The Dynamic Equation and Simulation of a Linkage Mechanism Fabricated from Three-Dimensional Braided Composite Materials*

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The work in this paper is concerned with a four-bar linkage mechanism with links fabricated from three-dimensional braided composite materials. The dynamic equation of the mechanism is established by finite element method (FEM), it includes composite materials parameters and structural parameters of the mechanism. The mass matrix of the beam element is obtained in light of the mass distribution characteristics of the composite materials. According to the 3-dimensional microstructure characters of the three-dimensional braided composite materials, the anisotropic beam element is considered to be made up of four parts, and then the stiffness matrix of the beam element is derived from the constitutive equations of each part and the relation between the strain distribution of each part and the node displacement of the beam element. Based on the damping element model and the expression of dissipation energy of the 3-dimensional braided composite materials, each modal damping value of the mechanism is calculated and the damping matrix of the mechanism is established. The dynamic responses and natural frequency of the mechanism are obtained by simulation, respectively. The work presented in the paper provides theoretical basis to a certain extent for the further research on nonlinear vibration characteristics and optimum design of this kind of mechanism.

Key Words: Elastic Mechanism, Finite Element Method, Composite Materials, Dynamic Analysis

1. Introduction

The development of mechanology has a close connection with the invention and application of new materials, and the research on the application of new materials is very important in the domain of modern mechanology. It is one of the valid strategies improving the elastodynamic properties of mechanical systems to exploit the advanced composite materials of which the high strength-to-weight ratios and damping properties are superior to that of the conventional metals.

In the 80's in last century, people began to study the problem about the application of composite laminates to

linkage mechanisms. And then, the mechanisms with links fabricated from composite laminates have been applied gradually into many fields such as high-speed production lines and the industrial robots, and it is shown in engineering practice that this kind of mechanism has some advantages such as lightweight and short residual vibration's time^{(1)–(5)}. But the composite laminates have a grievous disfigurement that the strength between layers is very low and their delaminations occur frequently. The three-dimensional braided fiber-reinforced composite material is a new type of composite material. In comparing with two-dimensional composite laminates, a key property of the innovative three-dimensional braided composite is its ability to reinforce composites in the thickness. Unlike convention laminates, it includes multi-directional fiber bundles which interconnect layers. This fiber construction reduces the usual tendency toward delamination^{(6)–(13)}.

As can be seen from above, there is high potential of three-dimensional braided composite applications in high-speed linkage mechanisms. In this paper, a four-bar link-

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age mechanism fabricated from three-dimensional braided composite materials is studied, and its dynamic equation is established by the FEM. The mass matrix of the beam element is obtained in light of the mass distribution characteristics of the composite materials. According to the 3-dimensional microstructure characters of the three-dimensional braided composite materials, the anisotropic beam element is considered to be made up of four parts, and then the stiffness matrix of the beam element is derived from the constitutive equations of each part and the relation between the strain distribution of each part and the node displacement of the beam element. Based on the damping element model and the expression of dissipation energy of the 3-dimensional braided composite materials, each modal damping value of the mechanism is calculated and the damping matrix of the mechanism is established. The dynamic responses and natural frequency of the mechanism are obtained by simulation, respectively. The work presented in the paper provides theoretical basis to a certain extent for the further research on nonlinear vibration characteristics and optimum design of this kind of mechanism.

In order to simplify the discussion and reveal the leading problems, the instantaneous structure hypothesis^{(14),(15)} of kineto-elastodynamics (KED) is adopted in analysis. At the same time, the damping of joints is neglected, for the KED is mainly used to study the elastic deformation of the links of mechanism

2. Mass Matrix and Stiffness Matrix of the Mechanism

The basic fiber structure of the 3-dimensional braided composite materials is of 3-dimension and 4-direction texture, as shown in Fig. 1, and the geometrical unit-cell can be adopted in analysis, as shown in Fig. 2. Four bundles of braided yarns are in diagonal direction of the unit-cell, and intersect at the center point O. The angle α between each braided yarn direction and x -axis direction is called the braided angle, and β is the angle between the direction of projection of each braided yarn on the Y - Z plane and y -axis direction. The α and β are shown in Fig. 2 are belong to the first direction braided yarn. According to the laminated plate analogy theory, the 3-dimesional braided composite materials can be regarded as the superposition of four unidirectional fiber composite materials, and the stiffness and damping of the materials are respectively the superposition of stiffness and damping of the four parts according to their contributing volume fractions⁽⁶⁾⁻⁽¹³⁾.

Consider the bending moment, shearing force and axial force, the displacement field of any point of composite link can be expressed as follows^{(11),(12),(16)}:

$$V(x, z) = V_0(x) + z\varphi(x), W(x, z) = W_0(x) \quad (1)$$

where $V_0(x)$ and $W_0(x)$ are the longitudinal and transverse displacements of the neutral layer of link, respectively.

Fig. 1 3-dimension and 4-direction structure of the 3-dimensional braided composite material

Fig. 2 Geometrical unit cell

$\varphi(x)$ is the angle displacement of cross-section of link. From elasticity theory, the strain-displacement relationship of composite links can be expressed as^{(11),(12),(16),(17)}:

$$\varepsilon_x = \frac{\partial V_0(x)}{\partial x} + z \frac{\partial \varphi(x)}{\partial x}, \gamma_{xz} = \frac{\partial W_0(x)}{\partial x} + \varphi(x) \quad (2)$$

2.1 Element shape function

In general, the links of the linkage mechanism are slim bars, so they are adequate to be simulated using the beam element. As shown in Fig. 3, the 3-dimensional braided composite beam element has two nodes (A and B), and the origin of the local coordinate system for the beam element is assumed to be at the point A. The displacements of any point and each cross-sectional rotation angle within the element can be expressed as in terms of the shape function and nodal displacements:

$$\begin{cases} V(\bar{x}, t) = \sum_i \phi_i(\bar{x}) u_i(t) & (i = 1, 5) \\ W(\bar{x}, t) = \sum_i \phi_i(\bar{x}) u_i(t) & (i = 2, 3, 4, 6, 7, 8) \\ \varphi(\bar{x}, t) = \sum_i \phi_i(\bar{x}) u_i(t) & (i = 9, 10) \end{cases} \quad (3)$$

Fig. 3 The beam element model of composite material

where $\bar{x}$ is the coordinate of the beam element in the local coordinate system, u_1 and u_5 are the longitudinal displacements of the nodes, u_2 and u_6 are the transverse displacements of the nodes, u_3 and u_7 are the elastic rotation angles of axes at the nodes, u_4 and u_8 are the curvatures of axes at the nodes, u_9 and u_{10} are the cross-sectional rotation angle at the nodes, and the generalized coordinates vector of the beam element is:

$$\mathbf{u} = [u_1 \ u_2 \ u_3 \ u_4 \ u_5 \ u_6 \ u_7 \ u_8 \ u_9 \ u_{10}]^T \quad (4)$$

The shape functions $\phi_i(\bar{x})$ in Eq. (3) are given by

$$\begin{cases} \phi_1(\bar{x}) = 1 - e \\ \phi_2(\bar{x}) = 1 - 10e^3 + 15e^4 - 6e^5 \\ \phi_3(\bar{x}) = L \times (e - 6e^3 + 8e^4 - 3e^5) \\ \phi_4(\bar{x}) = L^2 \times (e^2 - 3e^3 + 3e^4 - 5e^5)/2 \\ \phi_5(\bar{x}) = e \\ \phi_6(\bar{x}) = 10e^3 - 15e^4 + 6e^5 \\ \phi_7(\bar{x}) = L \times (-4e^3 + 7e^4 - 3e^5) \\ \phi_8(\bar{x}) = L^2 \times (e^3 - 2e^4 + e^5)/2 \\ \phi_9(\bar{x}) = 1 - e \\ \phi_{10}(\bar{x}) = e \end{cases} \quad (5)$$

where L is the length of the beam element, $0 \leq \bar{x} \leq L$, and $e = \frac{\bar{x}}{L}$.

2.2 Kinetic energy of element

Considering the moment of inertia, the total kinetic energy T of the element equals the sum of the translational kinetic energy T_1 and the rotational energy T_2 .

$$T = T_1 + T_2 \quad (6)$$

where

$$\begin{cases} T_1 = \frac{1}{2} \int_0^L m [\dot{V}_a(\bar{x}, t)]^2 + [\dot{W}_a(\bar{x}, t)]^2 d\bar{x} \\ T_2 = \frac{1}{2} \int_0^L I \dot{\varphi}_a^2(\bar{x}, t) d\bar{x} \end{cases} \quad (7)$$

where, $\dot{V}_a(\bar{x}, t)$ and $\dot{W}_a(\bar{x}, t)$ are respectively the instantaneous absolute velocities of the central point of any cross section of element in the longitudinal and transverse directions, $\dot{\varphi}_a$ is the absolute angular velocity. m and I are the cross sectional mass and moment of inertia, respectively, and

$$(m, I) = \iint_S (1, z^2) \rho dS \quad (8)$$

Where ρ is the average mass density of materials, and $\rho = \rho_f v_f + \rho_m v_m$, ρ_f and ρ_m are respectively the densities of

fiber and matrix, v_f and v_m are respectively the volume percent of fiber and matrix, respectively, S is the area of the cross-section.

Substituting Eqs. (3) and (5) into Eq. (7), the translational kinetic energy T_1 of the element is:

$$T_1 = \frac{1}{2} \dot{\mathbf{u}}_a^T \bar{\mathbf{M}}_1 \dot{\mathbf{u}}_a \quad (9)$$

where $\dot{\mathbf{u}}_a$ is the absolute velocity vector of nodes, $\bar{\mathbf{M}}_1$ is the inertia matrix related to the length L of the element.

$$\begin{cases} (\bar{m}_1)_{ij} = \rho S \int_0^L \phi_i(\bar{x}) \phi_j(\bar{x}) d\bar{x} \\ (i, j = 1, 2, 3, 4, 5, 6, 7, 8) \\ (\bar{m}_1)_{ij} = 0 \\ (i, j = 9, 10) \end{cases} \quad (10)$$

The rotational energy T_2 of the element is:

$$T_2 = \frac{I}{2} \int_0^L [\dot{\theta}(t) + \dot{\varphi}(\bar{x}, t)]^2 d\bar{x} = \frac{1}{2} I \dot{\theta}^2 L + \frac{1}{2} \dot{\mathbf{u}}^T \bar{\mathbf{M}}_2 \dot{\mathbf{u}} + \dot{\mathbf{u}}^T \mathbf{F} \quad (11)$$

where θ is the angular displacement of the rigid beam relative to the fixed coordinate system, φ is the elastic angular displacement of the cross-section relative to the moving coordinate system, $\dot{\mathbf{u}}^T \mathbf{F}$ is the coupling of rigid motion and elastic motion, it is neglected in this paper. The inertia matrix $\bar{\mathbf{M}}_2$ is

$$\begin{cases} (\bar{m}_2)_{ij} = \frac{I}{2} \int_0^L \phi_i(\bar{x}) \phi_j(\bar{x}) d\bar{x} \\ (i, j = 9, 10) \\ (\bar{m}_2)_{ij} = 0 \\ (i, j = 1, 2, 3, 4, 5, 6, 7, 8) \end{cases} \quad (12)$$

Therefore, inertia matrix of the beam element is:

$$\bar{\mathbf{M}} = \sum_{j=1}^2 \bar{\mathbf{M}}_j \quad (13)$$

2.3 Potential energy of element

As described above, the 3-dimensional braided composite materials can be regarded as the superposition of four unidirectional fiber composite materials. The strain energy of the k -th part in the beam element can be expressed as:

$$W_k = \frac{1}{2} \int_{\Omega} \boldsymbol{\varepsilon}_k^T \boldsymbol{\sigma}_k d\Omega \quad (14)$$

where Ω is a quarter of the element volume. $\boldsymbol{\varepsilon}_k$ and $\boldsymbol{\sigma}_k$ are respectively the strain and stress vector of the k -th part in its principal material orientation, respectively, and they can be expressed as⁽¹⁷⁾:

$$\begin{cases} \boldsymbol{\varepsilon}_k^T = [\varepsilon_{x'}, \varepsilon_{y'}, \varepsilon_{z'}, \gamma_{y'z'}, \gamma_{z'x'}, \gamma_{x'y'}] \\ \boldsymbol{\sigma}_k = [\sigma_{x'}, \sigma_{y'}, \sigma_{z'}, \tau_{y'z'}, \tau_{z'x'}, \tau_{x'y'}]^T \end{cases} \quad (15)$$

Each part can be considered as transversely isotropy, so the constitutive relation of the k -th part in its principal material orientation can be expressed as^{(13), (17)}

$$\boldsymbol{\sigma}_k = \mathbf{H} \boldsymbol{\varepsilon}_k \quad (16)$$

$$\mathbf{H} = \begin{pmatrix} H_{11} & H_{12} & H_{13} & & & \\ H_{12} & H_{22} & H_{23} & & & 0 \\ H_{13} & H_{23} & H_{33} & & & \\ & & & H_{44} & & \\ & 0 & & & H_{55} & \\ & & & & & H_{66} \end{pmatrix} \quad (17)$$

where^{(13), (17)}

$$\begin{aligned} H_{11} &= \frac{(1 - \mu_{23}\mu_{23})E_{11}}{K}; \quad H_{12} = H_{13} = \frac{(1 + \mu_{23})\mu_{21}E_{11}}{K}; \\ H_{22} &= H_{33} = \frac{(1 - \mu_{21}\mu_{12})E_{22}}{K}; \\ H_{23} &= \frac{(\mu_{23} + \mu_{12}\mu_{21})E_{22}}{K}; \\ H_{44} &= H_{23}; \quad H_{55} = H_{13}; \quad H_{66} = H_{12}; \\ K &= 1 - 2\mu_{12}\mu_{21}(1 + \mu_{23}) - \mu_{23}\mu_{23}. \end{aligned}$$

According to micromechanics of composite materials, the elastic constants E_{ij} , G_{ij} and μ_{ij} can be calculated from elastic constants of fiber and matrix, volume ratio of fiber and the microstructure parameters of composite materials^{(13), (17)}.

The strains of the k -th part in the beam coordinate system XYZ can be obtained from the transition relation of strain in elastic mechanics as^{(13), (17)}

$$\boldsymbol{\varepsilon}_k = \mathbf{L}\boldsymbol{\varepsilon} \quad (18)$$

where the transition matrix $\mathbf{L}$ of strain is determined by the direction cosine between the beam coordinate system XYZ and the coordinate system $X'Y'Z'$ of k -th part in its principal material orientation.

Substituting Eqs. (3)–(5) into Eq. (2), the relations between the strains and nodal displacements can be written as

$$\boldsymbol{\varepsilon} = \mathbf{S}\mathbf{u} \quad (19)$$

where

$$\left\{ \begin{aligned} \mathbf{S} &= [\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3, \mathbf{S}_4, \mathbf{S}_5, \mathbf{S}_6, \mathbf{S}_7, \mathbf{S}_8, \mathbf{S}_9, \mathbf{S}_{10}]^T \\ \mathbf{S}_1 &= \left[-\frac{1}{L}, 0, 0, 0, \frac{1}{L}, 0, 0, 0, -\frac{z}{L}, \frac{z}{L} \right] \\ \mathbf{S}_5 &= \left[0, -\frac{30x^2}{L^3} + \frac{60x^3}{L^4} - \frac{30x^4}{L^5}, 1 - \frac{18x^2}{L^2} \right. \\ &\quad \left. + \frac{32x^3}{L^3} - \frac{15x^4}{L^4}, x - \frac{9x}{2L} + \frac{12x^3}{2L^2} - \frac{25x^4}{2L^3}, 0, \right. \\ &\quad \left. \frac{30x^2}{L^3} - \frac{60x^3}{L^4} + \frac{30x^4}{L^5}, -\frac{12x^2}{L^2} + \frac{28x^3}{L^3} \right. \\ &\quad \left. - \frac{15x^4}{L^4}, \frac{3x^2}{L} - \frac{8x^3}{L^2} + \frac{5x^4}{L^3}, 1 - \frac{x}{L}, \frac{x}{L} \right] \\ \mathbf{S}_2 &= \mathbf{S}_3 = \mathbf{S}_4 = \mathbf{S}_6 = \mathbf{S}_7 = \mathbf{S}_8 = \mathbf{S}_9 = \mathbf{S}_{10} = \mathbf{0} \\ \mathbf{u} &= [u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}]^T \end{aligned} \right. \quad (20)$$

Substituting Eqs. (15)–(20) into Eq. (14), the strain energy of the j -th part is:

$$W_j = \frac{1}{2} \int_{\Omega} \mathbf{u}^T \mathbf{S}^T \mathbf{L}^T \mathbf{H} \mathbf{L} \mathbf{S} \mathbf{u} d\Omega = \frac{1}{2} \mathbf{u}^T \bar{\mathbf{K}}_j \mathbf{u} \quad (21)$$

Then the total potential energy of element can be expressed as

$$W = \sum_{j=1}^4 W_j = \frac{1}{2} \mathbf{u}^T \bar{\mathbf{K}} \mathbf{u} \quad (22)$$

where the stiffness matrix $\bar{\mathbf{K}}$ of element is:

$$\bar{\mathbf{K}} = \sum_{j=1}^4 \bar{\mathbf{K}}_j \quad (23)$$

2.4 Mass matrix and stiffness matrix of the linkage mechanism

Assuming that $\mathbf{R}_i$ is the transformation matrix between the i -th element coordinates and the global coordinates, the equivalent mass matrix and equivalent stiffness matrix of the i -th element in the global coordinate system can be written as^{(14)–(16)}

$$\bar{\mathbf{M}}_i = \mathbf{R}_i^T \bar{\mathbf{M}} \mathbf{R}_i, \quad \bar{\mathbf{K}}_i = \mathbf{R}_i^T \bar{\mathbf{K}} \mathbf{R}_i \quad (24)$$

Assuming that $\mathbf{B}_i$ is the coordinate compatible matrix of the i -th element between the local number and the global number, the mass and stiffness of the i -th element in the global coordinates are, respectively^{(14)–(16)}:

$$\mathbf{M}_i = \mathbf{B}_i^T \bar{\mathbf{M}} \mathbf{B}_i, \quad \mathbf{K}_i = \mathbf{B}_i^T \bar{\mathbf{K}} \mathbf{B}_i \quad (25)$$

Therefore, the mass matrix and stiffness matrix of the elastic linkage mechanism can be expressed as:

$$\mathbf{M} = \sum_{i=1}^{N_e} \mathbf{M}_i, \quad \mathbf{K} = \sum_{i=1}^{N_e} \mathbf{K}_i \quad (26)$$

where N_e is the element number.

2.5 Natural frequency and vibration mode of the mechanism

Under the free vibration circumstances, the Lagrange equation is established by Eq. (26), and the standard eigenvalue equation can be written as

$$(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{U} = \mathbf{0} \quad (27)$$

Each order modal natural frequency ω_r and the vibration mode $\mathbf{U}$ can be obtained by Eq. (27). $\mathbf{U}$ is the vector of elastic displacements of the nodes in the global coordinates. Using $\mathbf{U}$, the strain energy W_m of mechanism can be obtained as follows:

$$W_m = \frac{1}{2} \mathbf{U}^T \mathbf{K} \mathbf{U} \quad (28)$$

3. Damping Matrix of the Mechanism

Material damping loss factor ψ (SDC) is defined as^{(18), (19)}:

$$\psi = \frac{\Delta W}{W_m} \quad (29)$$

where ΔW is the dissipated energy during a cycle of vibration and W_m is the maximum strain energy in the cycle.

3.1 Element damping model

The 3-dimensional braided composite materials can be regarded as the superposition of four part of unidirectional fiber composite materials. The unit dissipation energy of each part should divide into six components, namely

$$\Delta W_k = \Delta W_1 + \Delta W_2 + \Delta W_3 + \Delta W_4 + \Delta W_5 + \Delta W_6 \quad (30)$$

According to the definition of SDC, every term in Eq. (30) can be expressed as

$$\Delta W_i = \frac{1}{2} \psi_i \epsilon_i \sigma_i \quad (31)$$

where, $\psi_1, \psi_2 = \psi_3$ are respectively the SDC of the unidirectional composite laminates in the direction 0° and 90° when it is bending, ψ_4 is the SDC of its transverse shearing, $\psi_5 = \psi_6$ is the SDC of its vertical shearing, these ψ_i are determined by experiment or selected from the concerned data.

3.2 Expression of dissipation energy

Considering Eq. (31), the dissipation energy of the k -th part in the beam element can be expressed as

$$\Delta W_k = \frac{1}{2} \int_{\Omega} \epsilon_k^T \psi \sigma_k d\Omega \quad (32)$$

where the damping matrix is

$$\psi = \text{diag}[\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6] \quad (33)$$

Assuming that $\mathbf{Q}_k = \mathbf{S}^T \mathbf{L}_k^T \psi \mathbf{H}_k \mathbf{L}_k \mathbf{S}$, then Eq. (32) can be written as

$$\Delta W_k = \frac{1}{2} \int_{\Omega} \mathbf{u}^T \mathbf{Q}_k \mathbf{u} d\Omega = \frac{1}{2} \mathbf{u}^T \mathbf{K}_{dk} \mathbf{u} \quad (34)$$

where

$$\mathbf{K}_{dk} = \int_{\Omega} \mathbf{Q}_k d\Omega$$

Therefore, the total dissipation energy of the beam element can be expressed as

$$\Delta \bar{W} = \sum_{k=1}^4 \Delta W_k = \frac{1}{2} \mathbf{u}^T \mathbf{K}_{di} \mathbf{u} \quad (35)$$

The damped stiffness matrix of mechanism can be expressed as

$$\mathbf{K}_d = \sum \mathbf{B}_i^T \mathbf{R}_i^T \mathbf{K}_{di} \mathbf{R}_i \mathbf{B}_i \quad (36)$$

Each order modal dissipated energy can be obtained by the characteristic vector $\mathbf{U}$ derived from Eq. (27) as

$$\Delta W = \frac{1}{2} \mathbf{U}^T \mathbf{K}_d \mathbf{U} \quad (37)$$

When damping is small, the damping ratio ξ_r of each order mode of a linear system can be expressed as^{(16), (18), (19)}

$$\xi_r = \frac{\psi_r}{4\pi} \quad (38)$$

Marshaling all modal vectors, the modal matrix of the mechanism can be written as

$$\mathbf{U} = [\mathbf{U}^{(1)} \mathbf{U}^{(2)} \dots \mathbf{U}^{(n)}] \quad (39)$$

The normalized modal matrix satisfies the orthogonal conditions as

$$\mathbf{U}^T \mathbf{M} \mathbf{U} = \mathbf{I} \quad (40)$$

$$\mathbf{U}^T \mathbf{K} \mathbf{U} = \begin{bmatrix} \backslash & & \\ & \omega_r^2 & \\ & & \backslash \end{bmatrix} \quad (41)$$

When damping is small ($\xi_r \leq 0.2$), the linear system

$$\mathbf{U}^T \mathbf{C} \mathbf{U} = \begin{bmatrix} \backslash & & \\ & 2\xi_r \omega_r & \\ & & \backslash \end{bmatrix} \quad (42)$$

Next we Premultiply Eq. (42) by $(\mathbf{U}^T)^{-1}$ and postmultiply Eq. (42) by $\mathbf{U}^{-1}$, the damping matrix of mechanism can be expressed as

$$\mathbf{C} = (\mathbf{U}^T)^{-1} \begin{bmatrix} \backslash & & \\ & 2\xi_r \omega_r & \\ & & \backslash \end{bmatrix} \mathbf{U}^{-1} \quad (43)$$

considering Eq. (40), we obtain

$$\mathbf{C} = \mathbf{M} \mathbf{U} \begin{bmatrix} \backslash & & \\ & 2\xi_r \omega_r & \\ & & \backslash \end{bmatrix} \mathbf{U}^T \mathbf{M} \quad (44)$$

4. Dynamic Equation of the Mechanism

Assuming that $\mathbf{P}$ and $\mathbf{Q}$ are the external exciting force vector and inertial force vector of rigid body, respectively, let $\mathbf{U}$ represent the elastic displacement vector in the global coordinates. Incorporating the damping matrix term, the dynamic equation of mechanism can be expressed as:

$$\mathbf{M} \ddot{\mathbf{U}} + \mathbf{C} \dot{\mathbf{U}} + \mathbf{K} \mathbf{U} = \mathbf{P} + \mathbf{Q} \quad (45)$$

The Eq. (45) can be solved by the mode superposition method.

5. Illustrative Example

The four-bar linkage with links fabricated from 3-dimensional braided composites is shown in Fig. 4. The 3-dimensional braided composite materials consist of carbon fiber and epoxy matrix, the material performance parameters are as follows:

$G_m = 1.26 \text{ GPa}$, $\mu_f = 0.25$, $\mu_m = 0.35$, $\psi_1 = 0.0045$, $\psi_2 = \psi_3 = 0.042$, $\psi_4 = 0.01$, $\psi_5 = \psi_6 = 0.0705$, $\rho_f = 1.76 \text{ g/cm}^3$, $\rho_m = 1.36 \text{ g/cm}^3$, the total volume ratio of fiber is 0.6, the braiding structural parameters are $\alpha = 30^\circ$ and $\beta = 45^\circ$

The geometrical parameters of the four-bar linkage mechanism: L , b and h represent the length, width and thickness of the links, respectively; the cross sectional parameters of every link: $b = 20 \text{ mm}$, $h = 25 \text{ mm}$, crank $L_1 = 260 \text{ mm}$, coupler $L_2 = 425 \text{ mm}$, rocker $L_3 = 460 \text{ mm}$, frame $L_4 = 620 \text{ mm}$.

Fig. 4 Diagram of linkage mechanism

Table 1 The first four orders natural frequency ω_r and the modal damping ψ_r of the mechanism while the angle of crank $\theta_1 = 80^\circ$

ω_r (Hz)	90.5	225	453	1294.1
ψ_r	0.0761	0.0518	0.0137	0.0428

Fig. 5 Simulation time-domain dynamic response characteristics curve of the midpoint of the coupler while $\omega_1 = 300$ rpm

We regard the crank as one beam element, and regard the rocker and the coupler link as two elements respectively. 1,2,3,4,5 and 6 express the serial number of nodes, 1', 2', 3', 4' and 5' express the serial number of elements, and (1), (2) and (3) express the serial number of bars.

The single arrows represent the elastic displacements and the axis inclination, the dual arrows express the curvature of axis, and the triple arrows stand for the cross-sectional rotation angle, as shown in Fig. 4.

The natural frequency ω_r and the modal damping ψ_r of the mechanism can be obtained by using the dynamic equation of the mechanism presented in this paper, Table 1 shows the from first to fourth order ω_r and ψ_r while the angle of crank $\theta_1 = 80^\circ$.

Don't consider the external exciting force, namely $P = 0$, the simulation calculation of dynamic response of the mechanism is obtained by the real mode superposition

Fig. 6 Simulation time-domain dynamic response characteristics curve of the midpoint of the rocker while $\omega_1 = 300$ rpm

method according to the dynamic equation presented in this paper. The computer simulation curves of the time domain responses of the midpoint of coupler and rocker while the angular velocity of link L_1 $\omega_1 = 300$ rpm are shown in Figs.5 and 6 respectively. The exciting force is mainly the self-excitation inertia force. Because the self-excitation inertia force of the midpoint of coupler is greater than that of midpoint of rocker, the dynamic responses of the coupler midpoint are greater than that of the rocker midpoint obviously.

6. Conclusions

Because the 3-dimensional braided composite materials overcome the shortcomings of the two-dimensional composite laminates (such as low strength between the layers), they are the ideal materials used in high-speed linkage mechanisms. The stiffness and damping characteristics of this material linkage are analyzed according to the microstructure of the 3-dimensional braided composite materials, and the dynamic equation of the mechanism is established by the FEM. The dynamic equation includes the microstructure parameters and constituent material parameters of the composite materials and structural parameters of the linkage. The work presented in the paper provides theoretical basis to a certain extent for the further research on nonlinear vibration characteristics and optimum design of this kind of mechanism.

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