the geometric constraints yet.

The Lagrangian approach is employed to derive the system equation of motion. It requires the formulation of the shaft kinetic energy \((T_s)\) and the shaft potential energy \((U_s)\). A uniform shaft with circular cross-section is assumed in the present study. Then, the kinetic energy of the shaft is given by

\[
T_s = \frac{1}{2} \int_0^l \rho A(\dot{U}^2 + \dot{V}^2 + \dot{W}^2)dx + \frac{1}{2} \int_0^l \rho I_d(\dot{B}^2 + \dot{\Gamma}^2)dx + \frac{1}{2} \rho I_p \Omega^2
\]

(3)

where \(\rho\) is the mass per unit volume, \(A\) is the cross-sectional area of the beam, \(I_d\) and \(I_p\) are the diametral and polar moments of inertia, and \(\Omega\) is the rotating speed of the shaft.

The shaft potential energy, which consists of the components due to pure bending, axial deformation, shear deformation, and lateral deformation, is expressed as

\[
U_s = \frac{1}{2} \int_0^l EA(U')^2dx + \frac{1}{2} \int_0^l EI \left( (V'_b)^2 + (W'_l)^2 \right) dx
\]

Fig. 1 A rotating shaft subjected to moving skew force (hinged-hinged boundary conditions)

where $E$ is the Young’s modulus, $I$ is the area moment of inertia of the shaft, $G$ is the shear modulus, $k$ is the shear coefficient and $P_n$ is the axial force. It should be noted that other axial forces could also be taken into account in the expression of $P_n$. Moreover, it is noted that the upper bound $x$ in the last term presents the position of the moving load, i.e. $x = vt$.

The Lagrangian approach is expressed as following

$$\frac{d}{dt} \left[ \frac{\partial}{\partial q} (T_s - U_s) \right] - \frac{\partial}{\partial q} (T_s - U_s) = Q$$

(5)

where the generalized coordinates vector $q = \{a \ b \ c \ d \ e\}^T$ represents the coefficient vectors with coefficients $a_i$, $b_i$, $c_i$, $d_i$ and $e_i$ for $i = 1$ to $N_p$. In the condition of the constant rotating speed, the equations of the motion in matrix form are derived as following

$$\begin{bmatrix}
\rho AA_{ij} & 0 & 0 & 0 & 0 \\
0 & \rho AA_{ij} & 0 & 0 & 0 \\
0 & 0 & \rho AA_{ij} & 0 & 0 \\
0 & 0 & 0 & \rho I_d A_{ij} & 0 \\
0 & 0 & 0 & 0 & \rho I_d A_{ij}
\end{bmatrix} \begin{bmatrix}
\ddot{a} \\
\ddot{b} \\
\ddot{c} \\
\ddot{d} \\
\ddot{e}
\end{bmatrix} + \begin{bmatrix}
EAB_{ij} & 0 & 0 & 0 & 0 \\
0 & (kGAB_{ij}) & 0 & 0 & 0 \\
0 & 0 & (kGAB_{ij}) & 0 & 0 \\
0 & 0 & 0 & (kGAC_{ij}^T) & 0 \\
0 & 0 & 0 & 0 & (kGAC_{ij}^T)
\end{bmatrix} \begin{bmatrix}
\dot{a} \\
\dot{b} \\
\dot{c} \\
\dot{d} \\
\dot{e}
\end{bmatrix}
$$

$$+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & (-P_n D_{ij}) & 0 & 0 & 0 \\
0 & 0 & (-P_n D_{ij}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c \\
d \\
e
\end{bmatrix} = \begin{bmatrix}
P_n E_i \\
P_n E_i \\
P_n E_i \\
P_n E_i \\
P_n E_i
\end{bmatrix}$$

(6)

where

$$A_{ij} = \int_0^d x^{i+j-2} dx$$

$$B_{ij} = \int_0^d (i-1)(j-1)x^{i+j-4} dx$$

$$C_{ij} = \int_0^d (i-1)x^{i+j-3} dx$$

$$D_{ij}(t) = \int_0^{x_{out}} (i-1)(j-1)x^{i+j-4} dx$$

$$E_i = x^{-1}(x = vt)$$

(7)

For simplicity, Eq. (6) can be expressed as

$$[M][\ddot{q}] + [\Omega][C][\dot{q}] + [K + \Phi(t)][q] = [F(t)]$$

(8)

It is noted that no geometrical boundaries have been satisfied in the equations of motion, Eq. (8). The complete equation of motion satisfying boundary conditions will be introduced in the next section via the derivation of the transformation matrix.

### 3. The Transformation Matrix for Boundary Conditions

Three general geometric boundaries of a shaft are introduced and the corresponding transformation matrices from these boundary conditions are derived. Thus, the equations of motion with the combination of boundary conditions can be formulated via the transformation matrix.

In the following derivation of the transformation matrix, the parameters $x_r$ and $x_l$ are used to denote the position of the supports on the right side and the left side of the rotating shaft, respectively. Because of the symmetric property of the boundary conditions, only the deflections $V$ and $B$ need to be derived from the four deflections $V$, $W$, $B$ and $\Gamma$.

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3.1 The hinged-hinged boundary condition

When the shaft is hinged-hinged, the boundary conditions are satisfied and expressed as follows:

\[ U(x_i,t) = 0; \quad V(x_i,t) = V(x_r,t) = 0; \]

\[ W(x_i,t) = W(x_r,t) = 0; \quad U'(x_i,t) = 0; \]

\[ B'(x_i,t) = B'(x_r,t) = 0; \quad I'(x_i,t) = I'(x_r,t) = 0 \]

(9)

Substitution of the above equation into Eq. (2) yields

\[
\begin{bmatrix}
1 & x_i & x_i^2 & \ldots & x_i^{N_p-1} \\
1 & x_r & x_r^2 & \ldots & x_r^{N_p-1}
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_{N_p}
\end{bmatrix}
= 0
\]

(10.a)

for \( V(x_i,t) = V(x_r,t) = 0 \) and

\[
\begin{bmatrix}
1 & 2x_i & \ldots & (N_p-1)x_i^{N_p-2} \\
1 & 2x_r & \ldots & (N_p-1)x_r^{N_p-2}
\end{bmatrix}
\begin{bmatrix}
d_2 \\
d_3 \\
\vdots \\
d_{N_p}
\end{bmatrix}
= 0
\]

(10.b)

for \( B'(x_i,t) = B'(x_r,t) = 0 \).

From Eq. (10.a) and Eq. (10.b), the following equations can be obtained respectively as follows

\[
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_{N_p}
\end{bmatrix}
= [R_b][b_I]
\]

(11.a)

\[
\begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
d_{N_p}
\end{bmatrix}
= [R_d][d_I]
\]

(11.b)

where

\[
[R_b] = \begin{bmatrix}
1 & x_i & x_i^2 & \ldots & x_i^{N_p-1} \\
1 & x_r & x_r^2 & \ldots & x_r^{N_p-1}
\end{bmatrix}
\]

(12.a)

\[
[R_d] = \begin{bmatrix}
1 & x_i & x_i^2 & \ldots & x_i^{N_p-1} \\
1 & x_r & x_r^2 & \ldots & x_r^{N_p-1}
\end{bmatrix}
\]

(12.b)

\[
[R_2] = \begin{bmatrix}
1 & 2x_i & \ldots & (N_p-1)x_i^{N_p-2} \\
1 & 2x_r & \ldots & (N_p-1)x_r^{N_p-2}
\end{bmatrix}
\]

(12.c)

3.2 The clamped-clamped boundary condition

If the shaft is clamped at both ends, the boundary conditions are specified as

\[ U(x_i,t) = U(x_r,t) = 0; \quad V(x_i,t) = V(x_r,t) = 0; \]

\[ W(x_i,t) = W(x_r,t) = 0; \quad U'(x_i,t) = U'(x_r,t) = 0; \]

\[ V'(x_i,t) = V'(x_r,t) = 0; \quad W'(x_i,t) = W'(x_r,t) = 0 \]

(13)

Similarly, substitution of the above equation into Eq. (2), one can obtain the following equations

\[
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_{N_p}
\end{bmatrix}
= [R_b][b_I]
\]

(14)

Where

\[
[R_1] = \begin{bmatrix}
1 & x_i & x_i^2 & x_i^3 & \ldots & x_i^{N_p-1} \\
1 & x_r & x_r^2 & x_r^3 & \ldots & x_r^{N_p-1}
\end{bmatrix}^{-1}
\]

(15)

3.3 The clamped-hinged boundary condition

The boundary conditions are expressed as follows:

\[ U(x_i,t) = 0; \quad V(x_i,t) = V(x_r,t) = 0 \]

\[ W(x_i,t) = W(x_r,t) = 0; \quad U'(x_i,t) = 0 \]

\[ V'(x_i,t) = V'(x_r,t) = 0; \quad W'(x_i,t) = W'(x_r,t) = 0 \]

(16)

After substitution of the above equation into Eq. (2), one can obtain the following equations

\[
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_{N_p}
\end{bmatrix}
= [R_b][b_I]
\]

(17.a)

\[
\begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
d_{N_p}
\end{bmatrix}
= [R_d][d_I]
\]

(17.b)

where

\[
[R_b] = \begin{bmatrix}
1 & x_i & x_i^2 & x_i^3 & \ldots & x_i^{N_p-1} \\
1 & x_r & x_r^2 & x_r^3 & \ldots & x_r^{N_p-1}
\end{bmatrix}
\]

(18.a)

\[
[R_2] = \begin{bmatrix}
1 & 2x_i & \ldots & (N_p-1)x_i^{N_p-2} \\
1 & 2x_r & \ldots & (N_p-1)x_r^{N_p-2}
\end{bmatrix}
\]

(18.b)

Based on the above formulation, the matrices \( R_b \) and \( R_d \) are obtained by considering \( V \) and \( B \) to satisfy each geometrical boundary. Using the same formulation, the matrices \( [R_b], [R_d] \) and \( [R_2] \) can also be obtained by using the other deflections \( U, W \) and \( I \), respectively. The matrices \( [R_b], [R_d], [R_2] \) and \( [R_2] \) are combined to obtain the transformation matrix \( R \) as
\[ R = \begin{bmatrix} R_c & 0 & 0 & 0 & 0 \\ 0 & R_c & 0 & 0 & 0 \\ 0 & 0 & R_c & 0 & 0 \\ 0 & 0 & 0 & R_c & 0 \\ 0 & 0 & 0 & 0 & R_c \end{bmatrix} \]  

Recall Eq. (8), a new equation of motion satisfying geometric boundaries is obtained as
\[
[R]^T[M][R][\dot{q}] + [\Omega]C[R][\dot{q}] + [F] = [R]^T[K + \Phi(\dot{t})][R][\dot{q}] = [R]^T[F(\dot{t})] 
\]  
where \( \{q\} = \{a_l \ b_l \ c_l \ d_l \ e_l\}^T \)

For simplicity, Eq. (20) can be rewritten as
\[
[M]^*\{\ddot{q}\} + \Omega[C]^*\{\dot{q}\} + [F]^*\{F(\dot{t})\]  
\]  
where
\[
[M]^* = [R]^T[M][R] \\
[C]^* = [R]^T[C][R] \\
[K]^* = [R]^T[K][R] \\
[\Phi(\dot{t})]^* = [R]^T[\Phi(\dot{t})][R] \\
[F]^* = [R]^T[F] 
\]

\([M]^*\) is the mass matrix due to translation and rotation. \([C]^*\) is the gyroscopic matrix. \([K]^*\) is the stiffness matrix due to shear-bending and axial strain. \([\Phi(\dot{t})]^*\) is a time-dependent matrix due to the axial force and \([F]^*\) is a forcing vector due to the moving skew force.

4. Transient Responses Formulation

The transient responses governed by Eq. (21), can be analyzed by the form of first-order differential equations as follows and be obtained by using the Runge-Kutta method with specified initial conditions.
\[
[y] = [A(\dot{t})][y] + [\dot{F}(\dot{t})] 
\]  
where \(y^T = [q \ \dot{q}]\).

\[
[A(\dot{t})] = \begin{bmatrix} 0 & I \\ -[M]^{-1}([K]^* + [\Phi(\dot{t})]^*) & -\Omega[M]^{-1}[C]^* \end{bmatrix} 
\]  

\[
[\dot{F}(\dot{t})] = \begin{bmatrix} 0 \\ [M]^{-1}[F]^* \end{bmatrix} 
\]  

5. Numerical Results and Discussions

To gain an insight into the transient responses of the system, the influences of the parameters will be discussed. Such parameters include the spin speed of the shaft, the moving velocity of the skew force, the Rayleigh beam coefficient and the skew angle. The above parameters are described by using the following symbols.

(a) \( \alpha = \omega / \omega_s \) is the non-dimensional moving speed of the force, \( \omega_s = (\pi / l) \sqrt{EI / \rho A} \).

(b) \( \beta = \pi r_0 / l \) is the Rayleigh beam coefficient for a circular cross section. The Rayleigh beam coefficient relates to the ratio of the diameter of the shaft and its length.

(c) \( \Omega = \omega / \omega_{10} \) is the non-dimensional rotational speed, where \( \omega_{10} \) is the fundamental natural frequency of the stationary Timoshenko beam.

(d) \( \theta \) is the skew angle. The magnitude of the skew force is introduced by \( p = 0.5 \rho_{cr} \), where \( \rho_{cr} \) is the Euler critical buckling load, \( \pi^2 EI / l^2 \).

(e) \( \chi / l \) is the non-dimensional position along the shaft.

(f) \( V / V_s \) and \( W / W_s \) are the non-dimensional deflections along the shaft caused by the moving loads in the Y and Z direction, respectively. \( V_s \) and \( W_s \) are the static deflections as the lateral force with magnitude of \( p \) is applied at the midpoint of the shaft with the corresponding geometric boundary. The non-dimensional axial deflection is represented as \( U / U_s \), in which \( U_s \) is the static deflection of the shaft when the axial force with magnitude of \( p \) is applied.

In the numerical study, the following parameter values have been used: \( l = 1.0 \) (m), \( \rho = 7700 \) (kg/m\(^3\)), \( E = 207 \) (GPa), \( G = 77.6 \) (GPa), \( K = 0.9 \). The values for the non-dimensional velocity, the rotating speed, the Rayleigh beam coefficient, and the skew angle are given in the range of \( 0 \leq \alpha \leq 2 \), \( 0 \leq \Omega \leq 2 \), \( 0 \leq \beta \leq 0.20 \) and \( 0 \leq \theta \leq 30 \), respectively.

In order to provide a check of solutions obtained by the present method, the non-dimensional maximum responses \( V / V_s \) of the rotating shaft with hinged-hinged boundary under a moving lateral force is compared to those obtained by using modal analysis\(^{(3)}\). The numerical results obtained by GAMM demonstrate the accuracy as shown in Table 1. It can be found that an increase of \( N_p \) will induce more accurate numerical results. In the case of \( N_p = 10 \), the percentage difference between GAMM and the exact solution is smaller than 0.3\%. Therefore, the number of terms of the polynomial \( N_p \) in GAMM analysis is chosen as \( N_p = 10 \).

The influence of the system parameters \( \alpha \), \( \theta \), \( \beta \) and \( \Omega \) on the dynamic responses for the three boundary conditions is investigated. In Fig. 2, the three non-dimensional deflections \( U / U_s \), \( V / V_s \) and \( W / W_s \) are plotted at vari-

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Table 1  Comparison of the maximum transient responses \( \theta = 0 \), \( \Omega = 0 \), \( \beta = 0.03 \).

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( V_{max} )</th>
<th>( V_{max} )</th>
<th>( V_{max} )</th>
<th>( V_{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>( \Omega )</td>
<td>( \beta )</td>
<td>( \Omega )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>( \text{GAMM} )</td>
<td>( \text{Calc} )</td>
<td>( \text{Calc} )</td>
<td>( \text{Calc} )</td>
<td>( \text{Calc} )</td>
</tr>
<tr>
<td>Np = 6</td>
<td>1.0222</td>
<td>1.5935</td>
<td>0.854786</td>
<td>0.596403</td>
</tr>
<tr>
<td>Np = 8</td>
<td>1.037473</td>
<td>1.592033</td>
<td>0.856628</td>
<td>0.600421</td>
</tr>
<tr>
<td>Np = 10</td>
<td>1.037069</td>
<td>1.60087</td>
<td>0.872086</td>
<td>0.6111</td>
</tr>
</tbody>
</table>
Fig. 2  Transient responses with various moving velocity $\alpha$ at $\Omega = 2.5$, $\theta = 30$, $\beta = 0.15$.

It is clear that the perturbation motion will be increased in the axial deflection $U/U_s$ for all three boundary conditions especially when $\alpha$ increases. This is because the virtual work of axial force acting on system will increase rapidly with the increase of $\alpha$. Then the perturbation motion in the axial deflection will also be increased. And it can also be found that the magnitude of the lateral deflections is the largest in the case of the clamped-clamped boundary condition and the smallest in the hinged-hinged case. Because of the more degrees of freedom restrained at the support, the effect of axial force will be more obvious. However, it is the contrary in the case of the axial deflection. Furthermore, with the increase of $\alpha$, the effect of axial force will decrease the system rigidity and the movement of moving force is more. Then the position of maximum deflections $V/V_s$ will move to
the right side of the shaft. Figure 3 indicates the influence of the skew angle \( \theta \) of the force on the three deflections at \( \Omega = 2.5 \), \( \alpha = 0.11 \) and \( \beta = 0.15 \). As it is shown, the axial deflection \( \frac{U}{U_s} \) will become large as the skew angle increases. In the meanwhile, the deflections \( \frac{V}{V_s} \) and \( \frac{W}{W_s} \) tend to decrease as the skew angle increases. It is clear that the axial component of the skew force will become large with an increase in the skew angle.

The influence of the Rayleigh beam coefficient \( \beta \) on the three deflections at \( \Omega = 2.5E \), \( \alpha = 0.5 \), \( \theta = 0 \) is sketched in Fig. 4. It can be found that the amplitudes of perturbation oscillations in the axial deflection become larger with an increase of \( \beta \). For the lateral deflections, an increase of \( \beta \) will increase the responses \( \frac{V}{V_s} \) and \( \frac{W}{W_s} \). The non-dimensional deflections with various rotating speed parameter \( \Omega \) are given in Fig. 5 at \( \alpha = 0.5 \), \( \theta = 30 \), \( \beta = 0.15 \). The results, show that the deflections, \( \frac{U}{U_s} \) and \( \frac{V}{V_s} \) for each boundary condition appear to be independent of the shaft rotating speed. However, this phenomenon is not valid for the \( \frac{W}{W_s} \) deflections as it does exhibit a strong
dependency on the rotating shaft. This is because the gyroscopic effect will be increased as the rotational speed increases. Then the deflection $W/W_s$ will also be increased.

6. Conclusions

In this study, the GAMM is first employed to analyze the dynamic response of a rotating shaft with general boundary conditions when the shaft is subjected to a moving skew force. The effects of parameters on system dynamic response are investigated. Such parameters include the moving velocity, the skew angle, the rotating speed and the Rayleigh beam coefficient. The results can be summarized as follows:

(1) The axial deflection $U/U_s$ will become large as skew angle increases, whereas the deflections $V/V_s$ and $W/W_s$ have a tendency to decrease.

(2) It will induce oscillation motion in the axial response for fast moving speeds or large skew angles of the shaft.

Fig. 4 Transient responses with various Rayleigh beam coefficient $\beta$ at $\Omega = 2.5$, $\theta = 30$ and $\alpha = 0.5$. 
force. The larger the Rayleigh beam coefficient is, the larger the amplitude of the perturbation oscillations will be. However, the influence of moving speed on the amplitude of axial response is insignificant.

(3) The maximum peak of the lateral response $V/V_s$ in the beam will move to the right side of the shaft as $\alpha$ increases.

(4) The lateral response $V/V_s$ is not significantly affected by the spin speed, but an increase of $\beta$ will increase the response $V/V_s$.

(5) The oscillation of responses $W/W_s$ is significantly affected by the moving speed and the Rayleigh beam coefficient. The faster the moving speed is, the simpler oscillation of responses $W/W_s$ will be. However, the amplitudes of responses $W/W_s$ will increase as an increase of the Rayleigh beam coefficient.

(6) The amplitudes of lateral responses $V/V_s$ and $W/W_s$ in the hinged-hinged case are the smallest in the three boundary conditions. In addition, the axial response $U/U_s$ in a hinged-hinged case is the largest as compared...
with the other boundary conditions.

(7) The trend of lateral responses $V/V_s$ and $W/W_s$ in the hinged-hinged case, clamped-clamped case, and the clamped-hinged case are the same. However, the responses $W/W_s$ in the clamped-clamped case incur more violent oscillations than those in the hinged-hinged case.

References


