Reduction of Vibratory Stress of Compressor Vane by Use of Friction Damper*

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The objective of this study is to verify the reduction effect of the friction damper on the vibratory stress of the compressor vane both theoretically and experimentally. First, an analysis method for predicting the damping characteristics of the compressor vane with a friction damper, which applies the substructure synthesis method and the harmonic balance method, is proposed. Secondly, an excitation test of the damper vane is carried out to verify the validity of the analysis method proposed here, and to confirm the damping characteristics of the damper vane. In the excitation test, a newly developed magnetic exciter, which applies the feature of the resonant circuit, is used to vibrate the vane with large excitation force. Vibration characteristics of the damper vane predicted by the analysis show good agreement with the measured results. Finally, to verify the effect of the friction damper, the field test of the actual gas turbine is carried out, where the vibratory stress of the compressor vane with and without the friction damper is measured.

Key Words: Friction Damper, Damping, Substructure Synthesis Method, Modal Analysis, Non-Contact Magnetic Exciter

1. Introduction

Recently, in order to improve the thermal efficiency of the gas turbine, the thickness of the compressor vane has become thinner. Therefore, for the purpose of reducing the vibratory stress, the vane with a friction damper has been widely used. In order to develop a damper vane, its damping characteristics should be predicted at the design stage to get the optimal damping effect.

Since the prediction of the damping characteristics of the blade with a friction damper is essential to improve the reliability of turbomachinery, many researches have been done(1)–(5). Kaneko et al. developed the analysis method adopting the cyclic symmetry method and the harmonic balance method in order to predict the damping characteristics of the turbine blade with a friction damper, and have applied it to the actual blade design(6), (7). In this method, the substructure synthesis method is applied to the cyclically symmetric system, and the damping characteristics of the friction damper are analyzed efficiently, and it is shown that analysis results are in good agreement with measured results(5)–(7).

In this paper, first, the analysis method of the damping characteristics of the damper vane is proposed applying the method of Ref. (7). Secondly, the damping characteristics of the damper vane are measured using the newly developed magnetic exciter, and the validity of the analysis method is verified comparing the calculated results with the measured results. Lastly, to verify the effect of the friction damper, the field test of the actual machine is carried out, where the vibratory stress of the compressor vane with and without the friction damper is measured.

2. Analysis Method

2.1 Mechanical model of friction damper

Figure 1 shows the mechanical model of the damper vane. The damper vane has an inner shroud and a seal holder, and the frictional damping is added using relative displacement between the shroud and the seal holder. To analyze the damping characteristics of the damper vane, the friction damper is modeled by the spring with stiffness $k_{kl}$ and coulomb friction (reaction force $N_{kl}$ and friction coefficient $\mu_{kl}$), as shown in Fig. 1. Subscripts $k$ and $l$ denote the number of dampers and the degree of freedoms, respectively. Hereafter, these subscripts will be omitted to simplify the expression of the equation unless confusion occurs. The element in Fig. 1, which consists of the spring...
and the coulomb friction, is called a damper element.

Based on the concept of the harmonic balance method, it is assumed that the vane is vibrating with the same frequency as the excitation force. When the displacement of the damper element is expressed with the following equation[6]–[8].

\[
F_d = \mu N - k_d B (1 - \cos \theta) \\
= -\mu N \quad (0 < \theta \leq \theta^*) \\
= -\mu N + k_d B (1 + \cos \theta) \quad (\pi < \theta \leq \pi + \theta^*) \\
= \mu N \quad (\pi + \theta^* < \theta \leq 2\pi)
\]  
(1)

Where, \(B\) is the amplitude of the vane at the damper position, \(\theta\) is the phase angle between the displacement of the damper and the excitation force, and \(\theta^*\) is the phase angle where the damper begins to slip. \(\theta^*\) is expressed with Eq. (2).

\[
\cos \theta^* = 1 - \frac{2\mu N}{k_d B} \quad (0 < \theta^* \leq \pi)
\]  
(2)

If \(F_d\) is approximated by the first term in the Fourier series, Eq. (3) is obtained from Eqs. (1) and (2).

\[
F_{d1} = F_c \cos \theta + F_s \sin \theta \\
F_c = \frac{1}{\pi} \int_0^{2\pi} F_d \cos \theta d\theta = \frac{k_d B}{\pi} \left( \theta - \frac{\sin 2\theta^*}{2} \right)
\]  
(3)

Equations (1)–(3) represent the friction characteristics of the damper vane of Fig. 1.

2.2 Equation of motion of damper vane

In the turbomachinery with multi-stages, the interaction between the vane and the blade generates the excitation force on the vane, which comes from the wake of the upstream blade or the potential field of the upstream/downstream blade. The frequency of the excitation force is the rotor speed multiplied by the vane count, and the equation of motion of the damper vane is expressed with Eq. (4).

\[
[M][\ddot{U}] + [K][U] = [P] \cos \omega t + \sum_{k \ell} [F^k_{\ell}] 
\]  
(4)

Where, \([M]\), \([K]\), \([U]\), \([P]\) are the mass matrix, stiffness matrix, displacement vector and external force vector of the vane. And \([F^k_{\ell}]\) is the friction force vector, which acts on the damper element. Based on the concept of the harmonic balance method, the friction force vector is expressed with Eq. (5).

\[
[F^k_{\ell}] = [F^k_{\ell}] \cos(\omega t - \phi^k_{\ell}) + [F^k_{\ell}] \sin(\omega t - \phi^k_{\ell})
\]  
(5)

Where, \([F^k_{\ell}]\) and \([F^k_{\ell}]\) are the amplitude of the friction force, while \(\phi^k_{\ell}\) is the phase between the excitation force and the friction force. These are unknown variables. As for the damping except for the friction damping, the viscous damping is introduced into Eq. (4) later in the form of the modal damping. Therefore, the damping term is omitted from Eq. (4).

Representing the solution of Eq. (4) by Eq. (6) using the modal coordinate \(q_m(t)\) and natural mode \(\psi\), substituting Eq. (6) into Eq. (4), Eqs. (7) and (8) are obtained.

\[
[U] = \sum_{m=1}^{M} [q_m(t)] \psi^m
\]  
(6)

\[
M_m \ddot{q}_m + 2\omega_m \omega_m \dot{q}_m + \omega_m^2 q_m = P_m \cos \omega t + \sum_{k \ell} F^{m}_k \cos(\omega t - \phi^k_{\ell}) R^{m}_{k\ell} + \sum_{k \ell} F^{m}_k \sin(\omega t - \phi^k_{\ell}) R^{m*}_{k\ell}
\]  
(7)

\[
M_m = [\psi^m]^T [M] [\psi^m], \quad P_m = [\psi^m]^T [P], \quad R^{m}_{k\ell} = [\psi^m]^T [1_{k\ell}]
\]  
(8)

[\psi^m] is the eigenvector, which is obtained from the eigenvalue equation where the right-hand side of Eq. (4) is set to zero. The superscript or subscript \(m\) denotes the order of vibration modes. And \([1_{k\ell}]\) is the vector for indicating the direction of the friction force (direction of the slip of the damper).

From Eqs. (6) and (8), the displacement in the \(L\)-th direction of the \(K\)-th damper is expressed with the following equation.
\[ \Delta L^K = \sum_{m=1}^{M} R_{KL}^m p_m^m \]  
(9)

Solving Eq. (7) with respect to \( q_m(t) \), and substituting it into Eq. (9), Eq. (10) is obtained.

\[
\Delta L^K = a_{KL}^K \cos \omega t + a_{KL}^L \sin \omega t \\
+ \sum_{k,l} F_{LK}^k \cos(\omega t - \phi_l^k) b_{KL}^k b_{kl}^L \\
+ \sum_{k,l} F_{LK}^k \sin(\omega t - \phi_l^k) b_{KL}^k b_{kl}^L 
\]
(10)

Where,

\[ a_{KL}^K = \sum_{m=1}^{M} R_{KL}^m x_m^m \]  
\[ a_{KL}^L = \sum_{m=1}^{M} R_{KL}^m p_m^m \]  
(11)

\[ b_{KL}^K = \sum_{m=1}^{M} R_{KL}^m x_s^m \]  
\[ b_{KL}^L = \sum_{m=1}^{M} R_{KL}^m x_s^m \]  
(12)

\[ x_c^m = \frac{1}{[1 - \beta_m^2 + (2\zeta_m\omega_m)^2]} M_m \omega_m^2 \]  
\[ x_s^m = \frac{1}{[1 - \beta_m^2 + (2\zeta_m\omega_m)^2]} M_m \omega_m^2 \]  
\[ \beta_m = \omega / \omega_m \]

2.3 Frequency response analysis of damper vane

When the friction damper is modeled as shown in Fig. 1, the displacement in the L-th direction of the K-th damper is expressed with Eq. (13).

\[
\Delta L^K \equiv B_L^K \cos \theta \quad \theta = \omega t - \phi_L^K
\]
(13)

Where, \( B_L^K \) is the amplitude in the L-th direction of the K-th damper. Equation (3), which represents the relationship between the damper forces \( F_{KL}^K \) and \( F_{SL}^K \) and the amplitude \( B_L^K \), is rewritten as Eq. (14) in order to simplify the expression.

\[ F_{KL}^K = q_{KL}^K (\theta_L^K) B_L^K \]  
\[ F_{SL}^K = q_{SL}^K (\theta_L^K) B_L^K \]  
(14)

Equation (15) is obtained by substituting Eqs. (13) and (14) into Eq. (10).

\[ B_L^K \cos(\omega t - \phi_L^K) = a_{KL}^K \cos \omega t + a_{KL}^L \sin \omega t \\
+ \sum_k (q_{KL}^K b_{KL}^K - q_{KL}^L b_{KL}^L) B_L^K \cos(\omega t - \phi_l^k) \\
+ \sum_k (q_{KL}^K b_{KL}^K - q_{KL}^L b_{KL}^L) B_L^K \sin(\omega t - \phi_l^k) \]  
(15)

From the prerequisite condition in Eq. (15), where the left-hand side should be equal to the right-hand side at arbitrary time, Eq. (16) can be derived.

\[
\sum_{k,l} (C_{KL}^K - S_{KL}^K) B_L^K \cos \phi_l^k + \sum_{k,l} S_{KL}^K B_L^K \sin \phi_l^k = a_{KL}^K \\
- \sum_{k,l} S_{KL}^K B_L^K \cos \phi_l^k + \sum_{k,l} (C_{KL}^K - S_{KL}^K) B_L^K \sin \phi_l^k = a_{KL}^L
\]
(16)

Where,

\[ C_{KL}^K = q_{KL}^K \theta_L^K - q_{KL}^L \theta_L^L \]
\[ S_{KL}^K = q_{KL}^K \theta_L^K + q_{KL}^L \theta_L^L \]

Equation (16) is the simultaneous equation whose size \( 2N_d \) when the total degrees of freedoms of dampers (the number of dampers \( \times \) degrees of freedom of a damper) is \( N_d \). On the other hand, Eq. (2) represents the relationship between the amplitude of the damper B and the phase angle \( \theta^* \) where the damper begins to slip. Therefore, the simultaneous equation of 3\( N_d \) is obtained for 3\( N_d \) unknown variables (3 unknown variables of B, \( \theta^* \), and \( \phi \) per one degree of freedom of the damper). In order to analyze the frequency response of the damper vane, the friction force of the damper \( \mu N \), the excitation force \( P \) and the excitation frequency \( \omega \) are given as known variables, and then B, \( \theta^* \), and \( \phi \) are obtained by solving Eqs. (2) and (16). Next, Eq. (7) is solved with respect to \( q_m \) by substituting them into Eq. (7), and the response of arbitrary points of the damper vane can be calculated by substituting \( q_m \) into Eq. (6).

3. Experimental Setup

Generally, an exact measurement of the damping characteristics of the vane of the turbomachinery is difficult because the damping of the vane is very small. If a contact type exciter (e.g. electrodynamics shaker) is used in an excitation test, it will give the additional stiffness and damping to the vane. It may be possible to excite the vane using the shaking table if the vane tested is small and light such a vane used in an aero-engine. However, as for the large vane used in the industrial gas turbine, it is difficult to use the shaking table due to the limitation of its capacity. In the excitation test using the shaking table, it is also difficult to excite the higher vibration modes of the vane. On the other hand, in the excitation test with a non-contact type exciter such as an air-siren or a magnetic exciter, the exact damping characteristics of the vane can be measured because it can excite the arbitrary vibration modes without affecting the vibration characteristics of the vane. However, as for a non-contact type exciter, it is difficult to test the large-sized vane with large excitation force. For the damper vane, it is essential to carry out the excitation test with larger excitation force than that for the actual vane under operation because the vibration characteristics of the damper vane are dependent on the amplitude of the excitation force due to its non-linearity.

For this reason, the non-contact magnetic exciter, which can excite the vane with large force to the high frequency region, was developed and the excitation test of the damper vane was carried out. In the conventional magnetic exciter, since the impedance of a circuit increases rapidly as the excitation frequency becomes high, it is difficult to get the large excitation force in the high frequency region by increasing the current of the circuit. In order to
overcome this problem, the condenser with variable electrostatic capacity $C$ is inserted in the circuit as shown in Fig. 2. Adjusting electrostatic capacity $C$ and resonating the circuit electrically, it becomes possible to increase the excitation force because the impedance of a circuit becomes small.

The impedance of the whole circuit $Z$ is expressed with Eq. (18), if the electrostatic capacity $C$ is inserted in the circuit of the exciter.

$$Z = R + j(\omega L - 1/\omega C) = R + jX$$  \hspace{1cm} (18)

Where, $R$ is the resistance of the circuit and $L$ is the reactance of the coil. If the electrostatic capacity $C$ is adjusted so that the imaginary part of the impedance becomes zero, the impedance $Z$ is equal to $R$ even in the high frequency region. Therefore it becomes possible to get the large excitation force in the high frequency region by increasing the current of the circuit.

The experimental setup is shown in Fig. 3. The frequency response of each vibration mode is obtained by the sine-wave sweep excitation test using the above-mentioned magnetic exciter, and the logarithmic decrement is evaluated by applying the half-power method to the measured frequency response. The response of the vane is measured with the small accelerometer and the strain gauge. In the excitation test, the actual compressor vane, whose height and code length is around 300 mm and 100 mm, is used. Also, the boundary condition of the actual compressor vane is simulated using the dummy casing.

4. Analysis and Measurement Results

Figure 4 shows the vibration modes of the damper vane, which are calculated under the so-called stick condition, where the damper of the inner shroud is completely stuck (Both ends of the vane are fixed). In Fig. 4, the $B_1$ mode indicates the 1st bending mode and the $T_1$ mode indicates the 1st torsion mode. And the $U_1$ mode is one of the stripe modes in which the middle section of the vane deforms like “U”. In the analysis of the damping characteristics of these vibration modes, first, the eigenvalue analysis of the vane is carried out under the boundary condition in which the inner shroud is free (the damper is slipped freely) and the outer shroud is fixed. And then, the vibration modes used for the modal analysis (Eq. (6)) are obtained. Next, the stiffness and the friction coefficient of the seal holder are decided from the calculation, simple element test, empirical value, and so on.

Then, the frequency response analysis is carried out according to the procedure described in chapter 2, and the logarithmic decrement and the peak frequency of the vane are evaluated.

In the following analysis, 16 damper elements with 2 degrees of freedoms per one element are used. Figures 5 – 7 show the examples of the calculated frequency response for the different excitation forces. In this analysis, the excitation force is applied on the position of the maximum amplitude of each vibration mode. Figure 8 shows the example of the measured frequency response of the 1st bending mode. Also in the excitation test, the excitation position is changed for each mode so that the position of the maximum amplitude is excited. Figures 9 – 11 show comparison of the measured and calculated logarithmic decrement and peak frequency. From these results it is shown that the damping effect of the lower modes like the $B_1$ and $T_1$ mode becomes very large as the excitation force increases because the relative displacement of the damper position becomes large. Especially for the $B_1$ mode, in which the relative displacement between the shroud and the seal holder tends to occur, the peak fre-
quency decreases and the logarithmic decrement increases remarkably as the excitation force increases. On the other hand, as for the U1 mode in which only the blade profile of the vane vibrates, the peak frequency and the logarithmic decrement scarcely change, even if the excitation force increases, and it is shown that the additional damping with the friction damper is improbable. In other words, the amount of the additional damping of the damper vane...
Fig. 12 Measured results of B₁ mode

Fig. 13 Measured results of T₁ mode

depends on the vibration mode, that is, the damping effect is large for the lower modes with the large relative displacement between the shroud and the seal holder, and small for the higher modes in which only the blade profile of the vane vibrates. As shown in Figs. 9 – 11, the measured result is in good agreement with the calculated result qualitatively and quantitatively. Therefore, it is concluded that the vibration characteristics of the damper vane can be predicted by the method proposed in this paper.

5. Verification Test

As the final verification test of the damper vane, the field test of the actual gas turbine was carried out. In this verification test, the vibratory stress of the compressor vane with and without the friction damper was measured. Figures 12 and 13 show the measured logarithmic decrements versus vibratory stress for the B₁ mode and T₁ mode. From these results, it can be said that the logarithmic decrements of the damper vane increases and the vibratory stresses decreases comparing to those of the vane without the friction damper. Especially for the B₁ mode, the damping increases remarkably. The result of the verification test of the actual vane is in agreement with the results obtained from the analysis and the excitation test.

6. Conclusion

The analysis method for predicting the damping character-

istics of the friction type damper vane was proposed and the excitation test of the actual damper vane was carried out in order to verify the validity of the proposed analysis method. In the excitation test, the non-contact magnetic exciter, which applied the feature of the resonant circuit, was developed to vibrate the vane with large excitation force. The measured results of the damping characteristics were in good agreement with the calculated results, and it is confirmed that the damping characteristics of the damper vane can be predicted by the proposed method. And the calculated and measured results show that the damping effect of the damper vane depends on the vibration mode, that is, the damping effect is large for the lower modes with the large relative displacement between the shroud and the seal holder, and small for the higher modes in which only the blade profile of the vane vibrates. Finally, in the field test of the actual gas turbine, the reduction effect of the damper vane on the vibratory stress was verified, comparing the measured vibratory stress of the vane with and without the friction damper.

References