Study on an Oil Damper with a Variable Damping Mechanism*

(Dynamic Characteristics of a Damper with a Piston Having a Fan-Shaped Orifice)

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This paper proposes an oil damper with a variable damping mechanism. The piston of the damper has a fan-shaped orifice as a flow channel for the oil. The magnitude of damping can be regulated continuously by varying the area of the orifice. Another merit of this damper is that its damping can be regulated as viscous, enabling a theoretical approach to be used in its study. We have analyzed the oil damper by assuming the flow of oil to be laminar and oscillatory. However, it is too difficult to obtain an analytical solution from the basic equation, so the analysis was accomplished numerically using the finite difference method. It was confirmed that the theoretical results agree well with the experimental ones.

Key Words: Vibration, Vibration Isolation, Oil Damper, Fluid Vibration, Variable Damping, Fan-Shaped Orifice, Finite Difference Method

1. Introduction

We name those kinds of oil dampers which have a variable damping mechanism 'variable oil dampers'. The merits of variable oil dampers are as follows.

(1) The dependence of damping on temperature, which is a demerit of conventional oil dampers, can be corrected completely.

(2) The variable oil damper contributes much to improve the performance of the vibration isolator⁽¹⁾.

In order to best utilize these characteristics, it is desirable that the variable oil dampers satisfy the following conditions.

(1) The dynamic characteristics of the oil damper can be analyzed theoretically.

(2) The magnitude of the damping coefficient can be regulated continuously and over a wide range.

(3) The oil damper provides viscous damping.

In the first report⁽²⁾ of this work, we proposed a variable oil damper whose piston has several circular

orifices. This oil damper satisfied conditions (1) and (3), but did not satisfy condition (2).

In the present report, we propose another type of variable oil damper whose piston has a fan-shaped orifice, as illustrated in Fig. 1. The damping coefficient of the oil damper is regulated by varying the opening

Fig. 1 Structure of the piston having a fan-shaped orifice

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angle of the orifice; this is performed by rotation of the inner piston. This oil damper will satisfy all of the conditions mentioned above, if the piston does not make contact with the inner surface of the cylinder. We assume the piston and the cylinder to have a common center, so the clearance between them forms a concentric annular circle. Therefore, this type of oil damper has two different channels of oil; the fan-shaped orifice and the annular channel.

If an oil damper has different shaped channels, then the damping characteristics of the oil damper are great influenced by the frequency of the piston⁽²⁾. The classic theory is useless for this type of oil damper, because it is based on the assumption that the oil flow

is steady, i. e., the magnitude of damping is independent of the frequency.

Assuming the oil flow to be unsteady and laminar, we have derived the fundamental equations of the oil damper, and solved them by means of the finite difference method. The finite difference method proposed here can be used to investigate other types of oil dampers.

2. Theoretical Analysis Based on Steady Flow Assumption

Although the theoretical analysis based on the steady flow assumption cannot precisely express the damping characteristics of the oil damper with a fan-

Table 1 Steady flow analysis of an oil damper having fan-shaped orifices and an annular channel

Equation of continuity	$Ac v_{crms} - A_p v_{prms} = 2\pi \int_{r_o}^{r_c} x v_0(x) dx + n \int_{-\theta_1}^{\theta_1} \int_{r_i}^{r_o} r v_1(r, \theta) dr d\theta$ [1.1]
Equations of motion	$\frac{dp}{dy} = \mu \left(\frac{d^2 v_0}{dx^2} + \frac{1}{x} \frac{dv_0}{dx} \right)$ [1.2], $\frac{dp}{dy} = \mu \left(\frac{\partial^2 v_1}{\partial r^2} + \frac{1}{r} \frac{\partial v_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_1}{\partial \theta^2} \right)$ [1.3]
Boundary conditions	$\left. \begin{aligned} v_0(r_p) &= v_{prms} \\ v_0(r_c) &= v_{crms} \end{aligned} \right\}$ [1.4], $\left. \begin{aligned} v_1(r_i, \theta) &= v_1(r_o, \theta) = v_{prms} \quad (-\theta_1 \leq \theta \leq \theta_1) \\ v_1(r, \pm \theta_1) &= v_{prms} \quad (r_i \leq r \leq r_o) \end{aligned} \right\}$ [1.5]
Solution	$v_0(x) = -U_r \left[\left(1 + R_p^2 + \frac{n\Omega}{1-R_p^2} \right) \ln \left(\frac{x}{r_c} \right) + 1 - \left(\frac{x}{r_c} \right)^2 \right] v_{rrms} + v_{crms}$ [1.6] $v_1(r, \theta) = 2U_r \left[\frac{(\cos^2 \theta_1 - \cos^2 \theta) (r/r_c)^2}{\cos 2\theta_1} + \frac{8}{\pi} \sum_{j=1}^{\infty} \frac{(-1)^j \cos m_j \theta}{(2j-1)(4-m_j^2)} \right.$ $\times \frac{(R_o^2 - R_i^2) (R_i/R_o)^{m_j} R_o^{-m_j} (r/r_c)^{m_j} - (R_o^2 (R_i/R_o)^{m_j} - R_i^2) R_i^{-m_j} (r/r_c)^{-m_j}}{1 - (R_i/R_o)^{2m_j}}$ $\left. \times v_{rrms} + v_{prms} \right]$ [1.7]
Pressure drag	$D_{ps2} = 4\pi\mu U_r \left[R_p^2 - \frac{n\Omega}{\pi} (R_o^2 - R_i^2) \right] v_{rrms}$ [1.8] $D_{ps1} = -4\pi\mu U_r v_{rrms}$ [1.9]
Friction drag	$D_{fs2} = 2\pi\mu U_r \left[1 - R_p^2 + \frac{n\Omega}{1-R_p^2} + \frac{2n\theta_1}{\pi} (R_o^2 - R_i^2) \right] v_{rrms}$ [1.10] $D_{fs1} = 2\pi\mu U_r \left[1 - R_p^2 - \frac{n\Omega}{1-R_p^2} \right] v_{rrms}$ [1.11]
Damping coefficient	$cs = 2\pi\mu U_r [1 + R_p^2 + n\Omega/(1-R_p^2)]$ [1.12]
Added mass	$mAs = mc U_r^2 \left[\left(1 + R_p^2 + \frac{n\Omega}{1-R_p^2} \right)^2 \{ R_p^2 (1 - \ln R_p) \ln R_p + \frac{1}{2} (1 - R_p^2) \} \right.$ $\left. - (1 + R_p^2 + \frac{n\Omega}{1-R_p^2}) \{ R_p^2 (2 - R_p^2) \ln R_p + \frac{1}{4} (1 - R_p^2) (3 - R_p^2) \} \right.$ $\left. + \frac{1}{3} (1 - R_p^2)^3 \right] + mc \frac{2n}{\pi} \int_{\theta_1}^{\theta_1} \int_{R_i}^{R_o} R V^2 dR d\theta$ [1.13]
$U_r = - \left[\left(1 + R_p^2 + \frac{n\Omega}{1-R_p^2} \right) \ln R_p + 1 - R_p^2 \right]^{-1}$ $\Omega = (R_o^4 - R_i^4) (\tan 2\theta_1 - 2\theta_1) / (2\pi)$ [1.14] $+ \frac{128\theta_1}{\pi^3} \sum_{j=1}^{\infty} \frac{(2-m_j) [R_o^2 - R_i^2] (R_i/R_o)^{m_j} + (2+m_j) [R_o^2 (R_i/R_o)^{m_j} - R_i^2]}{(2j-1)^2 (4-m_j^2) [1 - (R_i/R_o)^{2m_j}]}$ $m_j = (2j-1)\pi / (2\theta_1), (j=1, 2, \dots)$	
$R = r/r_c$ $R_p = r_p/r_c$ $R_i = r_i/r_c$ [1.15] $R_o = r_o/r_c$ $V = v_1/v_{prms}$	r_i, r_o : Inner and outer radii of fan-shaped channels n, θ_1 : Number and half angle of the channels r_p, l : Radius and length of piston r_c : Inner radius of cylinder ρ, μ, ν : Density, viscosity and kinematic viscosity of fluid $v_{rrms} = v_{prms} - v_{crms}$, $mc = \pi r_c^2 \rho l$ [1.16]
[1.17]	

shaped orifice, it is useful for obtaining the theoretical equation of the variable oil damper. Further, it can be used for a test of the accuracy of the numerical solution obtained from the unsteady flow assumption.

2.1 Fundamental equations

Table 1 shows the fundamental equations and their solution in the steady flow analysis of the variable oil damper. Each of the equations is given a bracketed number. We shall explain the analytical procedure by referring to the respective numbers.

A cross-sectional view of the oil damper is shown in Fig.[1.17] in Table 1. The piston has n numbers of fan-shaped orifices of the same size. Let the inner radius of the orifices be r_i , the outer radius r_o and a half of the opening angle θ_1 . The equations of motion of the fluid through the annular and the fan-shaped channels are represented by the cylindrical coordinates x - y and r - θ - y , respectively, where x and r refer to the distance from the origin located at the center of the piston. We assume that the piston and the cylinder move at constant velocities v_{prms} and v_{crms} , respectively, and that the oil flow through the channels is fully developed laminar flow. Letting the velocities through the annular and the fan-shaped channels be $v_0(x)$ and $v_1(r, \theta)$, respectively, the equations of motion of the fluid are expressed by differential equations [1.2] and [1.3]. Equations [1.4] and [1.5] are boundary conditions of the differential equations. The equation of continuity, [1.1], is derived from the condition that the volume of the incompressible fluid excluded by the piston and cylinder must be equal to that of the flow passing the channels.

2.2 Damping coefficient and added mass

The solutions of the equations of motion, [1.2] and [1.3], may be written in the form of the infinite series as shown in Eqs.[1.6] and [1.7] (see Refs.(3) and (4)). Note that we assume the particular integral of Eq.[1.3] to be

$$v_1(r, \theta) = (dp/dy)r^2(\cos^2\theta_1 - \cos^2\theta) / (2\mu \cos 2\theta_1) + v_{prms} \quad (1)$$

so Eqs.[1.6] and [1.7] cannot be calculated for an angle of $2\theta_1 = \pi/2$ rad.

As a result of the theoretical analysis, we obtain the pressure drags D_{ps2} and D_{ps1} and the friction drags D_{fs2} and D_{fs1} , as given by Eqs.[1.8]~[1.11], where the subscripts 2 and 1 refer to the drag acting on the piston and cylinder, respectively. Equation [1.13] contains a double integral which represents the kinetic energy of the fluid through the fan-shaped orifice; this term is integrated numerically.

3. Theoretical Analysis Based on Unsteady Flow Assumption

3.1 Fundamental equations and their transformation to nondimensional form

The left-hand column of Table 2 shows the fundamental equations in the unsteady flow analysis. They are composed of the equations of continuity, [2.1], the equations of motion, [2.2] and [2.3], boundary conditions [2.4] and [2.5], and initial condition [2.6]. They will be varied under the conditions that the piston vibrates at the velocity of $v_p \sin \omega t$ and the cylinder is at rest. We have solved the equations of motion by means of the finite difference method with initial

Table 2 Fundamental equations and their dimensionless form in unsteady flow analysis

	Fundamental equations	Dimensionless form of fundamental equations
Equation of continuity	$-[\pi r_p^2 - n\theta_1(r_o^2 - r_i^2)]v_p \sin \omega t$ $= 2\pi \int_{r_o}^{r_c} x v_0(x, t) dx$ $+ 2n \int_0^{\theta_1} \int_{r_i}^{r_o} r v_1(r, \theta, t) dr d\theta \quad (2.1)$	$-[R_p^2 - \frac{n\theta_1}{\pi}(R_o^2 - R_i^2)] \sin 2\pi T$ $= 2 \int_{R_o}^{R_c} X U dX + \frac{2n}{\pi} \int_0^{\theta_1} \int_{R_i}^{R_o} R V dR d\theta \quad (2.7)$
Equations of motion	$\frac{\partial v_0}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v_0}{\partial x^2} + \frac{1}{x} \frac{\partial v_0}{\partial x} \right) \quad (2.2)$ $\frac{\partial v_1}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v_1}{\partial r^2} + \frac{1}{r} \frac{\partial v_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_1}{\partial \theta^2} \right) \quad (2.3)$	$\frac{\eta_c^2}{2\pi} \frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial X^2} + \frac{1}{X} \frac{\partial U}{\partial X} - \eta_c^2 P \sin(2\pi T + \xi) \quad (2.8)$ $\frac{\eta_c^2}{2\pi} \frac{\partial V}{\partial T} = \frac{\partial^2 V}{\partial R^2} + \frac{1}{R} \frac{\partial V}{\partial R} + \frac{1}{R^2} \frac{\partial^2 V}{\partial \theta^2} - \eta_c^2 P \sin(2\pi T + \xi) \quad (2.9)$
Boundary conditions	$v_0(r_p, t) = v_p \sin \omega t, v_0(r_c, t) = 0 \quad (2.4)$ $v_1(r_i, \theta, t) = v_1(r_o, \theta, t) = v_1(r, \pm\theta_1, t) = v_p \sin \omega t \quad (2.5)$ $(-\theta_1 \leq \theta \leq \theta_1, r_i \leq r \leq r_o)$	$U(R_p, T) = \sin 2\pi T, U(1, T) = 0 \quad (2.10)$ $V(R_i, \theta, T) = V(R_o, \theta, T) = V(R, \pm\theta_1, T) = \sin 2\pi T \quad (2.11)$ $(-\theta_1 \leq \theta \leq \theta_1, R_i \leq R \leq R_o)$
Initial condition	$v_0(x, 0) = v_1(r, \theta, 0) = 0 \quad (2.6)$ $(r_p \leq x \leq r_c, r_i \leq r \leq r_o, -\theta_1 \leq \theta \leq \theta_1)$	$U(X, 0) = V(R, \theta, 0) = 0 \quad (2.12)$ $(R_o \leq X \leq 1, R_i \leq R \leq R_o, -\theta_1 \leq \theta \leq \theta_1)$

Table 3 Definition of dimensionless quantities

Definition of dimensionless quantities		
Distance	$X = x/r_c$, $R = r/r_c$	[3.1]
Radius	$R_p = r_p/r_c$, $R_i = r_i/r_c$, $R_o = r_o/r_c$	[3.2]
Velocity	$U = v_0/v_p$, $V = v_1/v_p$	[3.3]
Time	$T = \omega t/(2\pi)$	[3.4]
Frequency	$\eta_c = r_c \sqrt{\omega/\nu} = \sqrt{2} \lambda/(1-R_p)$	[3.5]
Pressure gradient	$P \sin(\omega t + \xi) = \frac{1}{\omega \rho v_p} \frac{\partial p}{\partial y}$	[3.6]
Velocity gradient	$F \sin(\omega t + \delta) = -\frac{r_p}{v_p} \frac{\partial v_0}{\partial x} \Big _{x=r_p} = -R_p \frac{\partial U}{\partial X} \Big _{X=R_p}$	[3.7]
	$F_i \sin(\omega t + \delta_i) = -\frac{r_i}{v_p} \int_0^{\theta_i} \frac{\partial v_1}{\partial r} \Big _{r=r_i} d\theta = -R_i \int_0^{\theta_i} \frac{\partial V}{\partial R} \Big _{R=R_i} d\theta$	[3.8]
	$F_o \sin(\omega t + \delta_o) = \frac{r_o}{v_p} \int_0^{\theta_i} \frac{\partial v_1}{\partial r} \Big _{r=r_o} d\theta = R_o \int_0^{\theta_i} \frac{\partial V}{\partial R} \Big _{R=R_o} d\theta$	
	$F_\theta \sin(\omega t + \delta_\theta) = \frac{1}{v_p} \int_{r_i}^{r_o} \frac{1}{r} \frac{\partial v_1}{\partial \theta} \Big _{\theta=\theta_1} dr = \int_{R_i}^{R_o} \frac{1}{R} \frac{\partial V}{\partial \theta} \Big _{\theta=\theta_1} dR$	

condition [2.6] where we set the initial velocity of the fluid to zero.

The fundamental equations [2.1]~[2.6] are transformed into nondimensional forms using the variables shown in Table 3. The nondimensional variables are denoted by capital or Greek letters. For example, X , R , R_p , R_i and R_o refer to the nondimensional variables x/r_c , r/r_c , r_p/r_c , r_i/r_c and r_o/r_c , respectively, where r_c is the inner radius of the cylinder. Further, U and V indicate the nondimensional velocities v_0/v_p and v_1/v_p , respectively, where v_p means the maximum piston velocity. Furthermore, T and η_c are nondimensional symbols of time t and frequency ω , respectively, as defined by Eqs.[3.4] and [3.5]. Previously, we used the symbol λ as a nondimensional frequency defined by $\lambda = (r_c - r_p)\sqrt{\omega/(2\nu)}$. The two nondimensional frequencies η_c and λ are connected by the relationship $\eta_c = \sqrt{2} \lambda/(1-R_p)$.

Equations [3.6]~[3.8] are definitions of the nondimensional pressure and velocity gradients of the fluid. The velocity gradient on the wall of the fan-shaped orifice varies with the location, so Eq.[3.8] is divided into three separate integrals. Nondimensionalized fundamental equations are listed in the right-hand column in Table 2.

3.2 Finite difference approximation to the equation of motion

We divide the annular channel into sets of concentric circles of spaces $\Delta X = (1-R_p)/n_x$, the fan-shaped orifice into polar mesh of sides $\Delta R = (R_o - R_i)/n_r$ and $\Delta \theta = \theta_1/n_\theta$ and the t -axis into equally spaced lines of distance $\Delta T = 1/n_t$, as shown in Table 4, where n_x , n_r , n_θ and n_t are the number of divisions. Let us denote

Table 4 A finite difference approximation to differential equations

Division of channel into polar mesh	
Crank-Nicolson equations	$U_{i,k+1} = b_1 U_{i-1,k+1} + b_2 U_{i+1,k+1} + b_3 U_{i,k} - b_4 P_k \quad (4.1)$ $V_{i,j,k+1} = a_1 V_{i-1,j,k+1} + a_2 V_{i+1,j,k+1} + a_3 (V_{i,j-1,k+1} + V_{i,j+1,k+1}) + a_4 V_{i,j,k} + a_5 (V_{i,j-1,k} + V_{i,j+1,k}) - a_5 P_k \quad (4.2)$
	$b_1 = \frac{g_0}{4(1+g_0)} \left(2 - \frac{\Delta X}{X}\right), \quad b_2 = \frac{g_0}{4(1+g_0)} \left(2 + \frac{\Delta X}{X}\right) \quad (4.3)$ $b_3 = \frac{1-g_0}{1+g_0}, \quad b_4 = \frac{2\pi\Delta T}{1+g_0}$
	$a_1 = \frac{g_1}{4(1+g_1+g_2)} \left(2 - \frac{\Delta R}{R}\right), \quad a_2 = \frac{g_1}{4(1+g_1+g_2)} \left(2 + \frac{\Delta R}{R}\right) \quad (4.4)$ $a_3 = \frac{g_2}{2(1+g_1+g_2)}, \quad a_4 = \frac{1-g_1-g_2}{1+g_1+g_2}, \quad a_5 = \frac{2\pi\Delta T}{1+g_1+g_2}$
	$g_0 = \frac{2\pi}{\eta_c^2} \frac{\Delta T}{(\Delta X)^2}, \quad g_1 = \frac{2\pi}{\eta_c^2} \frac{\Delta T}{(\Delta R)^2}, \quad g_2 = \frac{2\pi}{\eta_c^2} \frac{\Delta T}{(R\Delta\theta)^2} \quad (4.5)$

the values of U and V at the representative mesh point by

$$U(i\Delta X, k\Delta T) = U_{i,k}$$

$$V(i\Delta R, j\Delta\theta, k\Delta T) = V_{i,j,k} \quad (2)$$

where i , j and k are positive integers.

We have used the Crank-Nicolson method⁽⁵⁾ to calculate a numerical solution of the nondimensionalized equations of motion, [2.8] and [2.9]. The Crank-Nicolson equations can be written as Eqs.[4.1] and [4.2] in Table 4. In these equations, the meaning of $U_{i,k}$ and $V_{i,j,k}$ are as prescribed above, and their coefficients, $b_1 \sim b_4$ and $a_1 \sim a_5$, are shown in Eqs.[4.3] ~ [4.5]. Variable P_k denotes the values of non-dimensional pressure gradient $P \sin(2\pi T + \xi)$ at time $T = k\Delta T$.

3.3 Solution of the finite difference equations

When we have analytically solved the equations of motion, [1.2] and [1.3], we left the pressure term dp/dy unknown. The pressure term was determined by substituting the solution into the equation of continuity, [1.1], after the equations of motion had been

solved. The same procedure can be applied to solve the finite difference equations [4.1] and [4.2]. However, the numerical analysis cannot contain unknown quantities, in other words, some number must be assigned to every quantity in the finite difference equations. Therefore, first we separate the difference equations into two sub-equations, neither of which contain the unknown pressure term P_k , and then solve these sub-equations. Secondly, the value of P_k is determined so as to satisfy the equation of continuity, [2.7]. Finally, the solutions of the sub-equations are combined to form the solutions of the finite difference equations [4.1] and [4.2].

3.3.1 Separation of the difference equations

Velocities $U_{i,k}$ and $V_{i,j,k}$ are separated into two components $U_{c,i,k}$, $U_{p,i}$ and $V_{c,i,j,k}$, $V_{p,i,j}$ as shown in

Table 5 SOR iterative formulae for solving the Crank-Nicolson equation

Division of equations	$U_{i,k} = U_{c,i,k} + P_k U_{p,i} \quad , \quad (i=0 \sim n_x)$	[5.1]
	$V_{i,j,k} = V_{c,i,j,k} + P_k V_{p,i,j} \quad , \quad (i=0 \sim n_r, j=0 \sim n_\theta)$	[5.2]
SOR iteration equations for U_p and V_p	$U_{p,i}^{(\nu+1)} = \omega b [b_1 U_{p,i-1}^{(\nu+1)} + b_2 U_{p,i+1}^{(\nu)} - (\omega b - 1) U_{p,i}^{(\nu)}] \quad , \quad (i=1 \sim n_x-1)$	[5.3]
	$V_{p,i,j}^{(\nu+1)} = \omega a [a_1 V_{p,i-1,j}^{(\nu+1)} + a_2 V_{p,i+1,j}^{(\nu)} + a_3 (V_{p,i,j-1}^{(\nu+1)} + V_{p,i,j+1}^{(\nu)} - a_5) - (\omega a - 1) V_{p,i,j}^{(\nu)}]$ $V_{p,i,0}^{(\nu+1)} = \omega a [a_1 V_{p,i-1,0}^{(\nu+1)} + a_2 V_{p,i+1,0}^{(\nu)} + 2a_3 V_{p,i,1}^{(\nu)} - a_5] - (\omega a - 1) V_{p,i,0}^{(\nu)}$ $(i=1 \sim n_r-1, j=1 \sim n_\theta-1)$	[5.4]
Initial conditions of U_p and V_p	$U_{p,i}^{(0)} = 0 \quad , \quad (i=0 \sim n_x)$	[5.5]
	$V_{p,i,j}^{(0)} = 0 \quad , \quad (i=0 \sim n_r, j=0 \sim n_\theta)$	[5.6]
SOR iteration equations for U_c and V_c	$U_{c,i,k+1}^{(\nu+1)} = \omega b [b_1 U_{c,i-1,k+1}^{(\nu+1)} + b_2 U_{c,i+1,k+1}^{(\nu)} + b_3 U_{c,i,k}^{(\nu)}] - (\omega b - 1) U_{c,i,k+1}^{(\nu)}$ $(i=1 \sim n_x-1)$	[5.7]
	$V_{c,i,j,k+1}^{(\nu+1)} = \omega a [a_1 V_{c,i-1,j,k+1}^{(\nu+1)} + a_2 V_{c,i+1,j,k+1}^{(\nu)} + a_3 (V_{c,i,j-1,k+1}^{(\nu+1)} + V_{c,i,j+1,k+1}^{(\nu)} + a_4 V_{c,i,j,k}^{(\nu)}) - (\omega a - 1) V_{c,i,j,k+1}^{(\nu)}]$ $V_{c,i,0,k+1}^{(\nu+1)} = \omega a [a_1 V_{c,i-1,0,k+1}^{(\nu+1)} + a_2 V_{c,i+1,0,k+1}^{(\nu)} + 2a_3 V_{c,i,1,k+1}^{(\nu)} + a_4 V_{c,i,0,k}^{(\nu)}] - (\omega a - 1) V_{c,i,0,k+1}^{(\nu)}$ $(i=1 \sim n_r-1, j=1 \sim n_\theta-1)$	[5.8]
Constants	$b_1, b_2 = b_1 U_{i-1,k} + b_2 U_{i+1,k} + b_3 U_{i,k} \quad , \quad (i=1 \sim n_x-1)$	[5.9]
	$a_1, a_2, a_3 = a_1 V_{i-1,j,k} + a_2 V_{i+1,j,k} + a_3 (V_{i,j-1,k} + V_{i,j+1,k}) + a_4 V_{i,j,k}$ $a_4, a_5 = a_1 V_{i-1,0,k} + a_2 V_{i+1,0,k} + 2a_3 V_{i,1,k} + a_4 V_{i,0,k}$ $(i=1 \sim n_r-1, j=1 \sim n_\theta-1)$	[5.10]
Initial conditions of U_c and V_c	$U_{c,i,k+1}^{(0)} = U_{i,k} \quad , \quad (i=1 \sim n_x-1)$	[5.11]
	$V_{c,i,j,k+1}^{(0)} = V_{i,j,k} \quad , \quad (i=1 \sim n_r-1, j=0 \sim n_\theta-1)$	[5.12]
Boundary conditions of U_c and V_c	$U_{c,0,k} = \sin(2\pi k/n_t) \quad , \quad U_{c,n_x,k} = 0$	[5.13]
	$V_{c,0,j,k} = V_{c,n_r,j,k} = V_{c,i,n_\theta,k} = \sin(2\pi k/n_t) \quad , \quad (i=0 \sim n_r, j=0 \sim n_\theta)$	[5.14]
Relaxation factors	$\omega b = \frac{2}{1 + \sqrt{1 - \mu b^2}} \quad , \quad \mu b = \frac{g_0}{1 + g_0} \cos \frac{\pi}{n_x}$	[5.15]
	$\omega a = \frac{2}{1 + \sqrt{1 - \mu a^2}} \quad , \quad \mu a = \frac{1}{1 + g_1 + g_2} (g_1 \cos \frac{\pi}{n_r} + g_2 \cos \frac{\pi}{2n_\theta})$	[5.16]
Integrals	$I_p = 2 \int_{R_0}^1 X U_p dX \quad [5.17] \quad , \quad J_p = \frac{2}{\pi} \int_0^{\theta_1} \int_{R_i}^{R_0} R V_p dR d\theta \quad [5.18]$	
	$I_c, k = 2 \int_{R_0}^1 R V_{c,k} dX \quad [5.19] \quad , \quad J_c, k = \frac{2}{\pi} \int_0^{\theta_1} \int_{R_i}^{R_0} R V_{c,k} dR d\theta \quad [5.20]$	
Pressure gradient	$P_k = -\frac{1}{I_p + n J_p} [(R_0^2 - \frac{n \theta_1}{\pi} (R_0^2 - R_i^2)) \sin(2\pi k/n_t) + I_{c,k} + J_{c,k}]$	[5.21]

Eqs. [5.1] and [5.2] in Table 5. Of these, $U_{c,i,k}$ and $V_{c,i,j,k}$ represent fictitious velocities when the nondimensional pressure gradient P_k equals zero. Therefore, they are considered to be the complementary functions of nonhomogeneous equations [4.1] and [4.2].

On the other hand, $U_{p,i}$ and $V_{p,i,j}$ represent the fictitious velocities when the unit pressure gradient $P_k = 1$ is applied to the fluid at rest. Therefore, they are considered to be the particular integrals of Eqs.[4.1] and [4.2]. Since $U_{p,i}$ and $V_{p,i,j}$ are independent of time, they do not carry the subscript k .

3.3.2 SOR iterative method for solving the simultaneous equations Putting $P_k = 0$, Eq.[4.1] becomes a homogeneous equation

$$U_{i,k+1} - b_1 U_{i-1,k+1} - b_2 U_{i+1,k+1} = b_1 U_{i-1,k} + b_2 U_{i+1,k} + b_3 U_{i,k} \quad (3)$$

Further, putting $P_k = 1$ and $U_{i-1,k} = U_{i+1,k} = U_{i,k} = 0$, Eq.[4.1] reduces an equation

$$U_{i,k+1} - b_1 U_{i-1,k+1} - b_2 U_{i+1,k+1} = -b_4 \quad (4)$$

The left-hand side of Eqs. (3) and (4) contains three unknowns, and the right-hand side the known values of U . Since there are $n_x - 1$ internal mesh points along each time-level, Eqs. (3) and (4) give $n_x - 1$ simultaneous equations for the $n_x - 1$ unknown values along the $(k+1)$ th time-level in terms of the known values along the k th time-level and boundary values at $i=0, n_x$.

Equations (3) and (4) are solved by the SOR method; the SOR iteration equations of which are given in Eqs.[5.7] and [5.3], respectively. In the SOR method the factor ω , called the relaxation factor, is used for decreasing the number of iterations. The optimum values of ω , say ω_b , for the maximum rate of convergence is given in Eq.[5.15].

Equation [5.3] expresses the $(\nu+1)$ th approximation to $U_{p,i}$ in terms of four values, that is, the $(\nu+1)$ th approximation to $U_{p,i-1}$, ν th approximations to $U_{p,i+1}$ and $U_{p,i}$, and the constant b_4 . The iterative calculation of $U_{p,i}$ is start with an initial condition [5.5]. Similarly, Eq.[5.7] expresses the $(\nu+1)$ th approximation to $U_{c,i,k+1}$, in terms of four values, that is, the $(\nu+1)$ th approximation to $U_{c,i-1,k+1}$, ν th approximations to $U_{c,i+1,k+1}$ and $U_{c,i,k+1}$, and the constant $b_{i,k}$ given in Eq.[5.9]. The iterative calculation of $U_{c,i,k+1}$ is started with an initial condition [5.11] and a boundary condition [5.13].

Taking it into consideration that the velocity $V_{i,j,k}$ is symmetric about $j=0$, we obtain two $(n_{r-1}) \times n_\theta$ simultaneous equations [5.4] and [5.8] from Eq. [4.2]. The iterative calculation of Eq.[5.4] is started with an initial condition [5.6]; the iterative calculation of Eq.[5.8] is started with an initial condition [5.12] and a boundary condition [5.14].

3.3.3 Composition of solutions of the difference equations

Integrating the solutions U_p , V_p , U_c and V_c over the whole area of the channels, we obtain the flow rates I_p , J_p , $I_{c,k}$ and $J_{c,k}$, respectively, as shown in Eqs.[5.17]~[5.20] in Table 5. The integrals are accomplished numerically using Simpson's rule. Substituting these into Eq.[5.21], which is derived from the equations of continuity, we obtain the unknown pressure gradient P_k . Further, substituting the value of P_k into Eqs.[5.1] and [5.2], we obtain the actual velocities $U_{i,k}$ and $V_{i,j,k}$ at a new time-level. The value of P_k also contributes to the estimation of the pressure drag $D_p(t)$ acting on the piston (see Eq.[6.5] in Table 6).

3.4 Calculation of velocity gradient on the wall

In addition to the pressure drag, the piston of the oil damper is subject to the friction drag $D_f(t)$. The friction drag is proportional to the velocity gradient of the fluid on the wall. The velocity gradient is approximated by either the forward- or central-difference formula. We shall use the central-difference approximation because of its high accuracy. In order to employ the central-differences for the problem concerned, it is necessary to introduce the 'fictitious' velocities at the external mesh point by imagining the rod to be extended by a distance of ΔX , ΔR or $R\Delta\theta$ at these ends. The central-difference approximation of the velocity gradient are represented by Eqs.[6.1] and [6.2] in Table 6, where the fictitious velocities are designated as $U_{-1,k}$, $V_{-1,j,k}$, $V_{n_{r+1},j,k}$ and $V_{i,n_{\theta+1},k}$. These velocities are calculated by Eqs.[6.3] and [6.4], which are obtained from the iterative formulae [5.3], [5.4], [5.7] and [5.8].

3.5 Derivation of damping coefficient and added mass

The damping coefficient c and the added mass effect m_A of the oil damper are calculated by Eqs. [6.7] and [6.8], respectively, which are derived from Eqs. [6.5] and [6.6] of the pressure and friction drags. In these equations, P_1 , P_2 , F_1 and F_2 , etc., refer to the particular values of P_k and F_k etc.; these are defined by Eqs.[6.9] and [6.10]. The subscripts 1 and 2 correspond to the instantaneous maximum and zero velocity of the piston, respectively. For example, P_1 is the value of P_k at time $T = \nu + 1/4 + \Delta T/2$ where ν is a positive integer. At time $T = \nu + 1/4$, the piston moves at a maximum velocity; and we add a corrective term $\Delta T/2$ because the finite difference solution of pressure P_k always lags behind the analytical solution by one-half of the time step. On the other hand, P_2 is the value of P_k at time $T = \nu + \Delta T/2$. At time $T = \nu$, the piston is at rest. Similarly, F_1 and F_2 are the values of F_k at $T = \nu + 1/4$ and $T = \nu$, respectively, (the corrective term is not necessary about them).

Table 6 Central-difference approximation to the velocity gradient on the wall and equations of damping coefficient and added mass

Velocity gradients at wall	$F_k = F \sin(\omega t + \delta) = \frac{R_p}{2\Delta X} (U_{-1,k} - U_{1,k})$	[6.1]
	$F_{ik} = F_i \sin(\omega t + \delta_i) = \frac{R_i \Delta \theta}{2\Delta R} \sum_{j=0}^{n\theta} (V_{-1,j,k} - V_{1,j,k})$	[6.2]
	$F_{ok} = F_o \sin(\omega t + \delta_o) = \frac{R_o \Delta \theta}{2\Delta R} \sum_{j=0}^{n\theta} (V_{nr+1,j,k} - V_{nr-1,j,k})$	
	$F_{\theta k} = F_{\theta} \sin(\omega t + \delta_{\theta}) = \frac{\Delta R}{2\Delta \theta} \sum_{i=0}^{n_r} \frac{1}{R(i)} (V_{i,n\theta+1,k} - V_{i,n\theta-1,k})$	
Fictitious velocity at external mesh points	$U_{p,-1} = (b_4 - b_2 U_{p,1})/b_1$, $U_{c,-1,k} = (U_{c,0,k} - b_2 U_{c,1,k} - b_0, k-1)/b_1$	[6.3]
	$V_{p,-1,j} = (a_5 - a_4 V_{p,1,j})/a_1$	[6.4]
	$V_{c,-1,j,k} = [(1-2a_3)V_{c,0,j,k} - a_2 V_{c,1,j,k} - a_0, j, k-1]/a_1$	
	$V_{p,nr+1,j} = (a_5 - a_1 V_{p,nr-1,j})/a_2$	
	$V_{c,nr+1,j,k} = [(1-2a_3)V_{c,nr,j,k} - a_1 V_{c,nr-1,j,k} - a_{nr,j,k-1}]/a_3$	
	$V_{p,i,n\theta+1} = a_5/a_3 - V_{p,i,n\theta-1}$	
Pressure drag	$D_p(t) = \pi \mu l [R_p^2 - \frac{n\theta_1}{\pi} (R_o^2 - R_i^2)] \eta_c^2 \psi_p P_k$	[6.5]
	$D_f(t) = 2\pi \mu l [F_k + \frac{n}{\pi} (F_{ik} + F_{ok} + F_{\theta k})] \psi_p$	[6.6]
Damping coefficient	$c/(2\pi \mu l) = \frac{\eta_c^2}{2} [R_p^2 - \frac{n\theta_1}{\pi} (R_o^2 - R_i^2)] P_1 + F_1 + \frac{n}{\pi} (F_{i1} + F_{o1} + F_{\theta 1})$	[6.7]
Added mass	$m_A/(\pi R_c^2 \rho l) = [R_p^2 - \frac{n\theta_1}{\pi} (R_o^2 - R_i^2)] P_2 + \frac{2}{\eta_c^2} [F_2 + \frac{n}{\pi} (F_{i2} + F_{o2} + F_{\theta 2})]$	[6.8]
Pressure and velocity gradients at particular time	$P_1 = P_k _{T=\nu+1/4, \Delta T/2}$, $P_2 = P_k _{T=\nu, \Delta T/2}$, $(\nu = 0, 1, 2, \dots)$	[6.9]
	$F_1 = F_k _{T=\nu+1/4}$, $F_2 = F_k _{T=\nu}$	[6.10]
	$F_{i1} = F_{ik} _{T=\nu+1/4}$, $F_{i2} = F_{ik} _{T=\nu}$	
	$F_{o1} = F_{ok} _{T=\nu+1/4}$, $F_{o2} = F_{ok} _{T=\nu}$	
	$F_{\theta 1} = F_{\theta k} _{T=\nu+1/4}$, $F_{\theta 2} = F_{\theta k} _{T=\nu}$, $(\nu = 0, 1, 2, \dots)$	

3.6 Procedure for finite difference calculation

Figure 2 shows how to calculate the damping coefficient and the added mass of an oil damper having a fan-shaped orifice by means of the finite difference method.

First, the values of particular integrals, $U_{p,i}$ and $V_{p,i,j}$, are calculated in terms of initial velocities along $t=0$. The values of $U_{p,i}$ and $V_{p,i,j}$ are independent of time t or time counter k , so the calculation need not be repeated about k .

Second, the values of complimentary functions, $U_{c,i,k}$ and $V_{c,i,j,k}$, along the $(k+1)$ th time-level are calculated in terms of initial velocities along the k th time-level and boundary conditions along the $(k+1)$ th time-level. This calculation needs to be repeated about the different time t or k .

Finally, the nondimensional pressure gradient P_k is determined so as to satisfy the equation of continuity. The actual velocities, $U_{i,k}$ and $V_{i,j,k}$, along the $(k+1)$ th time-level are obtained from the value of P_k . If the time counter k coincides with the integral multiple of one-fourth of the time partitions, $n_i/4$,

then we can compute damping coefficient c and added mass m_A from the values of pressure gradient P_k and velocity gradients F_k , F_{ik} , F_{ok} and $F_{\theta k}$ at that time.

The previous calculation is repeated with a larger value of k until c and m_A converge to a certain value.

4. Dynamic Characteristics of the Oil Damper Having a Fan-Shaped Orifice

Figures 3(a) and (b) show plots of velocity distributions of the annular channel and the fan-shaped orifice, respectively, in the case of steady flow ($\lambda=0$). Open circles mean the finite difference solutions. On the other hand, solid lines mean the analytical solutions calculated by Eqs.[1.6] and [1.7] in Table 1. It is observed in Fig.3 that the finite difference method gives a solution as accurately as the analytical solution.

Figures 4(a) and (b) show the values of damping coefficient c and added mass m_A of the oil damper. These values are nondimensionalized by dividing the constants, and are plotted against the angle $2\theta_i$ of the orifice for various values of λ . Open circles are plots of

the finite difference solutions, and are connected with solid lines. And closed circles are plots of the analytical solutions.

Figure 4 shows an example of the characteristics of an oil damper having a fan-shaped orifice prescribed by parameters $R_p=0.95$, $R_o=0.7$, $R_i=0.5$ and $n=1$. However, it is obvious from the comparison between Figs. 4(a), (b) and 4(c) that the fundamental characteristics of the oil damper are independent of the shape of the orifice. Figure 4(c) shows plots of c and m_A of an oil damper having a cylindrical orifice of the same cross-sectional area as the fan-shaped orifice. In spite of considerable differences between the two orifices in shape, their characteristic curves resemble each other closely.

It is observed in Fig. 4 that the oil damper having a fan-shaped orifice possesses the following characteristics.

(1) If the impressed frequency is low, the added mass effect caused by the motion of fluid takes a maximum value for a certain condition of the opening angle, $2\theta_1=30^\circ$, of the orifice. For a higher impressed frequency, however, the added mass decreases monotonously by increasing the opening angle.

(2) The damping coefficient decreases monotonously by increasing the opening angle.

(3) As the impressed frequency increases, the

damping coefficient increases but the added mass decreases for all ranges of the angle.

(4) The oil damper having a fan-shaped orifice is much more susceptible to the frequency that the one not having the orifice.

5. Comparison Between Experimental and Theoretical Results

In order to verify the theoretical results discussed in the previous chapter, we constructed experimental equipment, the schematic of which is shown in Fig. 5. The equipment consists of two plates and three ring-type springs arranged in parallel around an oil damper. The piston of the oil damper is composed of two parts, as illustrated in Fig. 1.

The dimensions of the piston are that outer diameter $d_p=47.5$ mm, length $l=30$ mm and inner and outer radii of the orifice, $r_i=12.5$ mm and $r_o=17.5$ mm. The opening angle of the orifice can be changed

Fig. 2 Flowchart of the finite difference calculation

(a) Annular channel

(b) Fan-shaped orifice

Fig. 3 Velocity distributions in an annular channel and a fan-shaped orifice; comparison between analytical and finite difference solutions

Fig. 4 Characteristics of an oil damper having a fan-shaped orifice

from 0° to 135° by rotation of the inner piston driven by a stepping motor. The motor rotates by 1.8° per step; thus the magnitude of the damping coefficient can be regulated in 75 steps. The piston is inserted in a cylinder of inner diameter $d_c = 50$ mm. The cylinder is filled with silicone oil of kinematic viscosity $\nu = 100$ mm²/s at 23°C and density $\rho = 968$ kg/m³.

The experiment is performed by applying the sinusoidal vibration to the base plate of the system. Damping coefficient c and added mass m_A of the oil damper can be calculated from the measurement of the frequency response of the supporting mass or the upper plate. The equations to be used in the calculations have already been reported in ref. (7).

The experimental values are plotted against the angle $2\theta_1$ of the orifice for various impressed frequencies λ in Fig. 6. In this figure, the solid lines represent the finite difference solution calculated for the same conditions as the experiment. It is observed in Fig. 6 that the experimental values agree well with the theoretical ones. Thus, it was confirmed that the oil damper having a fan-shaped orifice satisfies the three conditions required for optimal use, and that the finite difference method is very useful for analyzing the oil damper.

6. Conclusions

We have discussed an oil damper having a fan-

Fig. 5 Experimental equipment

shaped orifice as a variable damping mechanism. It was confirmed from the theory and the experiment that this type of oil damper has three merits mentioned in the introduction. Other characteristics of the oil damper are as follows.

(1) The oil damper is greatly influenced by the frequency of the piston motion. Therefore the conventional analysis, which is based on the assumption that the piston moves at a constant velocity, is not helpful concerning this type of oil damper. Applying the finite difference method to the analysis of the oil damper,

(a) Damping coefficient

(b) Added mass

Fig. 6 Comparison between experimental and theoretical results

we obtained a good approximation of the dynamic characteristics of the oil damper subject to harmonic oscillation.

(2) The damping coefficient decreases by increasing the opening angle of the fan-shaped orifice.

(3) If the impressed frequency is low, then the added mass effect of the fluid in the oil damper takes a maximum value for certain conditions of the opening angle. As the impressed frequency increases, the maximum value decreases and finally dies out.

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