Identification of Nonlinear Multi-Degree-of-Freedom Systems*

(Identification Under Noisy Measurements)

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In a previous paper, the authors proposed a new technique for identifying nonlinear multi-degree-of-freedom systems. In this paper, the applicability of the proposed technique under noisy measurements is discussed using several examples. It is shown that a high accuracy of identification can be obtained when (1) the excitation frequency takes appropriate values, (2) the order of truncation in the Fourier-series expansion is two or three, and (3) the excitation has appropriate amplitudes.

Key Words: Vibration, Identification Technique, Nonlinear Multi-Degree-of-Freedom System, Principle of Harmonic Balance, Identification under Noisy Measurements

1. Introduction

Techniques for identifying linear vibratory systems based on their dynamic responses have been well developed and are used in many practical cases. However, most of the techniques developed so far cannot be applied to identification of nonlinear vibratory systems. These days, the need for identifying nonlinear vibratory systems is gaining great importance. To meet this need, several techniques^{(1)~(7)} have been proposed, but other various techniques are still needed.

In a previous paper⁽⁸⁾, the authors proposed a new technique of identifying nonlinear vibratory systems, and showed, by applying it to some typical systems, that the technique yields an accurate identification.

However, the data used there were such ones that were obtained numerically from the equations of motion of the systems and hence contain almost no noise. It was concluded that when data without noise

were used, the identification was always accurate. In practical cases, the data are obtained experimentally and inevitably contain noise. When such data are used, the result of identification is influenced by the values given to the excitation frequency, the order of truncation in the Fourier-series expansion, and so on. In this paper, conditions for obtaining an accurate identification under noisy measurements are investigated.

2. Summary of the Proposed Technique of Identification

First the technique proposed in the previous paper is summarized. For this purpose, a simple example is considered. The system is, as shown in Fig. 1, a single-degree-of-freedom vibratory system which consists of a body with mass m and a spring with spring constant k . The system is subjected to a viscous damping force with damping coefficient c . Its spring has nonlinear characteristic expressed by N , which is a function of displacement x . When an excitation q is applied to this system, the equation of motion becomes as follows:

$$m\ddot{x} + c\dot{x} + kx + N = q \quad (1)$$

* Received 22nd February, 1988. Paper No. 87-0589 A

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To identify this system, several procedures are followed. The first procedure is to assume that the nonlinearity N is approximated by a polynomial of x of the form

$$N = \alpha_2 x^2 + \alpha_3 x^3 + \dots \quad (2)$$

where $\alpha_2, \alpha_3, \dots$, are unknown coefficients.

The second procedure is to apply a periodic force q with period $T(=2\pi/\omega)$ to a system, and to induce in the system a steady-state response x with the same period as that of the excitation. Then, both the excitation q and the response x are measured in a period. Using these data, x and q are developed in Fourier series. Similarly, the terms $x^2, x^3, \dots$ appearing in Eq. (2) are determined by the operation of x , and are developed in Fourier series.

The last procedure is to substitute the Fourier series obtained above into Eq. (1) and to apply the principle of harmonic balance. This yields the equations with respect to unknown quantities $m, c, k, \alpha_2, \alpha_3, \dots$

When various values are given properly to ω , and when the order of truncation is taken appropriately in the Fourier-series expansion, the number of equations exceeds that of unknown quantities. Applying the least-squares method to these equations determines the unknown quantities.

This technique can be generalized in a straightforward manner to be applicable in multi-degree-of-freedom systems. It is this technique that was proposed in the previous paper.

3. Effects of the Values Given to the Excitation Frequency and the Order of Truncation in the Fourier-Series Expansion

First the criterion for determining the accuracy of the identification will be defined. In general, the nonlinearity can occur in spring and/or damping characteristics, so it is desirable that the accuracy be checked separately in inertia, damping and spring forces. Also, it is desirable that the accuracy be related to the amplitudes of the induced oscillations. For the first purpose, the criterion of accuracy will be defined by using the exact inertia, damping, and spring forces F_i, F_d, F_r , as well as the identified inertia,

damping, and spring forces $\tilde{F}_i, \tilde{F}_d, \tilde{F}_r$. For the second purpose, these forces will be evaluated using the oscillation induced by a certain harmonic excitation. As this excitation is specified here the one whose frequency is equal to the fundamental natural frequency and whose amplitude has a prescribed value or is determined indirectly by prescribing the amplitude of the induced oscillation.

Thus, as criteria for checking the accuracy, the quantities

$$\left. \begin{aligned} E_i &= \sqrt{\frac{\int (F_i - \tilde{F}_i)^2 dt}{\int F_i^2 dt}}, & E_d &= \sqrt{\frac{\int (F_d - \tilde{F}_d)^2 dt}{\int F_d^2 dt}} \\ E_r &= \sqrt{\frac{\int (F_r - \tilde{F}_r)^2 dt}{\int F_r^2 dt}} \end{aligned} \right\} \quad (3)$$

and

$$E_{\text{mean}} = (E_i + E_d + E_r)/3 \quad (4)$$

will be used, where integration is performed for a period using the induced oscillation.

Using these criteria, the effects of the values given to the excitation frequency or the order of truncation in the Fourier-series expansion will now be examined. Since the effects of the amplitude of the excitation are discussed later, for now it is taken to be the same as that used for obtaining the response data.

3.1 Systems having a nonlinearity in their spring characteristics

First a single-degree-of-freedom system, shown in Fig. 1, having a nonlinearity given by

$$N = \alpha x^3 \quad (5)$$

is considered. When a harmonic excitation $q = Q \cos \omega t$ is applied to the system, the equation of motion becomes

$$m\ddot{x} + c\dot{x} + kx + \alpha x^3 = Q \cos \omega t \quad (6)$$

To obtain the response data, the amplitude of the excitation Q is taken as $Q=1$, and the other system parameters are taken as

$$m=1, c=0.05, k=1, \alpha=0.1 \quad (7)$$

Then, an appropriate value is given to ω , and Eq. (7) is integrated numerically till steady-state responses are obtained. In this way the response data are obtained in the form of sequence numbers at discrete times. To each of these numbers, random numbers with standard deviation of 0.01 are added. The obtained results are regarded as the data of the response with noise.

To identify the system, the nonlinearity is assumed to be given by the first two terms in Eq. (2), i. e.,

Fig. 1 A single-degree-of freedom system

$$N = \alpha_2 x^2 + \alpha_3 x^3 \quad (8)$$

Now the effects of the values given to the excitation frequency and the order of truncation in the Fourier-series expansion are investigated. For this purpose, the response curves obtained analytically from Eq. (6) are shown in Fig. 2, in which R_1 and R_3 show the amplitudes of the harmonic and the third-order harmonic components, respectively. In addition, the values of R_1 and R_3 obtained numerically from Eq. (6) are represented by $\circ$ in Fig. 2. Identification is carried out using these data. Two cases are considered by changing what values are given to the excitation frequency. In the first case, values are selected from a narrow region around ω_m as shown by (a) in Fig. 2. In the second case, the values are selected from a wide region as shown by (b). The accuracy of the identification are evaluated by E_{mean} for these cases. The results obtained in this way for various orders of truncation are shown in Fig. 3. In the figure, (a) shows the case in which five values with interval 0.1 around ω_m are selected as the excitation frequency, and (b) the case in which ten values and thirty values between $\omega=0.1$ and 3.0, and twenty-three values between $\omega=0.1$ and 2.3, are given to the excitation frequency.

From Fig. 3, it is found that an accurate identification is obtained when values over a wide region are given to the excitation frequency and the order of truncation n is taken as $n=1$ or 2. It is also

found that, in the case in which values in a wide region are not available, an accurate identification is obtained when values near the resonance frequency ($\omega=1.0$) are selected and n is taken as $n=1$ or 2. Finally, it is noted that an accurate identification is obtained when ω_m is taken as 0.3 and n as 3. The reason for this is that third-order harmonics are involved in the response. Summarizing these, it may be concluded that an accurate identification can be obtained if the excitation frequency takes (1) values over a wide region, (2) values near the resonance frequency and (3) such values that induce higher order harmonics.

The results of identification when $n_\omega=12$ and $n=2$, which is the case yielding the best results in Fig. 3, are shown in Table 1.

Next, as an example of a multi-degree-of-freedom system, a two-degree-of-freedom system as shown in Fig. 2 is considered. All three springs of the system have the same nonlinearity given in terms of displacement x by

$$N = ax^3 \quad (9)$$

When a harmonic excitation $q = Q \cos \omega t$ is applied to the body with mass m_1 , the equation of motion becomes as follows:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} + \{N\} = \{q\} \quad (10)$$

where matrices $[M]$, $[C]$, $[K]$ are of the form

Table 1 The results and the accuracy of identification for $n_\omega=23$ and $n=2$

	Exact	Identified
m	1.0000	0.99993195
c	0.0500	0.04996767
k	1.0000	0.99898707
α_2	0.0000	0.00047901
α_3	0.1000	0.10003140

E_i	0.00006805
E_d	0.00064658
E_r	0.00092363
E_{mean}	0.00054638

Fig. 2 The response curve and chosen values for the excitation frequency ($\bullet: \omega_m$)

Fig. 3 The accuracy of identification

$$\left. \begin{aligned} [M] &= \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, [C] = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix}, \\ [K] &= \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix} \end{aligned} \right\} \quad (11)$$

and vectors $\{N\}$ and $\{q\}$ are of the form

$$\{N\} = \begin{Bmatrix} N_1 \\ N_2 \end{Bmatrix}, \{q\} = \begin{Bmatrix} Q \cos \omega t \\ 0 \end{Bmatrix} \quad (12)$$

Matrices $[C]$, $[K]$ and vectors $\{N\}$ are given in terms of c , k in Fig. 4 and α in Eq. (9), respectively.

To obtain the response data, the amplitude of the excitation Q is taken as $Q=1$ and other system parameters are taken as

$$m_1 = m_2 = 1, c = 0.05, k = 1, \alpha = 0.1 \quad (13)$$

As in the single-degree-of-freedom system, the response data are generated by integrating Eq. (10) numerically and by adding to the obtained steady-state response $\{x\}$, random numbers with standard deviation of 0.01.

To identify the system, the matrices $[M]$, $[C]$ and $[K]$ are put in the form of Eq. (11). The nonlinearity $\{N\}$ is considered to be given by $N_1 = \partial U / \partial x_1$ and $N_2 = \partial U / \partial x_2$ where U is a potential of the form

$$U = \alpha_{30}x_1^3 + \alpha_{21}x_1^2x_2 + \alpha_{12}x_1x_2^2 + \alpha_{03}x_2^3 + \alpha_{40}x_1^4 \\ + \alpha_{31}x_1^3x_2 + \alpha_{22}x_1^2x_2^2 + \alpha_{13}x_1x_2^3 + \alpha_{04}x_2^4 \quad (14)$$

Identification is carried out for the case where the excitation frequency is given (a) values from a narrow region around a certain values ω_m , and (b) values from a wide region. The accuracy of the identification in terms of E_{mean} for these two cases for various orders of truncation is shown in Fig. 5. In this figure, (a) shows the results when nine values with interval 0.1 around ω are selected and (b) the results when ten and thirty values in the wide region between 0.1 and 3.0 are selected.

From Fig. 5, it is found that an accurate identification is obtained when values in a wide region are selected and the order of truncation n is taken as $n=1$ or 2. It is also found that even when excitation frequency is given values in a narrow region, an accurate identification is still obtained when values near the resonant frequency ($\omega_m=1.0$ and $\omega_m=1.7$) are selected and the order of truncation is taken as $n=2$ or 3. It is also noted that, as in the single-degree-of-freedom system, an accurate identification is obtained when values around $\omega=0.3$ are selected, in which case

Fig. 4 A two-degree-of-freedom system

the third-order harmonics are in resonance. In any case, n need not necessarily be so large.

The results of identification for $n_\omega=30$ and $n=1$, which case yields the best results in Fig. 5, are shown in Table 2.

3.2 Systems having a nonlinearity in their damping characteristics

Here systems having a nonlinearity in their damping characteristics are considered.

First the system governed by the Van der Pol equation is considered. When a harmonic excitation is applied to this system, the equation of motion becomes as follows:

$$m\ddot{x} - \epsilon(1-x^2)\dot{x} + kx = Q \cos \omega t \quad (15)$$

To obtain the response data, the amplitude of the excitation force Q is taken as $Q=1$, and the other

Table 2 The results and the accuracy of identification for $n_\omega=30$ and $n=1$

	Exact	Identified		Exact	Identified
m_1	1.0000	0.99806453	α_{30}	0.0000	-0.00024226
m_2	1.0000	1.00253301	α_{21}	0.0000	0.00020737
c_{11}	0.1000	0.10010546	α_{12}	0.0000	-0.00023987
c_{12}	-0.0500	-0.04999911	α_{03}	0.0000	0.00029580
c_{22}	0.1000	0.09968134	α_{40}	0.0500	0.04934311
k_{11}	2.0000	1.99730087	α_{31}	-0.1000	-0.09832481
k_{12}	-1.0000	-0.99903572	α_{22}	0.1500	0.14946303
k_{22}	2.0000	2.00359572	α_{13}	-0.1000	-0.10120439
			α_{04}	0.0500	0.05055288

E_i	0.00219539
E_d	0.00420464
E_r	0.00193690
E_{mean}	0.00277897

Fig. 5 The accuracy of identification

system parameters are taken as

$$m=1, k=1, \varepsilon=0.01 \quad (16)$$

Then, for the excitation frequency, ω , for which the steady-state response occurs with the excitation frequency, the steady-state response data are obtained numerically. To the obtained data are added random numbers with the standard deviation of 0.01, and the results are regarded as the response with noise.

To identify the system, the nonlinearity N is supposed to be given by

$$N=(\beta_1 x + \beta_2 x^2) \dot{x} \quad (17)$$

The excitation frequency is taken as $\omega=0.8, 0.9, 1.0, 1.1$, and 1.2 . The accuracy of the identification is evaluated in terms of E_{mean} , and the results for various orders of truncation are shown in Fig. 6.

From Fig. 6, it is found that an accurate identification is obtained when the order n is taken as $n=2$. It is noted that the result for $n=1$ is extremely inaccurate as compared with those in Fig. 3(b). The reason for this is that, if n is taken as $n=1$ in the Fourier-series expansion for $x\dot{x}$, the conditions which will yield $\beta=1$ are excluded.

The results of identification for $n=2$ are shown in Table 3.

For the second example, a two-degree-of-freedom system with nonlinear damping as shown in Fig. 7, is considered. The nonlinear term N is supposed to be given by

$$N=\beta x_1^2 \dot{x}_1 \quad (18)$$

When a harmonic excitation $q=Q \cos \omega t$ is applied to the body with mass m_1 , the equation of motion becomes

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} + \{N\} = \{q\} \quad (19)$$

where matrices $[M]$, $[C]$, $[K]$ and vectors $\{N\}$ and $\{q\}$ are of the forms of Eqs.(10) and (11), respectively, and their elements are given in terms of c_1, c_2, k_1, k_2 in Fig. 7 and β in Eq.(18).

To obtain the response data numerically, the amplitude of the excitation force Q is taken as $Q=1$, and the other system parameters are taken as

$$m_1=1, m_2=0.1, k_1=1, k_2=0.1,$$

Fig. 6 The accuracy of identification

Fig. 7 A dynamic damper

$$c_1=-0.02, c_2=0.01, \beta=0.02 \quad (20)$$

The response data are generated by integrating Eq. (19) numerically and by adding random numbers with standard deviation of 0.01.

To identify the system, the matrices $[M]$, $[C]$, $[K]$ and the nonlinear terms $\{N\}$ are put in the form of Eq.(11), and in the form

$$\left. \begin{aligned} N_1 &= (\beta_{1,10}x_1 + \beta_{1,01}x_2 \\ &\quad + \beta_{1,20}x_1^2 + \beta_{1,11}x_1x_2 + \beta_{1,02}x_2^2) \dot{x}_1 \\ N_2 &= (\beta_{2,10}x_1 + \beta_{2,01}x_2 \\ &\quad + \beta_{2,20}x_1^2 + \beta_{2,11}x_1x_2 + \beta_{2,02}x_2^2) \dot{x}_2 \end{aligned} \right\} \quad (21)$$

Identification is carried out for the case where the excitation frequency takes values selected (a) from a narrow region around ω_m and (b) from a wide region. The accuracy of the identification in terms of E_{mean} for these two cases for various values of the order of truncation is shown in Fig. 8(a) and (b).

From Fig. 8, it is found that an accurate identification is obtained when the excitation frequency takes values near the resonance excitation ($\omega_m=1.0$) and the order n of truncation is taken as $n=2$ if available values are restricted to narrow region. It is also found that for the case where values from a wide region are available, an accurate result is obtained when the order n of truncation is taken as $n=2, 3$. The reason why the identification is inaccurate when $n=1$ can be explained similarly as in the system governed by the Van der Pol equation.

The results of identification for $n_\omega=30$ and $n=2$, which is the case yielding the best results in Fig. 5, are shown in Table 4.

3.3 Systems having a nonlinearity in both their spring and damping characteristics

Here, as a system having a nonlinearity in both their spring and damping characteristics, a single-degree-of-freedom system having the same damping characteristic as the system governed by the Van der Pol equation and having the spring characteristic given by a cubic function of the displacement, is considered. When a harmonic excitation is applied to the system, the equation of motion becomes as fol-

Table 3 The results and the accuracy of identification for $n=2$

	Exact	Identified
m	1.0000	1.00020363
ε	0.0100	0.01020630
k	1.0000	1.00015390
β_1	0.0000	-0.00016872
β_2	0.0100	0.01001840

E_i	0.00020363
E_d	0.00344568
E_r	0.00015390
E_{mean}	0.00226774

lows:

$$m\ddot{x} - \varepsilon(1-x^2)\dot{x} + kx + \alpha x^3 = Q \cos \omega t \quad (22)$$

To obtain the response data, the amplitude of the excitation force Q is taken as $Q=1$, and the other system parameters are taken as

$$\left. \begin{aligned} m=1, k=1, \varepsilon=0.01, \\ \alpha=0.1 \end{aligned} \right\} \quad (23)$$

Then, the excitation frequency ω is given values for which the steady-state response with the excitation frequency occurs, and the steady-state response data are obtained numerically. To the obtained data are added random numbers with the standard deviation of 0.01, and the results are regarded as the response with noise.

To identify the system, the nonlinearity N is assumed to be given by

$$N = \beta_1 x \dot{x} + \beta_2 x^2 \dot{x} + \alpha_2 x^2 + \alpha_3 x^3 \quad (24)$$

The excitation frequency is given the values $\omega = 0.8, 0.9, 1.0, 1.1, 1.2, 1.3$, and 1.4. The accuracy of the

Table 4 The results and the accuracy of identification for $n_w=30$ and $n=2$

	Exact	Identified		Exact	Identified
m_1	1.0000	0.99936759	$\beta_{1,10}$	0.0000	0.00027201
m_2	0.1000	0.09982970	$\beta_{1,01}$	0.0000	-0.00006128
c_{11}	-0.0100	-0.01071169	$\beta_{1,20}$	0.0200	0.02010565
c_{12}	-0.0100	-0.01001482	$\beta_{1,11}$	0.0000	-0.00001670
c_{22}	0.0100	0.00997145	$\beta_{1,02}$	0.0000	0.00000837
k_{11}	1.1000	1.09922197	$\beta_{2,10}$	0.0000	0.00000017
k_{12}	-0.1000	-0.09987042	$\beta_{2,01}$	0.0000	-0.00000041
k_{22}	0.1000	0.09982639	$\beta_{2,20}$	0.0000	0.00000557
			$\beta_{2,11}$	0.0000	0.00000020
			$\beta_{2,02}$	0.0000	-0.00000009

E_i	0.00128695
E_d	0.00463481
E_r	0.00130582
E_{mean}	0.00240920

Fig. 8 The accuracy of identification

identification is evaluated in terms of E_{mean} . The results are shown in Fig. 9 for various orders of truncation.

From Fig. 9, it is found that an accurate identification is obtained when the order of truncation, n , is taken as $n=2, 3$.

The results of identification for $n=3$, which is the case yielding the best identification in Fig. 9, is shown in Table 5.

4. Effects of the Order of Polynomials Expressing the Nonlinear Terms and the Amplitude of the Excitation

In this chapter, the effects of the order of polynomials used for expressing the nonlinear terms and the amplitude of the excitation are discussed.

For this purpose, single-degree-of-freedom systems having a nonlinearity in their spring characteristics are considered. Two systems are considered. One is that governed by Eq. (6), and the other is that governed by

$$m\ddot{x} + c\dot{x} + \sin x = Q \cos \omega t \quad (25)$$

The latter system has been taken up because it is an example whose nonlinearity is not given by a simple polynomial.

To obtain the response data, the system parameters of Eq. (6) are taken as

$$\left. \begin{aligned} m=1, c=0.05, k=1, \\ \alpha=0.1 \end{aligned} \right\} \quad (26)$$

and m and c of Eq. (25) are taken as the same values as in Eq. (26). The amplitude of the excitation Q is taken in both systems as $Q=0.05, 0.1, 0.2$. Random

Table 5 The results and the accuracy of identification

	Exact	Identified
m	1.0000	1.01801678
ε	0.0100	0.01110099
k	1.0000	1.00762264
α_2	0.0000	0.00021798
α_3	0.1000	0.10240832
β_1	0.0000	-0.00040921
β_2	0.0100	0.01030260

E_i	0.01801678
E_d	0.06000469
E_r	0.01273031
E_{mean}	0.03025059

Fig. 9 The accuracy of identification

numbers with standard deviation of 0.001 are used as noise. Twenty - nine values ($n_\omega=29$) in the range 0.1~3.0 are used as the excitation frequency. The order of truncation, n , in the Fourier-series expansion is taken as $n=1$.

The accuracy of the identification is evaluated in terms of quantities given in Eqs.(3) and (4). Because the accuracy is evaluated with related to the amplitudes, the harmonic oscillation with amplitudes $x_0=0.5, 1.0, 1.5$ are used for evaluating the quantities in Eq.(3).

Taking into account the symmetry of the restoring force, the nonlinear term N is considered to be given by a polynomial of odd order l of the form

$$N = a_3 x^3 + \dots + a_l x^l \quad (27)$$

To the systems given by Eqs.(6) and (25), E_{mean} is calculated for various l , and its result is shown in Figs. 10 and 11 with x_0 as the parameter.

From Fig. 10 it is found that, while for $l=3$ the identification is always accurate irrespective of the

magnitudes of the excitation, for $l \neq 3$ it is dependent on the amplitude of the excitation. From this, it can be concluded that, if the order of the nonlinear term is specified properly, the result of identification is accurate for wider range of displacement than is expected from the amplitude of the excitation. If the order is not specified properly, the result of identification is accurate only up to the region of displacement corresponding to the amplitude of the excitation. In this case, for obtaining an accurate identification, the magnitude of the excitation must be chosen as large as possible.

From Fig. 11, it is found that an accurate identification is obtained when the order of the nonlinear terms is taken large enough, but not too large. It is also found that, if the amplitude of the excitation is not large enough, the result of identification is accurate only for a range of displacement corresponding to the magnitude of the excitation. Thus, to obtain the result of identification which is accurate in a wide

Fig. 10 The accuracy of identification

Fig. 11 The accuracy of identification

Table 6 The results and the accuracy of identification

	Exact	Identified
m	1.0000	0.99932402
c	0.0500	0.05005294
k	1.0000	0.99954486
α_3	0.1000	0.09988735

x_0	0.5	1.0	1.5
E_i	0.00067598	0.00067598	0.00067598
E_d	0.00105873	0.00105873	0.00105873
E_r	0.00046741	0.00050259	0.00055365
E_{mean}	0.00073404	0.00074576	0.00076279

region of displacement, the amplitude of the excitation must be chosen as large as possible. In the case of Fig. 11, the order l is taken as $l=7$, and the excitation as large as $Q=0.2$, and the result is accurate in the region of displacement up to $x_0=0.1$.

Examples of the results of identification for the system of Eq. (6) with Q and l taken as $Q=0.2$ and $l=3$, and those for the system of Eq. (25) with Q and l taken as $Q=0.2$ and $l=7$, are shown in Tables 6 and 7.

5. Conclusions

In a previous paper the authors proposed a new technique for identifying nonlinear vibratory systems. In this paper the technique is applied using data with noise which is generated numerically, and conditions for obtaining an accurate identification under noisy measurements are discussed. First, the effects of the values of the excitation frequency and the order of truncation in the Fourier-series expansion are investigated for single-degree-of-freedom systems as well as multi-degree-of-freedom systems with different nonlinear characteristics. Next, the effects of the order of polynomials used as nonlinear terms and the amplitude of the excitation are discussed for single-degree-of-freedom systems. The obtained results may be summarized as follows:

(1) An accurate identification is obtained when the excitation frequency is given values selected from a wide region. If values in a wide region are not available, values near resonant frequency are preferred.

(2) The order of truncation which yields an accurate identification depends on the characteristics of the nonlinearity. For example, in a system governed by the Van der Pol equation, it is insufficient to take the order n as $n=1$. In cases considered in the paper, it is usually sufficient to take n as $n=2$ or 3.

Table 7 The results and the accuracy of identification

	Exact	Identified
m	1.0000	1.00042217
c	0.0500	0.05015518
k		1.00021473
α_3		-0.16685575
α_5		0.01127414
α_7		-0.00233367

x_0	0.5	1.0	1.5
E_i	0.00000267	0.00000267	0.00000267
E_d	0.00002311	0.00002311	0.00002311
E_r	0.00000281	0.00000268	0.00000689
E_{mean}	0.00000953	0.00000949	0.00001089

(3) If the order of the polynomials used for expressing the nonlinearity is specified properly, an accurate identification is obtained for a wider region of displacements than is expected from the amplitude of the excitation. In general, however, the amplitude of the excitation must be chosen large enough depending on the region of the displacement for which the identification is applicable.

References

- (1) Korenberg, M. J., Identification of Nonlinear Differential Systems, JACC, (1973), p. 597.
- (2) Ibanez, P., Identification of Dynamic Parameters of Linear and Non-linear Structural Models from Experimental Data, J. Nucl. Eng. Des., 25(1973), p. 30.
- (3) Beliveau, J. G., Identification of Viscous Damping in Structures from Modal Information, Trans. ASME, J. Appl. Mech., 43(1976), p. 335.
- (4) Masri, S. F. and Caughey, T. K., A Nonparametric Identification Technique for Nonlinear Dynamic Problems, Trans. ASME, J. Appl. Mech., Vol. 46 No. 2 (1979), p. 433.
- (5) Udawadia, F. E. and Kuo, C. P., Non-parametric Identification of a Class of Non-linear Close-Coupled Dynamic Systems, Earthqu. Eng. Struct. Dyn., 9 (1981), p. 135.
- (6) Horta, L. G., Nonlinear Effect on Model Data Analysis Method, NASA/LARC, Hampton, Virginia, CP-2266, (1982), p. 135.
- (7) Yang, Y. and Ibrahim, S. R., A Nonparametric Identification Technique for a Variety of Discrete Nonlinear Vibrating Systems, Trans. ASME, J. Vib. Acoust. Stress Reliab. Des., 107 (1985), p. 60.
- (8) Yasuda, K., Kawamura, S. and Watanabe, K., Identification of Nonlinear Multi-Degree-of-Freedom Systems (Presentation of an Identification Technique), JSME Int. J., Ser. III, Vol. 31, No. 1 (1983), p. 8.