A Study on Running Accuracy of an Externally Pressurized Gas Thrust Bearing* (Load Capacity Fluctuation Due to Machining Errors of the Bearing)

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The running accuracy characteristics of an externally pressurized gas thrust bearing will be theoretically studied in the course of this research in relation to the machining deviations of forms of the bearing and the rotor. For the first step of the research, the load-carrying capacity fluctuation of the bearing, which may result in axial run-out of the rotor, is analyzed in this report. As the factors causing the load capacity fluctuation, two aspects of machining error of the bearing are taken into consideration: one is the perpendicularity of the rotor end surface, and the other is the size deviation of the gas supply holes. The point source solution of the Reynolds equation yields the load capacity characteristics under these form errors of the bearing.

Key Words: Lubrication, Thrust Bearing, Externally Pressurized Gas Bearing, Running Accuracy, Deviation of Forms, Load Capacity Fluctuation, Point Source Solution

1. Introduction

Externally pressurized gas bearings have been intensively applied to high-precision spindles of measuring instruments and machine tools, due to the advantages of their low frictional properties and excellent running accuracies.

Generally speaking, the running accuracy characteristics of a rotor may depend on the machining accuracy of the bearing and the rotor. As for a rotor supported by an externally pressurized gas bearing, it is empirically known that one can obtain a remarkably better running accuracy of the rotor than the machining accuracy of the bearing due to the so-called "averaging effect." The averaging effect, however, has been discussed only from a qualitative aspect of bearing performance, and no papers have been published on its quantitative characteristics from

the viewpoint of precision-design consideration of the bearing system. It seems that, up to this time, the machining accuracy of the bearing, which would be required in order to attain the necessary running accuracy of the rotor, is determined on the basis of empirical data or from the economical standpoint of the machining cost.

It would be very important, therefore, to discuss and investigate theoretically the running accuracy characteristics of the bearing in relation to the machining accuracy of the bearing and the rotor for further developments of gas bearing applications with excellent high-precision performance in the future.

This research is the first step in the theoretical investigation of run-out characteristics of gas bearings. An externally pressurized gas-lubricated thrust collar bearing with multiple supply holes will be analyzed considering two aspects of machining deviations of forms for the rotor and the bearing, which may account for the run-out characteristics of the rotor: one is perpendicularity error of the rotor end surface to its axis, and the other is size deviation of supply holes.

In this paper, load-carrying capacity fluctuation

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due to these machining deviations is investigated theoretically. Load capacity fluctuation may result in an excitation force to the rotor, and cause an axial vibration or run-out of the rotor. The run-out characteristics of the bearing will be discussed in a following paper.

2. Fundamental Equations and Solving Procedure

We consider a shaft with a deviation of forms in perpendicularity of its end surface to the rotation axis. The shaft is supported by an externally pressurized gas-lubricated thrust collar bearing. If such a shaft rotates, a gap height distribution change takes place due to the form deviation of the shaft end surface, so that the pressure distribution variation may result, even when the shaft rotation keeps the central or reference gap height constant.

Figure 1 is a schematic diagram of the rotor and the bearing analyzed in this study. The inner and the outer radii of the bearing are r_i and r_o , respectively. The supply holes with inherent compensation are arranged on a radius r_a with equal circumferential spacings. The number and the diameter of the supply holes are k and $2r_s$, respectively. h_0 is the central gap height or reference clearance, and is assumed, in this paper, to be constant throughout the shaft rotation.

The end surface of the shaft has a perpendicularity error as shown in the figure, and the amount of maximum form deviation at the outer radius of the shaft is represented by ϵh_0 , taking a ratio to the reference clearance. The angular position of the maximum form deviation is measured from a particular supply hole, and is designated to be α .

The Reynolds equation governing the pressure distribution in the bearing clearance is expressed as follows in a dimensionless form:

$$\frac{1}{R} \frac{\partial}{\partial R} \left(H^3 R \frac{\partial P^2}{\partial R} \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left(H^3 \frac{\partial P^2}{\partial \theta} \right) = \Lambda \frac{\partial(PH)}{\partial \theta} \quad (1)$$

in which

$$\Lambda = 12\mu\omega r_o^2 / (p_a h_0^3), \quad H = h/h_0, \quad \left\{ \begin{array}{l} R = r/r_o, \quad P = p/p_a \end{array} \right\} \quad (2)$$

where h : bearing clearance height, r and θ : polar coordinates, p : pressure, p_a : ambient pressure, μ : viscosity coefficient, and ω : angular velocity of the shaft. The dimensionless bearing clearance distribution can be expressed as

$$H = 1 + \epsilon R \cos(\theta - \alpha) \quad (3)$$

In order to satisfy the purpose of this research, point source analysis should be employed in obtaining the pressure distribution from Eq. (1). We then apply a point source solution⁽¹⁾ using the Green function of Eq. (1). Equation (1) is rearranged into a Poisson equation as

$$\nabla^2 P^2 = \left\{ \frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} \right\} P^2 = -\frac{3}{H} \left\{ \frac{\partial H}{\partial R} \frac{\partial P^2}{\partial R} + \frac{1}{R^2} \frac{\partial H}{\partial \theta} \frac{\partial P^2}{\partial \theta} \right\} + \frac{\Lambda}{H^3} \frac{\partial(PH)}{\partial \theta} \quad (4)$$

The Green function $G(R, \theta; \rho, \varphi)$ is a function which satisfies the following three relationships:

$$(1) \quad \nabla^2 G = 0$$

$$(2) \quad \oint_C \frac{\partial G}{\partial n} ds = -1$$

where, C : integration path around a point (ρ, φ) , ds : line element of C , and dn : element of outward normal of C .

$$(3) \quad G = 0 \text{ on the inner and outer bearing peripheries.}$$

The Green function for the geometry of a thrust collar bearing with multiple supply holes, which we are concerned with in this paper, has already been reported in the literature⁽²⁾ as being

$$G(R, \theta; \rho, \varphi) = -\frac{1}{4\pi} \lim_{m \rightarrow \infty} \left\{ \sum_{j=-m}^m \log \frac{1 - 2R_i^{2j} (R/\rho) \cos(\theta - \varphi) + R_i^{4j} (R/\rho)^2}{1 - 2R_i^{2j} (R\rho/R_i^2) \cos(\theta - \varphi) + R_i^{4j} (R\rho/R_i^2)^2} - \log \left(\frac{\rho}{R_i} \right) \log \left(\frac{R}{R_i^{2m+2}} \right) / \log R_i \right\} \quad (5)$$

where $R_i = r_i/r_o$: ratio of inner and outer radii of the bearing.

The solution of Eq. (4) can be given as follows by using Green function, Eq. (5),

$$P^2 = \sum_{j=1}^k q_j G \left(R, \theta; R_a, \frac{2\pi(j-1)}{k} \right) + 1 + \iint \left[\frac{3}{H} \left\{ \frac{\partial H}{\partial R} \frac{\partial P^2}{\partial R} + \frac{1}{R^2} \frac{\partial H}{\partial \theta} \frac{\partial P^2}{\partial \theta} \right\} - \frac{\Lambda}{H^3} \frac{\partial(PH)}{\partial \theta} \right]_{R=\rho, \theta=\varphi} G \rho d\rho d\varphi \quad (6)$$

q_j is the strength of the source at each supply hole, and can be determined by the flow continuity condition at the supply holes as

$$H_j^2 \cdot (q_j / 2\pi) = \Gamma P_s \phi(P_{mj}) \quad (7)$$

where H_j : dimensionless bearing clearance at each

Fig. 1 Schematic diagram of bearing

supply hole position, $P_s (= p_s/p_a)$: dimensionless supply pressure, and Γ is feeding parameter defined by

$$\Gamma = 24\mu C_D r_s \sqrt{RT} / h_0^2 p_a \quad (8)$$

in which C_D : discharge coefficient, R : gas constant, and T : temperature. ϕ is a function for orifice flow used currently in the theoretical analysis of an externally pressurized gas bearing, and $P_{mj} (= p_{mj}/p_a)$ is the outlet pressure of each supply hole.

When one of the supply holes has a size deviation in its diameter so as to be $2r_s'$, we define the parameter for hole size accuracy as

$$\varepsilon_r = (r_s' - r_s) / r_s$$

Then, q_s for the particular supply hole should be determined by replacing Γ in Eq. (7) by the following Γ' ,

$$\Gamma' = \Gamma(1 + \varepsilon_r) \quad (9)$$

In the later calculations, we assume that only one of the supply holes has a hole size deviation. The angular coordinate θ will be measured from the position of this supply hole.

Iteration procedure is necessary for the final solution, because Eq. (6) contains the pressure P^2 also in the double integral term in its right hand side. This solution, however, gives a pressure distribution considering an exact bearing gap distribution, or in other words, non-linearity with respect to perpendicularity parameter ε is exactly taken into account in the solution. In this sense, the solution by Eq. (6) will be named "non-linear solution".

As the effect of parameter ε cannot be explicitly evaluated by this solution, the following "perturbation solution" is introduced in addition. Now, we expand the pressure P^2 in Eq. (4) in a power series with respect to ε and Λ as

$$\begin{aligned} P^2 \equiv Q &= Q_0 + \varepsilon Q_{10} + \varepsilon^2 Q_{20} + \cdots \\ &+ \Lambda \{ Q_{01} + \varepsilon Q_{11} + \varepsilon^2 Q_{21} + \cdots \} \\ &+ \cdots \end{aligned} \quad (10)$$

Substituting Eq. (10) into Eq. (4) and rearranging it, we can obtain the fundamental equations for each order of ε and Λ . For example, the governing equations for Q_0 , Q_{10} and Q_{20} are obtained as follows.

$$\begin{aligned} \varepsilon^0 : \nabla^2 Q_0 &= 0 \\ \varepsilon^1 : \nabla^2 Q_{10} &= 3 \left\{ -\cos(\theta - \alpha) \frac{\partial Q_0}{\partial R} \right. \\ &\quad \left. + \frac{1}{R} \sin(\theta - \alpha) \frac{\partial Q_0}{\partial \theta} \right\} \end{aligned}$$

*1 Note that the value that $\varepsilon=0.3$ may be considerably large compared with the one encountered in the actual bearing. We use this value to show the theoretical tendency more clearly. A bearing with only one supply hole ($k=1$) is also unreal, but we employ it as a typical theoretical model to show the bearing characteristics.

$$\begin{aligned} \varepsilon^2 : \nabla^2 Q_{20} &= 3 \left\{ R \cos^2(\theta - \alpha) \frac{\partial^2 Q_0}{\partial R^2} \right. \\ &\quad - \sin(\theta - \alpha) \cos(\theta - \alpha) \frac{\partial^2 Q_0}{\partial \theta^2} \\ &\quad \left. - \cos(\theta - \alpha) \frac{\partial^2 Q_{10}}{\partial R^2} + \frac{\sin(\theta - \alpha)}{R} \frac{\partial Q_{10}}{\partial \theta} \right\} \end{aligned} \quad (11)$$

The boundary condition at a supply hole can be obtained similarly for each order of pressure distribution by substituting Eq. (10) into Eq. (7).

The solutions of Eq. (11) can be given by the same expression as Eq. (6). The right hand sides of Eq. (11) are definite, so the solutions by the perturbation method can be obtained without the iteration process.

The load-carrying capacity of the bearing, w , can be obtained by integrating the pressure distribution over the bearing area. Dimensionless load capacity, W , is defined as

$$W = \frac{w}{\pi r_0^2 p_a} = \int_0^{2\pi} \int_{R_i}^1 (P-1) R dR d\theta \quad (12)$$

It should be noted that the load capacity W is a function of α , the relative angle of the direction of maximum perpendicularity deviation from the reference supply hole.

3. Theoretical Results

We consider typical bearing geometry as $R_i = r_i/r_0 = 1/3$, $R_a = r_a/r_0 = 2/3$ and $R_s = r_s/r_0 = 0.01$ for theoretical calculations. These dimensions are assumed throughout theoretical calculations in this paper.

Figure 2 shows examples of theoretical load capacity (non-linear solution) W versus the shaft rotation angle, α , for the bearings with $k=1$ and $k=3$, under the conditions that $P_{sg} \equiv (p_s - p_a)/p_a = 3.0$, $\Gamma = 1.0$, $\varepsilon = 0.3^{*1}$ and $\varepsilon_r = 0$.

Figure 2 clearly shows that load capacity fluctuation can take place along with the shaft rotation for both cases of $k=1$ and $k=3$, which may cause an axial

Fig. 2 Load capacity fluctuation

excitation or run-out of the shaft. The load capacity fluctuation has a period of $360^\circ/k$, and it is closely sinusoidal if $\varepsilon_r=0$, which is the case of Fig. 2. In the later, the full amplitude ΔW of load capacity fluctuation will be used as an evaluating parameter of bearing characteristics.

As stated above, the perturbation solution is also introduced to make clear the effect of form deviation parameter ε . The load capacity W can be expressed by the following form according to the perturbation solution,

$$\begin{aligned} W = & W_0 + \varepsilon a_{11} \cos \alpha \\ & + \varepsilon^2 (a_{20} + a_{22} \cos 2\alpha) \\ & + \varepsilon^3 (a_{31} \cos \alpha + a_{33} \cos 3\alpha) \\ & + \dots \end{aligned} \quad (13)$$

The numerical results for the perturbed terms of load capacity including Λ depend on the number of meshes in the circumferential direction in the numerical calculation. By evaluating the dependency, we reach the conclusion that the value of Λ does not affect the bearing load capacity or load capacity fluctuation. In the following calculations, then, we will assume that $\Lambda=0$.

Figure 3 shows the calculated results of W_0 , the load capacity of a bearing without machining errors, while Fig. 4 shows lower order coefficients of load capacity, a_{11} , a_{20} and a_{22} , for a typical bearing with only one supply hole ($k=1$). In Fig. 4, solid and broken curves correspond to positive and negative values of coefficients a_{ij} , respectively.

Summarizing the calculated results of coefficients a_{ij} , we can obtain the following:

(1) For the case with $k=1$ as shown in Fig. 4, a_{11} has the largest order of magnitude compared with other coefficients. As a_{11} corresponds to the term with the lowest order of ε , characteristics of load capacity

Fig. 3 Average load capacity

Fig. 4 Coefficients of load capacity fluctuation ($k=1$)

fluctuation are predominated by the term $\varepsilon a_{11} \cos \alpha$, if the perpendicularity error ε is not so much.

(2) The coefficients vary independently of each other for a change of Γ value. We cannot find a deductive relationship between the changes of coefficients.

(3) The coefficient a_{ij} (especially a_{11} for the case of $k=1$) may be either positive or negative; hence, at $\alpha=0$, either maximum or minimum load capacity would correspondingly result.

(4) For the case that $P_{sg}=3.0$, for example, the choking flow condition is achieved at the supply hole when $\Gamma=0.337$. The higher derivatives of the flow function ϕ become discontinuous there. All the a_{ij} coefficients except for a_{11} are calculated by using these higher derivatives of ϕ in the course of theoretical treatment, so that they also have a discontinuous point at $\Gamma=0.337$ in the figure. This may be a disadvantage for the application of a perturbation solution.

(5) For the case of $k=1$, the perturbation solution calculated with up to the 3-rd order of ε is very close to that by the non-linear solution, when ε is less than 0.3 or so. The relative difference between them is less than 0.2% at most for an example where $P_{sg}=3.0$ and $\varepsilon=0.3$.

Perturbation coefficients of load capacity are calculated for other than the case of $k=1$. Figure 5 is an example of the results for $k=3$. For this case, the coefficients a_{11} , a_{22} and a_{31} vanish to zero. In general, if $\varepsilon_r=0$, the lower coefficients, except a_{20} , become zero, when $k>1$. The lowest term for load capacity fluctuation (i. e. the term varying with α) may be $a_{kk} \varepsilon^k \cos k\alpha$, and all the terms lower than this will vanish. Thus, we can conclude that, if $\varepsilon_r=0$, the load capacity fluctuation decreases rapidly with an order of ε^k , when the number of supply holes increases.

Fig. 5 Coefficients of load capacity fluctuation ($k=3$)

We will now discuss the effect of size deviation in diameter of one of the supply holes (i. e. $\varepsilon_r \neq 0$). For this case, the characteristics of bearing load capacity may be inferred by the superposition of the solutions for $\varepsilon_r=0$ and for $k=1$. Thus, we can easily suppose that the coefficients a_{ij} including a_{11} cannot be zero for this case.

Figure 6 shows the calculated results of load capacity coefficients for $k=3$ and $\varepsilon_r=0.1$. Comparing Fig. 6 ($\varepsilon_r=0.1$) with Fig. 5 ($\varepsilon_r=0$), we can point out the following.

(1) Though the coefficient a_{11} is not zero for the case of $\varepsilon_r \neq 0$, the magnitude of its value is not so large. For the calculated conditions of Fig. 6, a_{11} is significantly less than a_{33} .

(2) The coefficient a_{33} is not so much affected by ε_r for its values in Figs. 5 and 6 are almost the same compared with each other.

(3) The coefficient a_{20} is affected by ε_r especially in the range of a large Γ value. These findings lead to the conclusion that the effect of ε_r can be mainly seen in terms lower than a_{33} , i. e. a_{11} , a_{20} , and a_{22} .

We next discuss the full amplitude of load capacity fluctuation, ΔW , obtained from the non-linear and the perturbation solutions of the bearing load capacity.

Figure 7 shows the calculated results of ΔW with P_{sg} as a parameter for the case that $k=1$ and $\varepsilon=0.3$. Figure 7 shows the non-linear solution. The perturbation solution coincides very well with this, if we take into account the perturbation terms up to the third order of ε .

In Fig. 7, partly because we assume that $\varepsilon=0.3$, which may be a rather large value for the perpendicularity error of a rotor, the amplitude ΔW is

Fig. 6 Coefficients of load capacity fluctuation ($k=3$, $\varepsilon_r=0.1$)

Fig. 7 Amplitude of load capacity fluctuation ($k=1$)

significantly large when $k=1$. The relative magnitude of it to W_0 in Fig. 3 becomes more than 50% at most. As shown later in Figs. 8 and 9, an increase of the number of supply holes reduces the load capacity fluctuation. The relative magnitude is about several percent for the case of $k=3$ in Fig. 8, for example.

The effect of ϵ_r to ΔW is shown in Fig. 8 for the case of $k=3$. As a matter of course, the magnitude of load capacity fluctuation ΔW increases with an increase of ϵ_r . It should be noted that the values of ϵ_r employed in the figure, which is intended to show clearly the general characteristics of load capacity fluctuation, may be unreally large compared with the practical hole size deviation. In Fig. 8, the perturbation solution is also shown for the purpose of reference for the case of $\epsilon_r=0.1$. This solution coincides comparatively well with the non-linear solution, though they show some deviation near the critical value of Γ .

The effect of the number of supply holes, k , to the load capacity fluctuation is shown in Fig. 9 for $\epsilon_r=0.1$. The figure shows that the fluctuation ΔW decreases with an increase of k also for a bearing with hole size deviation. It can be concluded that the so-called averaging effect may become possible to evaluate quantitatively by a theoretical analysis of this paper.

4. Conclusions

For the first step to discuss the quantitative relations between run-out characteristics of an externally pressurized gas bearing and machining accuracy of the bearing and the rotor, load capacity fluctuation due to machining errors is analyzed theoretically in this paper for an externally pressurized gas-lubricated thrust collar bearing. The perpendicularity error of the shaft end surface and the size deviation of supply

Fig. 8 Amplitude of load capacity fluctuation ($k=3$)

Fig. 9 Amplitude of load capacity fluctuation (Effect of number of supply holes)

holes are taken into account as the governing factors to run-out characteristics. The obtained results can be summarized as follows.

(1) Point source solution of Reynolds equation can be obtained by using Green function for the bearing geometry. The load capacity fluctuation characteristics are discussed through two kinds of formulations of the solution; one is the non-linear solution, which takes into exact consideration the gap height distribution, and the other is the perturbation solution, which can show more clearly the tendency of the solution for an increase of ϵ .

(2) For the case without size deviation of supply holes, the magnitude of load capacity fluctuation is of the order of ϵ^* , so that an increase of the number of supply holes reduces significantly the load capacity fluctuation.

(3) For the case with hole size deviation, the characteristics of load capacity fluctuation may be estimated by a superposition of the solution with only perpendicularity error of the rotor, and the one with hole size deviation effect. An increase of the number of supply holes reduces considerably the load capacity fluctuation for this case as well. This can be considered as a quantitative evaluation of the averaging effect in an externally pressurized gas bearing.

We will discuss in the next report the bearing stiffness and damping coefficient for a bearing with form deviations, and the run-out characteristics will further be investigated.

It is noted that the numerical calculations were

executed by FACOM VP-200 at the Kyoto University Data Processing Center.

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