A Study on the Numerical Analysis of Fluid Film Lubrication by the Boundary-Fitted Coordinates System*
(Fundamental Equations of DF Method and the Case of Incompressible Lubrication)

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A new method of direct numerical calculation for the fluid film lubrication problem is presented in this paper. The present method is derived by applying the boundary fitted coordinates system to the divergence formulation (DF) method. The present method makes it possible to deal with an arbitrary configuration of a lubricated surface, so the disadvantage of the DF method compared with the finite element method is solved. The present method has the following advantages: (1) An arbitrary configuration of a lubricated surface can be calculated. (2) Computer programming is easy. (3) The scale of computer program and cpu time are small, so calculation by a personal computer is possible.

Key Words: Numerical analysis, Lubrication Theory, Direct Numerical Solution Method, Divergence Formulation Method, Boundary-fitted Coordinates System, Incompressible Fluid

1. Introduction

The latest progress in the performance of computers is remarkable and has produced effects in every field. In the field of the fluid lubrication problem, there had been no effectual method except for the analytical method in former times. Recently, methods which solve direct-numerically the Reynolds equation by use of a high-performance computer have been developed. And these methods have been the mainstream methods\(^{(1)}\).

Concerning the fluid lubrication problem, the Navier-Stokes equation is simplified by the following assumptions\(^{(2)}\). (1) The lubricated fluid flows laminarly. (2) The fluid film thickness is less than the dimensions of the lubricated surface. (3) The external force is negligible. (4) The lubricated fluid does not slip on the surface of the bearings. (5) The inertia force is negligible. (6) The coefficient of viscosity does not vary. The velocity profile is obtained by integration of a simplified N-S equation, and the Reynolds equation is obtained by integrating the continuity equation in the direction of fluid film height.

The Reynolds equation has been solved by the finite difference method. However, the Reynolds equation is not applied to boundaries at which the fluid film height varies successively, for the reason that the Reynolds equation is derived by integrating in the fluid film height. The continuity condition of fluxes normal to these boundaries must be applied. Therefore, the finite difference method is not an advantageous method from the view of wide-usability and is therefore not used much. The finite element method and the divergence formulation (DF) method\(^{(3)}\) are used very often instead of the finite difference method. These methods are established to conserve mass fluxes on a small lubricated surface element by means of integrating the Reynolds equation. The DF method is inferior to the finite element method in view of the flexibility of the boundary configuration\(^{(4)}\). However, the DF method is easily understood physically, and is able to produce a computer program.

In this paper, the boundary fitted coordinates

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system which is used effectively in the calculation of intricate flow field, is applied to the DF method. Consequently, the widely useful computer program which deal with an arbitrary configuration of lubricated surface can be developed by the present method.

**Nomenclature**

\[ F_x, F_r, F_t : \text{load carrying capacity in the direction of } x, \theta = 0 \text{ and } \theta = \pi/2. \]

\[ h : \text{fluid film height.} \]

\[ i, j : \text{unit vectors in the } s - \text{ and } \theta - \text{direction, respectively.} \]

\[ M_r, M_t : \text{components of moment in the direction of } \theta = 0 \text{ and } \theta = \pi/2, \text{ respectively.} \]

\[ \rho : \text{fluid film pressure.} \]

\[ \rho_0 : \text{ambient pressure.} \]

\[ q^s, q^\theta : \text{mass fluxes per unit length in the direction of } s \text{ and } \theta, \text{ respectively.} \]

\[ \mathbf{q} = q^s \mathbf{i} + q^\theta \mathbf{j} : \text{mass flux vector per unit length.} \]

\[ Q : \text{mass flux.} \]

\[ r, \theta, z : \text{cylindrical coordinates.} \]

\[ s : \text{meridian coordinate.} \]

\[ t : \text{time.} \]

\[ T_s, T_t : \text{frictional moments of shaft and bearing.} \]

\[ \rho : \text{density.} \]

\[ \mu : \text{coefficient of viscosity.} \]

\[ \omega_0, \omega_2 : \text{angular velocities of shaft and bearing, respectively.} \]

\[ \omega_\theta : \text{angular velocity of } (s, \theta) \text{ coordinates.} \]

\[ \xi, \eta : \text{boundary fitted coordinates} \]

2. Fundamental Equations

The arbitrary axisymmetrical lubricated fluid film is considered, as shown in Fig. 1. \((r, \theta, z)\) are cylindrical coordinates and \(s\) is the meridian coordinate. The arbitrary coordinates system \((\xi, \eta)\) is considered on the fluid film surface. \((\xi, \eta)\) coordinates are parallel to boundaries which surround the lubricated surface and at which the fluid film height varies successively. \((\xi, \eta)\) coordinates are established as intervals of grids equal to 1. The complicated lubricated surface is transformed to the rectangular domain constructed of many regular squares by the above transformation. Therefore, it is easy to handle and the wide-usability increases.

In the DF method, the balance of mass fluxes which flow in and out of the small square element on the \((\xi, \eta)\) coordinates is considered. In this section, mass fluxes which pass through the \(\xi = \text{const.} \) line and the \(\eta = \text{const.} \) line, and the mass absorbed in the square element are derived.

The mass flux vector \(\mathbf{q}\) per unit length is expressed as

\[ \mathbf{q} = q^s \mathbf{i} + q^\theta \mathbf{j} \]

\[ q^s = -\frac{\rho h^2}{12\mu} \frac{\partial p}{\partial s} \]

\[ q^\theta = -\frac{\rho h^2}{12\mu} \frac{\partial p}{\partial \theta} + \rho kr \]

\[ \mathbf{q} = \frac{(\omega_\theta + \omega_s)/2 - \omega_\theta}{\sqrt{\mathbf{q}}} \]

where \(i\) and \(j\) are unit vectors in the direction of \(s\) and \(\theta\), respectively. The unit vector \(\mathbf{n}\) which is normal to the \(\xi = \text{const.} \) line and is directional to the increase of \(\xi\), is expressed as

\[ \mathbf{n} = \frac{\text{grad } \xi}{|\text{grad } \xi|} \]

\[ = (r\theta, i + s, j)/\sqrt{\mathbf{q}} \]

where \(c\) is shown in Eq. (8) and suffixes \(\xi\) and \(\eta\) mean partial differential movement. Therefore, \(\mathbf{n}\) directional mass flux component \(q^\xi\) per unit length is derived as

\[ q^\xi = \mathbf{q} \cdot \mathbf{n} \cdot (s = r\thetaq^s + s\etaq^\theta)/\sqrt{\mathbf{q}} \]

The \(\mathbf{n}\) directional mass flux component \(q^\eta\) per unit length is derived similarly.

\[ q^\eta = \mathbf{q} \cdot \mathbf{n} \cdot (s = r\thetaq^s + s\etaq^\theta)/\sqrt{\mathbf{q}} \]

\[ s \text{ and } \theta \text{ - directional pressure gradients are expressed by use of } \xi \text{ and } \eta \text{ - directional ones, as follows:} \]

\[ \frac{\partial p}{\partial s} = (r\theta p_r - r\theta p_\theta) \]

\[ \frac{\partial p}{\partial \theta} = (s p_s - s p_\theta) \]

Substituting Eq. (5) into Eq. (1), and substituting into Eqs. (3) and (4), \(q^\xi\) and \(q^\eta\) are expressed as

\[ q^\xi = \rho(\xi = A\beta + B\gamma + D)/\sqrt{\mathbf{q}} \]

\[ q^\eta = \rho(\xi = F\beta + G\gamma + H)/\sqrt{\mathbf{q}} \]

\[ a = s^2 + (r\theta)^2 \]

\[ \beta = s^2 + (r\theta)^2 \]

\[ \gamma = s^2 + (r\theta)^2 \]

\[ s = r\theta \]

\[ A = \frac{h^2}{12\mu} \frac{\partial q^s}{\partial s} \]

\[ B = \frac{h^2}{12\mu} \frac{\partial q^\theta}{\partial \theta} \]

\[ C = \frac{h^2}{12\mu} \frac{\partial q^s}{\partial \theta} \]

\[ D = -h\theta r \overline{\omega} \]

The mass flux \(Q^s\) which passes through the interval \(r^t\) between \(\eta = \eta_1 \) and \(\eta = \eta_2 \) on the \(\xi = \text{const.} \) line, is expressed as

\[ Q^s = \int_{r_1}^{r_2} q^s dr^t \]

The infinitely small length \(dr^t\) on the physical plane is

![Fig. 1 Arbitrary axisymmetrical lubricated surface](https://example.com/fig1.png)
given by
\[d^* = \sqrt{(ds)^2 + (rd\theta)^2} = \sqrt{\rho d\xi^2 + 2\beta d\xi d\eta + \alpha d\eta^2}\]
(9)
\[d^2\xi\]

is equal to zero on the \(\xi = \text{const.}\) line, so \(Q^*\) is derived as follows:
\[Q^* = \int_{\eta_1}^{\eta_2} \int_{\theta_1}^{\theta_2} \rho(\beta d\xi + \alpha d\eta) \, d\eta\]
(10)
The mass flux \(Q^*\) which passes through the interval \(I^*\) between \(\xi = \xi_1\) and \(\xi = \xi_2\) on the \(\eta = \text{const.}\) line, is obtained similarly, as follows:
\[Q^* = \int_{\theta_1}^{\theta_2} \int_{\eta_1}^{\eta_2} \rho d\xi d\eta\]
(11)
The mass \(Q^*\) absorbed into the region \(V\) bounded by four lines of \(\xi = \xi_1, \xi = \xi_2, \eta = \eta_1, \) and \(\eta = \eta_2\) is expressed as
\[Q^* = \int_{\theta_1}^{\theta_2} \int_{\eta_1}^{\eta_2} \frac{\partial (\rho h)}{\partial t} \, dV\]
(12)
where the partial differentiation by the time \(t\) means the partial differentiation for the case that \(s\) and \(\theta\) are constant. Transforming the surface integral of Eq. (12) into the surface interval on the \((\xi, \eta, \theta)\) plane, \(Q^*\) becomes
\[Q^* = \int_{\eta_1}^{\eta_2} \int_{\theta_1}^{\theta_2} \frac{\partial (\rho h)}{\partial t} \, dV\]
(13)
The axial load capacity \(F_a\), the radial component \(F_r\), and the tangential component \(F_t\) of the radial load capacity, the radial component \(M_r\), and the tangential component \(M_t\) of the moment frictional moments \(T_r, T_t\) which work on the shaft and the bearing, are given by the surface integral on the lubricated surface \(S\), as follows:
\[F_a = \int_{\theta_1}^{\theta_2} \int_{\eta_1}^{\eta_2} (p - \rho g_r) \, dr \, d\eta\]
\[F_r = \int_{\theta_1}^{\theta_2} \int_{\eta_1}^{\eta_2} (p - \rho g_r) \, dr \, d\eta\]
\[F_t = \int_{\theta_1}^{\theta_2} \int_{\eta_1}^{\eta_2} (p - \rho g_r) \, dr \, d\eta\]
\[M_r = \int_{\eta_1}^{\eta_2} (p - \rho g_r) \left( x \, dz \, ds + r \, dr \, ds \right) \sin \theta \, d\theta\]
\[M_t = \int_{\eta_1}^{\eta_2} (p - \rho g_r) \left( x \, dz \, ds + r \, dr \, ds \right) \cos \theta \, d\theta\]
\[T_r = \int_{\theta_1}^{\theta_2} \left( \frac{h}{2} \frac{\partial r}{\partial \theta} - \frac{r \, w}{h} \right) \, r^2 \, d\theta\]
\[T_t = \int_{\theta_1}^{\theta_2} \left( \frac{h}{2} \frac{\partial r}{\partial \theta} + \frac{r \, w}{h} \right) \, r^2 \, d\theta\]
(14)
The above equations are transformed easily to the case of Cartesian coordinates \((x, y)\) by the following relations:
\[x = s\]
\[y = r \theta\]
\[U = r \omega\]
(15)
where \(y\) is the coordinate in the direction of the bearing motion and \(U\) is the velocity of bearing motion.

3. Discretization in the Case of Incompressible Fluid Lubrication

In this section, the discretization method for the incompressible fluid \((\rho = \text{const.})\) is shown. The grid cell means the regular square region made by the neighboring four nodes, and the DF cell means the regular square region whose center is a node and whose size equals 1. Values of \(s, \theta, r\) are given at the node, and the fluid film thickness \(h\) and \(\partial h/\partial t\) are given on the center of the grid cell. \(p_{\omega}\) means the pressure at the node point \((i, j)\).

The pressure distribution on the grid cell is approximated by a linear distribution of four node pressures, as follows:
\[p(\xi, \eta) = (i - \eta)\frac{p_{i-1,j} + (\xi - i + 1)p_{i,j}}{2} + (\eta - j + 1)\frac{p_{i,j} + (\xi - i + 1)p_{i,j+1}}{2}\]
(16)
Differentiating Eq. (16) by \(\xi\) and \(\eta\) pressure gradients are given as
\[p_{\xi} = (i - \xi)(p_{i-1,j} - p_{i,j-1}) + (\xi - i + 1)(p_{i,j} - p_{i,j-1})\]
\[p_{\eta} = (j - \eta)(p_{i-1,j} - p_{i,j-1}) + (\eta - j + 1)(p_{i,j} - p_{i,j+1})\]
(17)
Substituting Eq. (17) into Eqs. (10) and (11), \(Q^*\) and \(Q^0\) are obtained as follows:
\[Q^* = p_{i-1,j} \left\{ \rho \int_{\theta_1}^{\theta_2} \left[ A(\eta - j) - \frac{B}{2} \right] \, d\eta \right\} + p_{i,j-1} \left\{ \rho \int_{\theta_1}^{\theta_2} \left[ A(\eta - j) + \frac{B}{2} \right] \, d\eta \right\} + p_{i-1,j} \left\{ \rho \int_{\theta_1}^{\theta_2} \left[ A(\eta - j + 1) + \frac{B}{2} \right] \, d\eta \right\} + p_{i,j+1} \left\{ \rho \int_{\theta_1}^{\theta_2} \left[ A(\eta - j + 1) - \frac{B}{2} \right] \, d\eta \right\} + \rho \int_{\theta_1}^{\theta_2} \int_{\eta_1}^{\eta_2} C(\xi - i) \, d\xi \, d\eta\]
(18)
\[Q^0 = p_{i-1,j} \left\{ \rho \int_{\theta_1}^{\theta_2} \left[ C(\xi - i + 1) - \frac{B}{2} \right] \, d\xi \right\} + p_{i,j-1} \left\{ \rho \int_{\theta_1}^{\theta_2} \left[ C(\xi - i + 1) + \frac{B}{2} \right] \, d\xi \right\} + p_{i-1,j} \left\{ \rho \int_{\theta_1}^{\theta_2} \left[ C(\xi - i) - \frac{B}{2} \right] \, d\xi \right\} + p_{i,j+1} \left\{ \rho \int_{\theta_1}^{\theta_2} \left[ C(\xi - i) + \frac{B}{2} \right] \, d\xi \right\} + \rho \int_{\theta_1}^{\theta_2} \int_{\eta_1}^{\eta_2} D(\xi - i) \, d\xi \, d\eta\]
(19)
It is difficult to require analytical values of integrals of Eqs. (18) and (19), so that these must be calculated by some approximate method. In this paper, \(A, B, C, D\) and \(D\) are approximated to values at the center \((\xi = i - 1/2, \eta = j - 1/2)\) of the grid cell. This method is equivalent to a successive stepped film thickness which has been used in the usual DF method.

Replacing \(\eta_1\) and \(\eta_2\) in Eq. (18) to \(j - 1/2\) and \(j\) respectively, the mass flux \(Q^0\) (Fig. 2) which passes through the interval \(\eta = j - 1/2 \sim j\) on the \(\xi = i - 1/2\) line, is given as follows:
\[ \begin{align*}
Q_i^w &= (p_{i-1/2} - p_{i+1/2}) + p_{i+1/2} - (A_i - 2B_i) + p_{i+1/2}(3A_i + 2B_i) + p_{i+1/2}(-3A_i + 2B_i))/8 + E_{i+1/2}/2 \\
\text{(20)}
\end{align*} \]

Replacing \( \xi_i \) and \( \xi_j \) in Eq.(19) to \( i \rightarrow 1/2 \) and \( i \) respectively, \( Q_i^m \) is given as follows:

\[ \begin{align*}
Q_i^m &= (p_{i-1/2} - P_{i-1/2}) + p_{i+1/2} - (C_i - 2B_i) + p_{i+1/2}(-C_i - 2B_i))/8 + E_{i+1/2}/2 \\
\text{(21)}
\end{align*} \]

\( Q_{nl}, Q_{nh}, \text{etc. are given similarly. Suffixes I, II, III and IV mean values at points (i-1/2, j-1/2), (i+1/2, j-1/2), (i-1/2, j+1/2) and (i+1/2, j+1/2), respectively.} \]

\( s_t, s_n, r_t, \text{and } r_n \) are calculated by finite differencing, as follows:

\[ \begin{align*}
s_t &= (s_{i-1/2} - s_{i+1/2})/2 \\
\text{(22)}
\end{align*} \]

We consider the mass which is absorbed in the DF cell by the unsteady motion of film thickness. The film thickness \( h \) may be discontinuous in the DF cell, but the fluctuation velocity \( \partial h/\partial t \) of \( h \) must be continuous in the DF cell. Therefore, approximating \( \partial h/\partial t \) to the linear distribution and substituting into Eq.(13), the absorbed mass \( Q_{l,i}^m \) in the \( (i, j) \) DF cell is expressed as follows:

\[ \begin{align*}
Q_{l,i}^m &= (p_{l+1/2} - p_{l-1/2})(9j_{l1} + 3j_{l2} + 3j_{f1} + j_{f2})(\partial h/\partial t) \\
\text{(23)}
\end{align*} \]

Substituting Eqs.(20), (21) and (23) into the continuity equation on the DF cell

\[ \begin{align*}
Q_{l,i}^m + Q_{l,n}^m - Q_{l,n}^m + Q_{l,n}^m + Q_{l,n}^m = Q_{l,n}^m \\
\text{(24)}
\end{align*} \]

the algebraic equation for node pressures is obtained as follows:

\[ \begin{align*}
\alpha_t p_{l-1/2} + \frac{\alpha_t p_{l+1/2}}{2} + \alpha_t p_{l+1/2} + \alpha_t p_{l+1/2} \\
= a_t p_{l-1/2} + a_t p_{l+1/2} + a_t p_{l+1/2} + a_t p_{l+1/2} \\
\text{(25)}
\end{align*} \]

Next, we consider the following four representative boundary conditions.

(1) The pressure distribution \( p^a \) along the boundary is already-known.

(II) The mass flux \( Q^a \) which passes through the boundary is already-known.

(III) The cyclic boundary condition.

(IV) The pressure along the boundary is constant and the total mass flux which passes through the boundary is already-known.

The boundary condition \((1)\) is applicable mainly to the ambient boundary by replacing \( p = p^a \). The boundary condition \((2)\) is applicable to the axis of symmetry by replacing \( Q^a = 0 \). In the case of this boundary condition, considering the region which is shown by the broken line in Fig. 3(a) as the DF cell, the continuity equation

\[ \begin{align*}
Q_{n1}^m - Q_{n2}^m - Q_{n2}^m + Q_{n2}^m - Q_{n2}^m = Q_{n2}^m \\
\text{(26)}
\end{align*} \]

is applied. In the case that the boundary condition \((3)\) is applied to the boundary \((2)\) in Fig. 3(b), the pressure distribution along boundary \((2)\) of \((3)\) equals to that along boundary \((3)\), and the outflow mass flux through boundary \((2)\) equals to the inflow through boundary \((3)\). Accordingly, regions I, II, III and IV of the DF cell are set up as shown in Fig. 3(b), and the continuity equation (25) for the inner DF cell is applied. In the case of boundary condition \((4)\), considering the region which is shown in Fig. 3(c) by the broken line as the DF cell, the continuity equation

\[ \begin{align*}
\frac{1}{2} \sum_{j=1}^{N} (Q_{l,j}^m + Q_{l,j}^m) = Q_{l,j}^m - \frac{1}{2} \sum_{j=1}^{N} Q_{l,j}^m \\
\text{(27)}
\end{align*} \]

and the condition that the pressure distribution along the boundary is constant

\[ p_{l,j} = p^a, j = 1, 2, \ldots, M_s + 1 \]

are applied.

Next, we consider bearing characteristics which are given by the surface integral on the lubricated surface. Approximating that integrands are constant on the grid cell, the surface integral on the lubricated surface \( S \) is transformed to the summation on all grid

\[ Q_{l,j}^m = p^a, j = 1, 2, \ldots, M_s + 1 \]
cells.
\[
\oint_{\gamma} f r d\theta d s = \sum_{j=1}^{n} \sum_{i=1}^{m} \{ f_{i+1/2,j-1/2} I_{i+1/2,j-1/2} \} 
\]
(29)

\( M_1 \) and \( M_2 \) mean the numbers of divisions in the \( \xi \)- and \( \eta \)-direction, respectively.

4. Examples

In this section, the results of numerical calculation by the widely-useful computer program which is developed by the present method is shown.

The external pressurized journal bearing shown in Fig. 4 is considered. The hole for the supply of oil is made on the bearing. \( c \) means the radial clearance and \( p_0 \) means the supply pressure. The grid configuration \( (15 \times 121) \) which is used for this calculation is shown in Fig. 5(a). This grid is generated by solving numerically alliance Poisson type partial differential equations for \( \xi \) and \( \eta \) by Thompson et al.\(^{40}\). Boundary condition (I) is applied to the bearing edge (\( \xi=1 \)) and circumferences of supply holes. The symmetric condition \( Q^0 = 0 \) by boundary condition (II) is applied to the center line. The cyclic condition (III) is applied to \( \eta=1 \) and \( M_2 \).

The result obtained by the Sommerfeld condition is shown in Fig. 5(b). The result obtained by the Gumbel condition that a negative node pressure is replaced with the ambient pressure by an iteration, is shown in Fig. 5(c). The result obtained by the equivalent flow model by Ikeyu and Mori\(^{40}\) is shown in Fig. 5(d). In the equivalent flow model, film rupture occurs in the negative pressure region and this rupture region has a strong compressibility. \( r \), which is a ratio of the lubricated fluid and the gas in the rupture region, is defined at the center of the grid cell, as follows:

\[
\begin{align*}
p_m &= (p_{i+1,j+1} + p_{i+1,j} + p_{i+1,j-1} + p_{i,j})/4 \\
p_m &\geq 0 : \eta_{i+1/2,j-1/2} = 1 \\
&\text{(continuous film region)} \\
p_m < 0 : \eta_{i-1/2,j-1/2} = 1 - ap_m^2 \\
&\text{(film rupture region)}
\end{align*}
\]  
(30)

Fig. 3 Boundary condition.

Fig. 4 Dimensions of journal bearing.

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The equation which obtained by replacing the right hand side $a_n$ of Eq.(25) with

$$a_n = -4\gamma(D_n + E_n) + \gamma(D_m + E_m)$$

is applied as the discretized equation for node pressure. This model is considered for the case of the static problem, so the mass absorbed into the DF cell is not considered. The shaded region in Fig.5 means the negative pressure region. Negative pressure regions in Fig.5(c) and (d) are film rupture regions.

These examples are calculated numerically by the successive line over-relaxation method. Triangular equations on the $\eta=\text{const.}$ line are solved by the LU-decomposition before the iterative calculation. Cpu times of the personal computer (PC 98 XA) are shown in Table 1. The judgement of convergence is made under the condition

$$\sum \Delta p_{n,j} < 10^{-4}$$

where $\Delta p_{n,j}$ means the pressure collection by an iteration. The initial pressure of iteration is set up to zero in Fig.5(b). The result of Fig.5(b) is used as the initial pressure in Fig.5(c). The result of Fig.5(c) is used as the initial pressure in Fig.5(d). Relaxation parameters in Fig.5(b) and (c) equal 1. The non-linearity in the case of Fig.5(d) is strong; accordingly, the relaxation parameter is very small (0.1). It is clarified from Table 1 that cpu times in the case of Fig.5(b) and (c) are less than 10 minutes in spite of many grids (1815).

5. Conclusion

Fundamental equations obtained by applying the boundary fitted coordinates system to the DF method have been shown. Discretization on the incompressible fluid lubrication has been shown, and the widely-useful computer program is developed by the present method. It is made possible by the present method to deal with an arbitrary configuration of a lubricated surface, so the disadvantage of the DF method compared with the finite element method has been solved. The present method has the following advan-

<table>
<thead>
<tr>
<th>Relaxation Parameter</th>
<th>Number of Iteration</th>
<th>Cpu Time</th>
</tr>
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<tbody>
<tr>
<td>(b) 1.0</td>
<td>409</td>
<td>8 m 10 s</td>
</tr>
<tr>
<td>(c) 1.0</td>
<td>156</td>
<td>3 m 33 s</td>
</tr>
<tr>
<td>(d) 0.1 (x=200)</td>
<td>1876</td>
<td>5 m 60 s</td>
</tr>
</tbody>
</table>

Fig. 5 Iso-bar lines
tages:
(1) An arbitrary configuration of a lubricated surface can be calculated.
(2) Programming is easy.
(3) The scale of the computer program and cpu time are small, so that calculation by a personal computer is possible.

References