High-Speed Forming of Metal Sheets by Electromagnetic Force*

Nobuo TAKATSU**, Masana KATO**,
Keijin SATO*** and Toshimi TOBE****

A numerical method to simulate the high-speed free forming of a clamped circular disk in an electromagnetic forming system using a flat spiral coil is presented. The method combines field penetration into the disk and dynamic elasto-plastic deformation of the disk. The spiral coil is approximated by coaxial circular loops carrying the discharge current from a capacitor bank. The penetration of the magnetic field of the coil into the disk is formulated as a boundary-value problem for the diffusion equation. Magnetic field, eddy currents and electromagnetic force density in the disk are calculated. An equation of motion for the disk loaded by the magnetic force is combined with the field equation as well as an equivalent circuit equation. Plane stress condition is assumed. Strain-rate effect on the work-hardening law of the disk material is taken into account. An experiment on the free bulging of annealed aluminum disks is also presented, showing fairly good agreement with the numerical solution.

Key Words: Forming, Electromagnetic Forming, Electromagnetic Force, Bulging, Sheet Metal, High-speed Photography, Eddy Current, Magnetic Field, Mutual Inductance.

1. Introduction

An electromagnetic forming system is essentially a mutual induction system composed of a work coil and a workpiece. Any change in geometry of the workpiece due to magnetic force alters the mutual inductance significantly, which in turn changes the coil current and resulting magnetic force. Thus, a numerical method which predicts the forming process (must consider) the interaction between the magnetic force and the deformation of the workpiece. Such a numerical prediction is important in designing an effective electromagnetic forming system. Several papers have been reported from this point of view but most of them have involved the forming of cylindrical workpieces by solenoid forming coils⁽¹⁾⁻⁽³⁾. Few reports

have been presented on the forming of sheet metals by flat coils and especially, on the analysis of the process.

A numerical method to describe the free-bulging process of thin metal sheets by a spiral flat coil is developed in this paper. A model of the system is shown schematically in Fig. 1. A circular disk is fixed at its periphery on a work coil. The coil is connected to a capacitor bank through a discharge switch. The analysis involves circuit analysis, the generation of

Fig. 1 A model of an electromagnetic forming system

* Received 22nd June, 1987. Paper No. 86-0790A, 86-1234A, 86-1344B

** Faculty of Engineering, Tohoku University, Aramaki-Aoba, Sendai, 980, Japan

*** Faculty of Engineering, Utsunomiya University, Ishii-cho 2753, Utsunomiya, 321, Japan

**** Tohoku Institute of Technology, Yagiyama-kasumi-cho, Sendai, 982, Japan

electromagnetic force and the dynamic bulging of the disk loaded by the force.

Circuit analysis for an electromagnetic forming system using a flat coil is the same as that using a solenoid coil but an evaluation of the mutual inductance has not yet been made. Several studies have been reported on the analysis of the magnetic field by flat coils. Moon et al.⁽⁴⁾ used a flat coil as a generator of stress waves along a semi-infinite bar, but the analysis was made on a one-dimensional model. Rajotte et al.⁽⁵⁾ reported on a high-speed circuit breaker which has a contactor driven by magnetic pulse. They obtained the steady-state solution but not a transient one.

In this paper, the penetration of the magnetic field applied by a coil current into the disk is formulated using the two-dimensional diffusion equation to obtain the eddy current and magnetic force induced in the disk. An equation of motion for an elasto-plastic disk loaded by magnetic pressure is combined with the field equation. Resulting simultaneous equations are solved numerically. An experiment is also presented to show the dynamic behavior of the disk and to examine the validity of the numerical method.

2. Electrical Equivalent Circuit

An electromagnetic forming system is a discharge circuit which consists of a capacitor bank C_0 , connecting lines with inductance L_c and resistance R_c , as well as a forming coil (L_1, R_1) coupled with the disk (L_2, R_2). Equations (1) and (2) describes the equivalent electrical circuit of the system;

$$(L_1 + L_c) \frac{di_1}{dt} + \frac{d}{dt}(Mi_2) + (R_1 + R_c)i_1 + \frac{1}{C_0} \int i_1 dt = 0 \quad (1)$$

$$\frac{d}{dt}(L_2 i_2) + \frac{d}{dt}(Mi_1) + R_2 i_2 = 0 \quad (2)$$

where i_1 is the coil current, M is the mutual inductance between the coil and the disk and i_2 is the sum of eddy currents induced in the disk. The initial conditions of Eqs. (1) and (2) are

$$i_1 = 0, (L_1 + L_c) \frac{di_1}{dt} = V_0, i_2 = 0 \quad (3)$$

with V_0 the initial voltage of the bank. The term Mi_2 in Eq. (1) is not a constant. It changes significantly with the deformation of the disk and, to a lesser degree, with the penetration of the field into the conductor. This term is determined numerically on the basis of magnetic energy W , and is discussed later in Section 3.3.

Hereafter, Eq. (2) will not be considered since the disk is treated as a distributed parameter system as will be shown in next chapter.

3. Magnetic Force Induced in a Circular Metal Sheet Generated by a Spiral Flat Coil

3.1 Analytical model and assumptions

The time-varying magnetic field generated by the coil carrying the discharge current from the capacitor bank penetrates into the disk and induces eddy currents inside the disk. Hence, magnetic force is produced. The penetration of the magnetic field is formulated based on the quasi-stationary Maxwell equations in which the following assumptions are made.

(i) The spiral coil is approximated with coaxial loop currents in a plane. Thus the problem is axisymmetric.

(ii) The disk is assumed to be nonmagnetic.

(iii) The skin depth is sufficiently small, compared to the thickness of the disk, and the field is shielded completely by the disk.

Details of the geometry of the coil and the disk are shown in Fig. 2. A set of N_i loop currents are fixed in a plane representing the coil. Only the i -th loop of radius a_i is shown in the figure. The thickness and the diameter of the disk are h and $2r_t$, respectively. The disk is mounted above the coil at a distance of d_g .

3.2 Field penetration and boundary conditions

The fundamental equations of the magnetic field within a conducting medium are as follows:

$$\nabla \cdot \mathbf{J}_f = 0 \quad (4)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f \quad (5)$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \quad (6)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (7)$$

$$\mathbf{B} = \mu \mathbf{H} \quad (8)$$

$$\mathbf{J}_f = \sigma_w (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (9)$$

where $\mathbf{J}_f$: current density, $\mathbf{H}$: magnetic intensity, $\mathbf{E}$: electric field, $\mathbf{B}$: magnetic flux density, μ : permeability, σ_w : conductivity, $\mathbf{v}$: velocity of the medium and t : time. By the assumption (ii), $\mu = \mu_0$.

Force density $\mathbf{F}$ is represented by

$$\mathbf{F} = \mathbf{J}_f \times \mathbf{B}. \quad (10)$$

Fig. 2 Details of the geometry of a coil and a disk

Combining the above equations, we obtain⁽⁶⁾

$$-\frac{1}{\mu_0 \sigma_w} \nabla^2 \mathbf{B} + \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \quad (11)$$

A cylindrical coordinate system is introduced for the present problem of a circular disk, as shown in Fig. 2. The magnetic field possesses the radial component B_r and axial one B_z since coil current is circumferential and given by

$$\left. \begin{aligned} & -\frac{1}{\mu_0 \sigma_w} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{1}{r^2} \right) B_r + \frac{\partial B_r}{\partial t} \\ & = -\frac{\partial}{\partial z} (v_z B_r - v_r B_z) \\ & -\frac{1}{\mu_0 \sigma_w} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) B_z + \frac{\partial B_z}{\partial t} \\ & = \frac{1}{r} \frac{\partial}{\partial r} \{ r (v_z B_r - v_r B_z) \}. \end{aligned} \right\} \quad (12)$$

Thus eddy current J_r has a circumferential component and force density has radial and axial components as

$$J_{r\theta} = \frac{1}{\mu_0} \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right) \quad (13)$$

$$F_r = J_{r\theta} \cdot B_z, \quad F_z = -J_{r\theta} \cdot B_r. \quad (14)$$

Axial component F_z is considered to contribute mainly to the bulging of the disk.

Boundary conditions for Eq.(12) are given based on the continuity of the magnetic flux density at the boundaries. At the bottom surface of the disk, the field is a sum of the coil field $\mathbf{B}_0$ and the field $\mathbf{B}_1$ generated by eddy currents and is given by

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1 \text{ at } z = d_g + w(t) \quad (15)$$

with $w(t)$ the vertical deflection of the disk. $\mathbf{B}_0$ and $\mathbf{B}_1$ can be determined using the closed solution for a loop current if coil current i_1 and eddy current density $J_{r\theta}$ are known. At the top surface, the field is zero according to the assumption (iii);

$$\mathbf{B} = 0 \text{ at } z = d_g + w(t) + h. \quad (16)$$

3.3 Mutual inductance

Consider a small loop current of sectional area $d\rho d\zeta$ within the disk, as shown in Fig. 2, where ζ is the depth from the bottom surface of the disk. Let $M_{i\rho}$ be the mutual inductance between the loop current and one of the primary loop currents. Stored field energy between two loops can then be given by

$$dW_{i\rho} = M_{i\rho} \cdot i_1 \cdot J_{r\theta}(\rho, \zeta) d\rho d\zeta. \quad (17)$$

Total field energy W is obtained by integrating $dW_{i\rho}$ for an entire cross section of the disk and summing for all primary loops. On the other hand, W can be defined by

$$W = M i_1 \cdot i_2. \quad (18)$$

The total eddy current i_2 is defined by

$$i_2 = \int_0^h \int_0^{r_c} J_{r\theta} d\rho d\zeta. \quad (19)$$

After eddy current $J_{r\theta}$ is determined, the term $M i_2$ in Eq.(1) can be evaluated from Eq.(18). Mutual inductance M can also be determined using Eq.(19).

4. Free Bulging of a Clamped Disk Loaded by Magnetic Pressure

To predict the dynamic behavior of the disk loaded by impulsive magnetic force, a simulation code for the dynamic bulging of a circular clamped disk considering the strain-rate sensitivity of the material has been developed. The code was originally applied to investigate the dynamic bulging under impulsive pressure generated by an underwater wire explosion⁽⁷⁾.

4.1 Assumptions

A disk bulging under magnetic force is illustrated in Fig. 3. Axisymmetrical deformation is assumed and a cylindrical coordinate system is introduced, with its origin at the center of the disk. Bending moments are ignored and the plane stress condition is assumed since the thickness of the disk is very small compared with its diameter. Magnetic pressure is then introduced by integrating the magnetic force density over the thickness of disk. Material of the disk is assumed to obey the Prandtl-Reuss stress-strain relation. Strain-rate effect is considered in the work-hardening law.

4.2 Equations of motion

Equations of motion of the disk in the radial (r) and axial (z) directions are

$$\left. \begin{aligned} \rho_0 h_0 r_0 \frac{\partial^2 u}{\partial t^2} &= h \frac{\partial s}{\partial r_0} p_r + \frac{\partial}{\partial r_0} \left(\sigma_s h r \frac{\partial r}{\partial s} \right) \\ &- \sigma_\theta h \frac{\partial s}{\partial r_0} \\ \rho_0 h_0 r_0 \frac{\partial^2 w}{\partial t^2} &= h \frac{\partial s}{\partial r_0} p_z + \frac{\partial}{\partial r_0} \left(\sigma_s h r \frac{\partial w}{\partial s} \right) \end{aligned} \right\} \quad (20)$$

where s is meridian distance measured from the center of the disk, u and w are displacements in the r and

Fig. 3 Analytical model of a clamped circular thin disk loaded by electromagnetic pressure

z directions, h is thickness, σ is true stress and ρ_0 is density. Subscripts s , θ and h denote the meridian, circumferential and thickness directions, respectively, and o denotes the initial value. p_r and p_z are magnetic pressures defined by

$$p_r = \int_0^h F_r \cdot d\zeta, \quad p_z = \int_0^h F_z \cdot d\zeta \quad (21)$$

4.3 Work-hardening properties of the material

Strain-rate effect on the properties of the material are taken into account. The following work-hardening law including logarithmic strain-rate dependency is used.

$$\sigma_{ef} = \begin{cases} \sigma_0^* (\epsilon_{ef}^p)^n & (0 < \dot{\epsilon}_{ef}^p \leq \dot{\epsilon}_{ef,o}^p) \\ \sigma_0^* (\epsilon_{ef}^p)^n + \sigma_1^* (\epsilon_{ef}^p)^m \log \frac{\dot{\epsilon}_{ef}^p}{\dot{\epsilon}_{ef,o}^p} & \dots\dots (\dot{\epsilon}_{ef}^p > \dot{\epsilon}_{ef,o}^p) \end{cases} \quad (22)$$

where σ_{ef} is effective stress, ϵ_{ef}^p is effective plastic strain, $\dot{\epsilon}_{ef}^p$ is effective plastic strain rate and σ_0^* , n , σ_1^* , m , $\dot{\epsilon}_{ef,o}^p$ are the material constants.

5. Numerical Results

The simultaneous equations for the discharge circuit, the field penetration and the motion of the disk were solved numerically. Numerical procedures are as follows: At each small time interval, coil current is calculated using Eq. (1) and the coil field is obtained. The magnetic field within the disk is determined by

Fig. 4 Spiral flat work coil

solving Eq. (12) so that the boundary conditions, Eqs. (15) and (16), are satisfied. The extent of the disk in the z -direction was assumed to be three times the skin depth to satisfy the boundary condition at the top surface of the disk. Eddy currents and magnetic force density are then calculated using Eqs. (13) and (14). Magnetic pressure is evaluated and the velocity and displacement of the disk are obtained by integrating the equation of motion (20) as well as the stresses and strains.

Conditions identical to those used in the experiment were used in the calculation. The work coil is illustrated schematically in Fig. 4. It is a "pancake" coil consisting of a single layer, 5 turn windings. Its effective diameter is 80 mm. Workpieces are circular plates of annealed aluminum alloy JIS A 1050 of 110 mm diameter. Other specifications of the system and work conditions are shown in Table 1.

5.1 Preliminary calculation

In order to examine the validity of the electro-magnetic field analysis, a preliminary calculation was made when the disk is at rest, that is, $v=0$ in Eq. (11). Bank voltage is 2 kV, initial thickness is 3.0 mm and gap distance is 2.9 mm in this case.

Figure 5 shows the magnetic field distribution at the gap between the coil and the disk. The solid lines show the calculated results. When a disk is mounted on the coil, (b), the radial component of the magnetic field increases significantly. The plots in the figures represent the experimental values obtained by search coils, showing fairly good agreement with the calcu-

Table 1 System parameters and working conditions

Coil (#050)	No. of windings N_t	5
	Radius of max. ring	32 mm
	Pitch p_c	5.5 mm
	Self-inductance L_1	0.86 μ H
Circuit conditions	Capacity of condenser bank C_0	40 μ F
	Charge voltage V_0	6.0 kV
	Circuit inductance	2.0 μ H
	Circuit resistance	25.5 m Ω
Workpiece	Material	Aluminum (annealed)
	Thickness h_0	0.5 mm
	Diameter $2r_t$	ϕ 110 mm
	" (bulged) $2r_b$	ϕ 80 mm
	Conductivity σ_w	36 MS/m
	Permeability μ_0	$4\pi \times 10^{-7}$ H/m
	Density ρ_0	2.75×10^3 kg/m ³
	Young's modulus	80.7 GPa
	Constant σ_0^*	118 MPa
	" n	0.27
	" σ_1^*	15.7 MPa
	" m	0.54
	" $\dot{\epsilon}_{ef,o}^p$	10^{-3}
	Gap distance d_g	1.6 mm

lated results. Figure 6 shows the distribution of the magnetic force density inside the disk, indicating that magnetic force is limited near the surface.

5.2 Forming process

A simulation of the bulging process was attempted according to the condition shown in Table 1. Material constants σ_1^* , m and $\dot{\epsilon}_{ef,o}^p$ representing strain-

rate sensitivity were determined on the basis of Lindholm's experiment⁽⁷⁾. Figure 7 shows coil current and total magnetic force, which is defined by the integral of F_z over the whole disk volume, as functions of time. Figure 8 shows the radial distribution of magnetic pressure p_z at $t=15.4\mu s$ when coil current reaches its maximum value. It is evident that the

Fig. 5 Radial distribution of magnetic field in the air gap. $E_0 = C_0 V_0^2/2$.

Fig. 6 Distribution of magnetic force density inside the disk

Fig. 7 Calculated coil current (a) and total magnetic force (b)

Fig. 8 Radial distribution of the magnetic pressure $t = 15.4 \mu s$

Fig. 9 Profiles of the disk during the deformation (Calculated)

Fig. 10 High-speed photographs of a disk during deformation

Fig. 11 Profiles of the disk during the deformation (Experiment)

annular part of the disk is loaded heavily.

Profiles of the bulging disk are shown in Fig. 9, which demonstrate the characteristics of the process clearly. The initial stage of the deformation is affected strongly by the radial distribution of the magnetic pressure generated by the flat coil. The annular part is accelerated initially, and the kinetic energy promotes the bulging of the central part.

6. Experiment

An experiment on free bulging was made to examine the numerical method. The work conditions have already been shown in Table 1.

6.1 Experimental apparatus

High-speed photography was used for the measurement of the bulging process of the disk. A signal from a current coil triggers a delay circuit, which in turn triggers a Xenon tube after a preset time delay. The duration of the light pulse is $3 \mu\text{s}$. Time resolution of the system is $1 \mu\text{s}$, which is fine enough for the present experiment. Conventional film (ASA 200) was used and its effective speed was raised to ASA 1 600 in the developing process. Repeating many shots at different time delays, the bulging process was obtained.

6.2 Forming process

Typical photos are shown in Fig. 10. The deformation is recognized $19 \mu\text{s}$ after discharge, which

Fig. 12 Deflection of the disk at specified radial locations as a function of time.

begins at the annular part of the disk. It took about $250 \mu\text{s}$ to accomplish the whole deformation process.

A profile of the specimen during the bulging was then determined using two measuring scales which were simultaneously taken into the picture. Figure 11 shows the profiles of the disk. The dynamic behavior is the same as predicted by the calculation. Deflections at specified radial points are shown in Fig. 12 as functions of time after discharge. Circular and triangular marks are the experimental values. Solid lines are the calculated results obtained in section 5.2, which agree well with the experimental values. The dashed lines represent the calculation, ignoring the strain-rate effect. It is obvious that the calculation overestimates the final deflection at the center of the disk by 23%, in this case, if the strain-rate effect is ignored.

7. Conclusions

The numerical method developed in this paper can predict the bulging process of a clamped circular disk loaded by electromagnetic pressure generated by a flat spiral coil. The observed motion of the disk can be explained by the nonuniformity of the radial distribution of magnetic pressure.

References

- (1) Jansen, H., Some Measurements of the Expansion of a Metallic Cylinder with Electromagnetic Pulses, IEEE Trans., Vol. IGA 4, No. 4 (1968), p. 428.
 - (2) Jablonski, J. and Winkler, R., Analysis of the Electromagnetic Forming Process, Int. J. Mech. Sci., Vol. 20, No. 5 (1978), p. 315.
 - (3) Negishi, H., Murata, M., Suzuki, H. and Maeda, T., An Analysis of Forming Process for Tube Expansion, J. Jpn. Soc. for Technol. of Plas., (in Japanese), Vol. 21, No. 254 (1980), p. 642.
 - (4) Moon, F. C. and Chattopadhyay S., Magnetically induced Stress Waves in a Conducting Solid, Trans. ASME, J. Appl. Mech., Vol. 41, No. 3 (1974), p. 641.
 - (5) Rajotte, R. J. and Drouet, M. G., Experimental Analysis of a Fast-Acting Circuit Breaker Mechanism, IEEE Trans., Vol. PAS 94, No. 89 (1975), p. 89.
 - (6) Woodson, H. H. and Melcher, J. R., Electromechanical Dynamics, Part II, John Wiley & Sons, Inc.
 - (7) Tobe, T., Kato, M. and Obara, H., Metal Forming by Underwater Wire Explosion (1), Bull. JSME, Vol. 22, No. 164 (1979), p. 271.
-