Compensation and Control of Dither-Smoothed Nonlinearities

Kuo-Kai SHYU∗∗ and Yun-Yao LEE∗∗∗

A novel approach to linearize nonlinearities commonly inherent in actuators is proposed in this paper. This approach solves the inverse of the nonlinearity without requiring its I/O relations as a one-on-one map, which is necessary for the current inverse-model method. By introducing the concept of the equivalent gain, the proposed method systematically finds the inverse model of the nonlinearity by solving a zero-finding problem. Discussions on the existence and uniqueness of the solution are given in this paper. A simulation example is presented to demonstrate how the task in finding the inverse of a complicated nonlinearity can be simplified by the proposed method. Experimental evaluations on a position servo system with conspicuous friction reveal that the dither-smoothed nonlinearities are still nonlinear, and compensating action like the proposed method is necessary for control systems using the dithering technology.

Key Words: Compensation, Dither Techniques, Friction, Inverse Problems, Nonlinearities, Linearization

1. Introduction

Industrial motion control systems usually suffer nonlinearities such as dead-zone, backlash, hysteresis, saturation, etc., from actuators. These nonlinearities deteriorate the system performance and pose some challenges for controller design. One important job for the control engineer is to design proper compensation schemes to reduce the influence of these nonlinearities upon the system.

Dead-zone and saturation are two types of nonlinearities commonly encountered in practical control systems, but the impact of the former is more serious. The reason is clear: Dead-zone diminishes controllability of systems for small input signals. At present, the typical approach to resolve the dead-zone problem is the inverse-model method(1), which has been augmented by a variety of schemes like the adaptive(2), neural network(3), and fuzzy logic controllers(4) for more complicated applications. Backlash essentially comes from the difference between the driving and driven parts in gear trains. Although it is necessary for satisfactory meshing in motion, backlash is not desired in precise servo systems for its liability to induce hunting. Dither signals have also been used to eliminate the backlash-induced hunting. Most studies on backlash compensation focus on the switching and inverse controllers(1),(5),(6). Hysteresis is a distinctive feature of some types of electromagnetic devices and piezoelectric actuators. Typical compensating schemes for this nonlinearity include the dithering technology(7) and the piecewise inverse model(1). In addition, the term ‘nonlocal memory hysteresis’ is used to describe the relations between friction and displacement of contacts in the presliding regime(8). The control methods for this peculiar type of hysteresis can be found in Refs. (8) and (9).

The inverse-model-based scheme and the dithering technology are the two main methods in compensating for the nonlinearities inherent in actuators(1),(5),(6). However, a quite complicated algorithm is unavoidable to handle the output or sandwiched nonlinearities, which dramatically burdens the designing task(1). In addition, this method requires that the I/O characteristic curve must be a one-on-one map. For nonlinearities with features of hysteresis, their I/O relations must be split into piecewise one-on-one maps for the existence of their inverse(1). On the other side, the dithering technology also has its problem in practice. This technology is useful in smoothing hard or discontinuous nonlinearities, but control engineers usually ignore the fact that the smoothed elements are still nonlinear. Furthermore, some types of nonlinearities cannot be effectively smoothed unless high-amplitude dither signals are applied. But dither signals with high amplitude

∗ Received 16th December, 2005 (No. 05-5130)
∗∗ Department of Electrical Engineering, National Central University, No. 300, Jungda Rd., Jungli City, Taiwan, 320, R.O.C. E-mail: kkshyu@ee.ncu.edu.tw
∗∗∗ Department of Electrical Engineering, National Central University; Lecturer, Chin-Yun University of Technology, Taiwan, R.O.C. E-mail: yunyao@cyu.edu.tw
will introduce undesired side effects like saturation, abrasion, vibration, noise, overheat, etc. Another example is that static friction can be hardly eliminated by the dither because a complete elimination of it implies conspicuous relative motion between contacts. This is hardly acceptable for most servo systems.

To solve the problems mentioned above, we propose an algorithm to linearize nonlinearities in actuators allowing them as multi-on-one maps. When the dithering technology is applied, the proposed algorithm allows the dither amplitude as small as possible. Numerical studies and experimental evaluations are presented to demonstrate how the tedious task in finding the inverse of a complicated nonlinearity can be simplified by the proposed method.

The rest of this paper is arranged as follows. Section 2 introduces the dither-smoothing technology. Linearization of arbitrary nonlinearities by the proposed method is introduced in section 3. The numerical example is given in section 4, and experimental evaluations, section 5. The last section states the conclusion of this paper.

2. The Dither-Smoothing Technology

In this section, we briefly summarize the principle of the dither-smoothing technology and introduce the concept of the equivalent gain. In Fig. 1, a nonlinear element \( N \) is excited by \( i(t) \), the combination of a sinusoidal dither signal \( D_s \sin(\omega t) \) and a constant \( U_b \),

\[
i(t) = U_b + D_s \sin(\omega t) \tag{1}
\]

Then the equivalent gain of this dither-injected element is defined as

\[
G_e(U_b) = \frac{\bar{y}}{U_b} \tag{2}
\]

where \( \bar{y} \) is the average of \( y(t) \) within one period of the dither signal\(^{(10)}\). When \( U_b \) is replaced by a time varying signal \( u(t) \), the concept of the equivalent gain is readily applicable if \( u(t) \) can be considered to be constant within each cycle of the dither signal. Practically, choosing \( \omega t \geq 20\alpha_b \) fulfills the condition. Here \( \alpha_b \) is the bandwidth of \( u(t) \). This introduces the equivalent gain \( G_e(u) \) for a slowly (compared to the dither signal) varying input \( u(t) \),

\[
G_e(u) = \frac{\bar{y}}{u}, \quad u \neq 0 \tag{3}
\]

The equivalent gain \( G_e(u) \) focuses our attention on the averaged behavior of the dithered element. For example, \( G_e(u) \) of symmetric dead-zone smoothed by a sinusoidal dither signal is\(^{(11)}\)

\[
G_e(u) = -\frac{m}{\pi} \left[ \varphi_1 + \varphi_2 + \frac{D_s}{u} \left( \cos \varphi_2 - \cos \varphi_1 \right) + \frac{d}{u} (\varphi_2 - \varphi_1) \right] \tag{4}
\]

where \( m \) is the slope of the linear portion, \( d \) specifies the region of dead-zone (zero output when \( -d \leq u \leq d \)), and

\[
\varphi_1 = \begin{cases} 
\sin^{-1} \left( \frac{d-u}{D_s} \right), & \text{when } |d-u| < D_s \\
\pm \frac{\pi}{2}, & \text{when } d-u \geq D_s \\
\pm \frac{\pi}{2}, & \text{when } d-u \leq -D_s 
\end{cases} \tag{5}
\]

\[
\varphi_2 = \begin{cases} 
\sin^{-1} \left( \frac{d+u}{D_s} \right), & \text{when } |d+u| < D_s \\
\pm \frac{\pi}{2}, & \text{when } d+u \geq D_s \\
\pm \frac{\pi}{2}, & \text{when } d+u \leq -D_s 
\end{cases}
\]

Figure 2 depicts the dither-smoothed I/O map and the normalized equivalent gain \( G_e(u)/m \) of symmetric dead-zone. For nonlinearities with memory, such as backlash, hysterical relay (see Fig. 3), and hysteresis, the dither can change their looped I/O maps into piecewise continuous functions, as shown in Fig. 3 (c). Note that backlash in Fig. 3 (a) can be removed completely by a dither signal with \( D_s \geq b \), where \( D_s \) is the dither amplitude and \( b \) is a half of the deadband width of backlash, as defined in Fig. 3 (a).

For the inverse-model method, the input-output relation or \( G_e(u) \) of the nonlinear element must be described by piecewise one-on-one functions\(^{(1)}\). In particular cases, closed forms of these functions as those in Eqs. (4) and (5) can be found, but in general they are not available. Therefore the look-up-table method has been used in industrial applications. The table representing the input-output relation also has to be invertible, or it must be divided into piecewise one-on-one functions. However, for hysterical or memoryless nonlinearities with complicated I/O relations, this is not a simple task\(^{(1)}\).

In the next section, we introduce an algorithm based on the gain-scheduling method to find the inverse model directly from the multi-on-one look-up table of the nonlinearity. It will be shown that this algorithm is very powerful in linearizing complicated nonlinearities.
Fig. 2 (a) Dither-smoothed input-output maps of symmetric dead-zone; (b) Normalized equivalent gain \( G_e(u) \)

3. The Proposed \( \beta \)-Compensation Algorithm

In this section, we introduce the proposed \( \beta \)-compensation algorithm, which finds the inverse of the memoryless nonlinearity directly from its look-up table requiring only that the slope of the nonlinearity be bounded. For those with looped or discontinuous I/O relations, the dithering technology introduced in the previous section should be applied.

For a memoryless nonlinear element \( N \), its equivalent gain \( G_e(u) \) specifies how the control effort \( u \) is distorted by \( N \). If \( G_e(u) \) is available, one may try to recover the control effort by post-multiplying the gain \( 1/G_e(u) \). Figure 4 (a) indicates such a configuration, in which \( \bar{u}_2 \) restores the profile of \( u \) and the gain \( 1/G_e(u) \) apparently removes the nonlinear element from the plant, as long as \( G_e(u) \neq 0 \). Note that the upper bar in \( \bar{u}_2 \) denotes the averaged output of \( N \) if the dither signal is used in the system.

The configuration in Fig. 4 (a), however, cannot be implemented for practical systems. The reason is simply that we can only manipulate the control effort in the signal stage. This implies that we have to augment the controller by a pre-compensating amplifier, as indicated in Fig. 4 (b). This structure is the same as the inverse-model method\(^{(1),(3),(6)} \), where \( 1/G_e(u) \) is replaced by the inverse model of the nonlinear element.

There are two problems, however, in the structure shown in Fig. 4 (b). The first one occurs when \( G_e(u) = 0 \). This is the reason why the delta function is involved in the inverse model of backlash\(^{(1)} \). The second is that this structure in general does not exactly cancel the nonlinear element. This can be verified as follows. In Fig. 4 (b), we have

\[
\bar{u}_2 = u_1 G_e(u_1) = \frac{G_e(u_1)}{G_e(u)}
\]
In general $G_e(u)$ is not constant, that is,

$$G_e(u) \neq G_e(u_1) \text{ if } u \neq u_1$$

Therefore we have

$$\bar{u}_2 \neq u$$

Equation (8) implies that to achieve the design goal ($\bar{u}_2 = u$), we must adjust the control effort $u$ by some function other than $1/G_e(u)$. Suppose $u$ is multiplied by a gain $\beta$ before mixed with the dither signal, as shown in Fig. 4 (c). Then we have

$$u_1 = \beta u$$

and

$$\bar{u}_2 = u_1 G_e(u_1) = \beta u G_e(\beta u)$$

Our objective is to design $\beta$ such that

$$\bar{u}_2 = u$$

With this purpose, Eq. (10) becomes

$$\beta u G_e(\beta u) - u = 0$$

(12)

or

$$x G_e(x) - u = 0$$

(13)

where $x = u_1 = \beta u$ is the unknown to solve.

In solving Eq. (13), two issues may arise: The existence and the uniqueness of the solution. To investigate the existence, we rewrite Eq. (13) as Eq. (14) by applying the definition of $G_e(u)$ in Eq. (3).

$$x G_e(x) - u = 0$$

(14)

From Eq. (14), we can assure that the solution of Eq. (13) exists if for any given input $u$, we can find the corresponding output $y$ in the input-output map such that $y = u$. In this case, the solution $x$ of Eq. (13) is simply the input in the I/O map that yields the output $y$. Figure 5 depicts this concept. Given an input $u$, we first set the output $y_0 = u$, and then search for the input $x_0$ such that $y_0 = N(x_0)$, where $N(x)$ is the I/O map of the nonlinearity. Then $x_0$ is the solution of Eq. (13) for $u = u_0$. With Fig. 5, the answer to the uniqueness also becomes clear. For a one-on-one map, the solution is unique, because for every output $y$ there exist only one input $x$ that yields $y$. For a multi-on-one map, as shown in the left part of Fig. 5, the solution of Eq. (13) is not unique. In this case, the solution $x$ is chosen such that it is close to its initial guess.

The configuration introduced in Fig. 4 (c) is named the $\beta$-compensation algorithm. Note that in solving Eq. (13), the input-output relation shall be normalized be-
cause Eq. (13) is solved on the condition \( y = u \). An example demonstrating the procedures to solve Eq. (13) is presented in the next section.

4. A Numerical Example

Figure 6 depicts the block diagram of a typical position servo system and Table 1 lists the parameters for this numerical example. Here we conceive a nonlinear voltage amplifier. This amplifier is divided into a normalized nonlinear-gain block and a linear gain with \( K_A = 8 \), as shown in Fig. 6 (a). Table 2 describes how the normalized nonlinear gain is constructed and Fig. 7 depicts the I/O map and its equivalent gain. The I/O map is composed of 12 line segments. For the breaking points of the input \( u \), the slope between [0.0, 0.25] is zero, and that between [0.25, 0.50], 1.0, etc (see Table 2). The output is zero when the input \( u \in [-0.3, 0.25] \). With the \( u-y \) map specified, the equivalent gain of the nonlinear amplifier \( G_e(u) \) can be obtained by

\[
G_e(u) = \frac{y}{u}, \quad u \neq 0; \quad G_e(u) = 0
\]  

The curve of \( G_e(u) \) is also shown in Fig. 7. In this example, the incremental of \( u \) is 0.002. Therefore we use two 3000-by-2 matrices to describe the \( [u, y] \) and the \( [u, G_e(u)] \) maps. Since the I/O map in Fig. 7 contains segments of negative slope, we cannot just reverse the input and the output to form the inverse model of the amplifier. By the current inverse-model method, we have to divide the I/O map shown in Fig. 7 into 12 segments to form the piecewise invertible functions. This tedious task can be greatly simplified by the proposed \( \beta \)-compensation algorithm.

The proposed algorithm is achieved by solving the zero-finding problem in Eq. (13). For example, given \( u = 0.5 \), we search \( u_1 \) from the \( [u_1, G_e(u_1)] \) table (actually the \( [u, G_e(u)] \) table) such that

\[
u_1 G_e(u_1) - 0.5 = 0
\]  

Equation (16) is a typical zero-finding problem, which can be solved by, for example, the MATLAB function FZERO.m. An alternative way to solve Eq. (13) is to use the mapping concept revealed in Eq. (14) and Fig. 5. When multiple solutions exist, we choose the one closest to the initial guess, which is provided by the previous solution. By this procedure, for every interested control input \( u \), we
search for the corresponding \( u_1 \) that satisfies Eq. (13). The solved control effort \( u_1 \) and the \( \beta \)-compensated I/O map are also shown in Fig. 7. Note that the \( \beta \)-compensation algorithm can be solved in an off-line mode and the resulting table \([u, u_1]\) can be built in the controller to reduce the computational load in real-time applications.

With the \( u-u_1 \) map constructed, the augmented controller shown in Fig. 6 (c) then straightens the I/O map of the voltage amplifier, as shown in Fig. 7. Figure 8 depicts the errors in tracking a 2-Hz 1-degree sinusoidal command. In Fig. 8, the results of two dither signals with amplitude \( D_1 = 0.25 \) V and \( D_2 = 0.5 \) V are compared. The oscillations in \( e_1, e_2 \) and \( e_2 \) are caused by the negative-slope segments in the input-output map of the amplifier. To eliminate the oscillations, the dither amplitude must be large enough to smooth out the negative-slope region in the I/O map. And it can be noted from the curves \( e_1 \) and \( e_2 \) that the dithered system does not imply a smaller root-mean-square error than the undithered one \( (e_1) \). On the other side, the root-mean-square error of the \( \beta \)-compensated controller (without using the dither) is only a third of that with \( D_2 = 0.5 \) V.

5. Experimental Evaluations

Figure 9 gives the schematic and block diagrams of the electromechanical system for experimental evaluations. A common feature of the servo system shown in Fig. 9 is the friction-induced dead-zone nonlinearity inherent in the actuator. Figure 10 depicts the measured output velocity \( V_m \) versus the input \( u_1 \) to the voltage amplifier in three tests. The detailed plots in Fig. 10 reveal that the control voltage needed to start the movement of the table is about 0.5 Volts. We use ‘about’ here because it varies slightly in tests. Such variations are attributable to the varying maximum static friction force, which depends on factors like dwell time, position, temperature, lubrication, loading conditions, etc. As presented in Fig. 2 (a), the dithering technology is regarded as an effective tool to eliminate the dead-zone nonlinearity shown in Fig. 10. However, the experimental observations here reveal a different situation. Figure 11 depicts the measured \( u_1-V_m \) relations with various amplitudes of sinusoidal dither signals added. The dither frequency is 400 Hz, approximately 40 times of the bandwidth of the experimental system. It can...
be seen that, even applying a dither with $D_i = 1.5$ Volts, the dead-zone nonlinearity is still not removed. This evidence reveals that the static dead-zone model cannot capture the feature of the friction-induced dead-zone nonlinearity in high-frequency cycling motion.

Despite the problem revealed in Fig. 11, the proposed $\beta$-compensation method can remove the dead-zone nonlinearity shown in Fig. 10 whether dither signals are applied or not. Using the measured input-output relations given in Fig. 11, we calculated $G_C(u)$ by Eq. (3) and built up the table \([u, u_i]\) by solving Eq. (13). Figure 12 depicts the compensated $u-V_m$ relations obtained by the $\beta$-compensation method and the inverse-model method. As it can be seen in Fig. 12 (b), the $\beta$-compensated output velocity has a smaller root-mean-square error from the best-fitted line. This can be attributed to the fact that the proposed method uses all the information contained in the measured data and therefore improves the accuracy of the inverse model.

With the $\beta$-compensated control input, the velocity tracking performance of the experimental system is shown in Fig. 13. A well known fact is that it is difficult for servo systems with conspicuous friction to track a reversing low velocity command. But as shown in Fig. 13, the proposed $\beta$-compensation method gives uniform and satisfactory performance. Note that in the experimental system, applying the dither-smoothing technology alone does not produce satisfactory results. From the evidences shown in Figs. 11 and 13, we can highlight the fact that the dither-smoothed nonlinearities are still nonlinear, and the proposed $\beta$-compensation method is necessary to reduce the impact of the nonlinear element to the performance of the system. In addition, from the curve of $D_i = 0$ in Figs. 12 and 13, we also conclude that when the I/O map of the nonlinear element is continuous, the $\beta$-compensation method can accomplish the linearizing task without the need of the dithering method. This can be very helpful for applications suffering the side effects brought by the dither.
6. Conclusions

A novel algorithm conceived to find the inverse model of a nonlinear element has been presented in this paper. Unlike the current inverse-model method, the proposed β-compensation method solves the inverse of the nonlinearity directly from its I/O map without dividing it into segments of one-on-one maps. Using the look-up table enables the proposed method to cover all the details of the nonlinearity and improves the accuracy of the inverse model.

The proposed β-compensation algorithm is basically a zero-finding algorithm. The issues on the existence and uniqueness of the solution have also been investigated. In the numerical example, it was demonstrated how the tedious task in finding the inverse of a complicated nonlinearity can be simplified by the proposed method. In experimental evaluations, we highlight the fact that the dither-smoothed nonlinearities are still nonlinear, and the proposed β-compensation method is necessary to reduce the impact of the nonlinear element to the performance of the system. In addition, from the evidences demonstrated in the experimental evaluations, we also conclude that when the I/O map of the nonlinear element is continuous, the proposed β-compensation method can accomplish the linearizing task without the need of the dither. This can be very helpful for applications suffering the side effects brought by the dither.

References


