Parametrically Excited Oscillations of a Rotating Shaft Under a Periodic Axial Force

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The parametrically excited oscillations in a rotating shaft system where a periodic axial force is applied are studied theoretically. The treated model is a four-degree-of-freedom system where a disk is mounted on an elastic shaft with a circular cross section. We investigate the following: the forms of parametric excitation terms in the equations of motion, the kinds of parametrically excited oscillations which occur in this system, the influences of the shaft rotating speed on the width of the unstable regions, and so on. The differences from other kinds of parametrically excited systems, such as an unsymmetrical shaft system, an unsymmetrical rotor system, and a column system under a periodic axial force, are also clarified.

Key Words: Theory of Vibration, Vibration of Rotating Body, Periodic Axial Force, Parametrically Excited Oscillation, Unstable Vibration

1. Introduction

Unstable vibrations are often caused by parametric excitations in the rotating shaft systems. Among them, typical examples are systems with an unsymmetrical shaft or an unsymmetrical rotor. In these systems, parametrically excited oscillations occur at the major critical speed and near the rotating speed whose double value is equal to the sum of the values of the natural frequencies. These oscillations have already been studied in detail.

The problem of parametrically excited oscillations of a column under a sinusoidal axial force is important for engineering. As for this problem, unstable vibrations with a single mode have been studied by Utida and Sezawa, and Somerset and Evan-Iwanowski, and unstable vibrations of a summed-and-differential type by Iwatsubo and et al. Also in a rotating shaft system, the periodic axial force acts on the shaft due to various reasons. However, it is expected that the feature of a parametrically excited oscillation in a rotating shaft system is different from that in a column system. Keller studied a rotating shaft system subjected to a periodic axial force and a periodic torque. He treated a simple two-degree-of-freedom system where no gyroscopic moment worked and where the frequency of parametric excitation was proportional to the shaft speed.

In this paper, a parametrically excited oscillation of a rotating shaft under a sinusoidal axial force is discussed. We treat a system with four degrees of freedom where a gyroscopic moment is acting. In particular, we will examine the types of unstable vibrations, the width of each unstable region, and the difference from the corresponding oscillation in a column system under a periodic axial force.
2. Equations of Motion

As shown in Fig. 1, we treat a rotating shaft system where a disk is mounted on an elastic shaft with a circular cross section subjected to a sinusoidal axial force $F(t)$. A stationary rectangular coordinate system $O-xyz$ whose $z$-axis coincides with the center line between the upper and lower bearings is considered. We designate the component of the shaft deflection $r$ at the position of the disk in the $x$- and $y$-directions by $x$ and $y$ respectively, and the projectional angles of the inclination $\theta$ to the $xz$- and $yz$-planes by $\theta_x$ and $\theta_y$ respectively. Designating the force and the moment exerted on the shaft at the disk position by $P$ and $M$ respectively, we get the following equation:

$$
P = ax + y \theta,
$$
$$
M = yr + \delta \theta.)
$$
\begin{eqnarray}
\end{eqnarray}

where $a$, $r$, and $\delta$ are the spring constants of the shaft.

When the shaft is simply supported at both ends, the spring constants are given by the following equation:

$$
a = E I_0 \lambda (f(\lambda, a) + f(\lambda, b))
$$
$$
g = E I_0 \lambda (f(\lambda, b) - af(\lambda, a))
$$
$$
\delta = E I_0 \lambda (af(\lambda, a) \tan h \lambda a + b f(\lambda, a) \tan h \lambda b)
$$
\begin{eqnarray}
\end{eqnarray}

where $E$ is the Young's modulus, and $I_0$ is the second moment of area. From Eq. (2), one knows that the spring constants are the functions of the axial force $F$.

For example, Fig. 2 shows the values of $a$, $r$, and $\delta$ for the shaft length $l = 700$ mm ($a: b = 1: 4$), the shaft diameter $d = 12$ mm and $E = 206$ GPa. In this figure, the buckling load is about $-4.22$ kN, and the spring constants vary almost linearly in the range of the axial force smaller than the buckling load. Consequently, it follows that the spring constants can be approximately expressed by linear functions of $F$.

Also, assuming that the axial force varies periodically with time $t$ as $F(t) = F_0 \cos \omega t$, we can obtain the following equation:

$$
\alpha(F) = \alpha_0 + \alpha_1 \cos \omega t
$$
$$
\gamma(F) = \gamma_0 + \gamma_1 \cos \omega t
$$
$$
\delta(F) = \delta_0 + \delta_1 \cos \omega t
$$
\begin{eqnarray}
\end{eqnarray}

Let the mass of a disk be $m$, the polar and the diametral moments of inertia of a rotor be $I_p$ and $I$, respectively, the rotating speed of a shaft be $\omega$, and the damping coefficients be $c_u$ ($i, j = 1, 2$). Adopting the reference length $e_u = mg/a_0$ (gravity), we define the following dimensionless quantities:

$$
\begin{align*}
x' &= x/e_u, \quad y' = y/e_u, \quad \delta_x' = \delta_x/\sqrt{m/e_u},
\end{align*}
$$
$$
\begin{align*}
\lambda &= \lambda_0/\sqrt{a_0/e_u}, \quad \omega' = \omega/\sqrt{a_0/e_u}, \quad \Omega' = \Omega/\sqrt{a_0/e_u},
\end{align*}
$$
$$
\gamma_0 = \gamma_0/\sqrt{m|I_0|}, \quad \delta_0 = \delta_0/\sqrt{a_0|I_0|},
$$
$$
\Delta_I = \Delta_0/\sqrt{m|I_0|}, \quad \Delta_\Omega = \Delta_\Omega/\sqrt{a_0|I_0|}, \quad c_{11} = c_{11}/\sqrt{a_0|I_0|},
$$
$$
\begin{align*}
c_{12} &= c_{12}/\sqrt{m|I_0|}
\end{align*}
$$
\begin{eqnarray}
\end{eqnarray}

By using these quantities, we obtain the equations of motion as follows:

$$
\begin{align*}
x' + c_{11} x' + c_{12} \delta_x' + (1 + \Delta_I \cos \Omega') x + \gamma_0 (1 + \Delta_\Omega \cos \Omega') \delta_0 = 0
\end{align*}
$$
$$
\begin{align*}
y' + c_{11} y' + c_{12} \delta_y' + (1 + \Delta_I \cos \Omega') \delta_y^0 = 0
\end{align*}
$$
$$
\begin{align*}
\delta_x' + i \omega \delta_y + c_{12} x' + \gamma_0 (1 + \Delta_I \cos \Omega') x + \delta_0 (1 + \Delta_\Omega \cos \Omega') \delta_0 = 0
\end{align*}
$$
$$
\begin{align*}
\delta_y' - i \omega \delta_x + c_{12} \gamma_0 (1 + \Delta_I \cos \Omega') \delta_y + \gamma_0 (1 + \Delta_\Omega \cos \Omega') \delta_0 = 0
\end{align*}
$$
\begin{eqnarray}
\end{eqnarray}

where the prime is omitted. We know that the form of the parametric excitation terms in Eq. (5) is different from that in the unsymmetrical shaft system and the unsymmetrical rotor system(3)(4) which are well known as representative parametrically excited rotating shaft systems. Therefore, it is expected that

![Fig. 1](image1)

**Fig. 1** A model of a rotating shaft system

![Fig. 2](image2)

**Fig. 2** The variations of the spring constants $a$, $r$, and $\delta$ versus the axial force $F$
the types of unstable vibrations in this system are
different from those in the unsymmetrical shaft system
and the unsymmetrical rotor system. In the latter
system\(^{14} \) with natural frequencies \( p_i (i=1,4, p_i >\ p_i > p_i > p_i) \), the parametrically excited oscillations
appear near the rotating speeds \( \omega = p_i, \omega = p_i \) and \( \omega = (p_i + p_i)/2 \).

In numerical calculations, we use the following
values as for the concrete dimensions of the model
shown in Fig. 1. The dimensions of the shaft are the
same as used in Fig. 2. The dimensions of the rotor are
\( m=7.823 \) kg, \( I_p=0.225 1 \) kg \( \cdot \) m\(^2\) and \( l=0.112 6 \) kg \( \cdot \) m\(^2\).
As for the values of the parameters in Eq. (3), \( \alpha_0 =
23.72 \) N \( /\) m\(^2\), \( \Delta \alpha_i/\alpha_0=0.1536, \gamma_i = -30.13 \) kN \( /\) rad, \( \Delta \gamma_i/\gamma_0 =
-4.9 \times 10^{-4}, \delta_0 = 5.567 \) kN \( \cdot \) m \( /\) rad, and \( \Delta \delta_i/\delta_0 = 0.075 1 \)
are adopted. These are the average and fluctuating
values obtained from the values of \( a, \gamma, \delta \) when
\( F = \pm F_0 = (F_0=2.94 \) kN\). Considering the magnitudes
of these values, we assume in the following analysis
that the parameters \( \Delta i (i=1,2,4) \) in Eq. (4) have
the magnitude of the same order as that of the small
quantity \( e \). We designate this order by the symbol
\( O(e) \).

3. A Frequency Equation and a Natural Frequency

We consider the solution for a natural vibration in
the system obtained by neglecting the damping in Eq.
(5). If a component with the natural frequency \( p \)
appears, components with various frequencies are
derived due to the parametrically excited term.

Thus we can express the approximate solution with
an accuracy of \( O(e) \) as follows:

\[
\begin{align*}
\theta_1 &= \frac{1}{2} \left( \frac{\sin \alpha_t + \sin \alpha_t}{\cos \alpha_t + \cos \alpha_t} \right) \\
\theta_2 &= \frac{1}{2} \left( \frac{\sin \alpha_t + \sin \alpha_t}{\cos \alpha_t + \cos \alpha_t} \right)
\end{align*}
\]

where \( \alpha = p, \beta = \Omega, \alpha = \Omega, \alpha = p + 2 \Omega \) and \( \alpha = p - 2 \Omega \). The magnitudes of the amplitudes \( \alpha, \beta, \alpha' \) and \( \beta' \) are \( O(e) \) for \( i=1, O(e) \) for \( i=2 \) and 3, and
\( O(e) \) for \( i=4 \) and 5. Substituting Eq. (6) into Eq.
(5) with no damping, and equating the corresponding
coefficients in both sides with an accuracy of \( O(e) \),
we obtain the frequency equation as follows:

\[
\begin{align*}
H(p) \gamma_0 &= \Delta_1/2 \Delta_2/2 \Delta_3/2 \Delta_4/2 \\
&= 0 \\
G(p) \gamma_0 &= \Delta_1/2 \Delta_2/2 \Delta_3/2 \Delta_4/2 \\
&= 0 \\
H(p + \Omega) \gamma_0 &= \Delta_1/2 \Delta_2/2 \Delta_3/2 \Delta_4/2 \\
&= 0 \\
H(p - \Omega) \gamma_0 &= \Delta_1/2 \Delta_2/2 \Delta_3/2 \Delta_4/2 \\
&= 0 \\
G(p + \Omega) \gamma_0 &= \Delta_1/2 \Delta_2/2 \Delta_3/2 \Delta_4/2 \\
&= 0 \\
G(p - \Omega) \gamma_0 &= \Delta_1/2 \Delta_2/2 \Delta_3/2 \Delta_4/2 \\
&= 0
\end{align*}
\]

where

\[
\begin{align*}
H(p) &= -p^2, \quad G(p) = \delta_i - i \omega d p - \beta' \quad (7 \cdot a) \\
H(p + \Omega) G'(p + \Omega) - \gamma^2 &= 0 \quad (7 \cdot b) \\
H(p - \Omega) G'(p - \Omega) - \gamma^2 &= 0 \quad (7 \cdot c)
\end{align*}
\]

From Eqs. (7 \cdot a), (7 \cdot b) and (7 \cdot c), we obtain the value
of natural frequency \( p \) which is a function of the
frequency \( \Omega \) of the axial force and the shaft rotating
speed \( \omega \). If the value of \( p \) is complex, an unstable
vibration appears\(^{14} \). Since all roots of Eqs. (7 \cdot b) and
(7 \cdot c) are real and the amplitudes of the components
corresponding to these roots are small, we treat only
the frequency obtained from Eq. (7 \cdot a). Figure 3 shows
the \( p-\Omega \) diagram for \( \omega = 1.0 \). Equation (7 \cdot a)
is reduced with an accuracy of \( O(e) \) as follows:

\[
H(p) G'(p) - \gamma^2 = 0 \quad (9)
\]

We designate the four roots of this equation by \( p_1, p_2,
p_3 \) and \( p_4 \) in the order of magnitude. For the curves
corresponding to these values, the same symbols are
written in Fig. 3. When the frequency \( \Omega \) is close to
the values of \( p_1-p_2, p_1-p_3, p_1-p_4 \) and \( p_2-p_3 \) whose \( p_i \)
are calculated from Eq. (9), two real roots with a
magnitude of \( O(e) \) disappear in the frequency range
denoted by A, B, C and D as shown in Fig. 3. In these
positions, two of the roots are conjugate complex
numbers, and therefore unstable vibrations occur. We
call the unstable vibration appearing near \( \Omega = p_1 \),
the \( p_1 - p_i \) mode oscillation.

We will consider the effect of the gyroscopic
moment on the natural frequency. Owing to the
gyroscopic action, the values of \( p_1 \) and \( p_2 \) become large as
the rotating speed \( \omega \) increases, and the absolute values
of \( p_3 \) and \( p_4 \) become small\(^{19} \). Thus, when the value of
\( \omega \) is larger than that of Fig. 3, the \( p-\Omega \) diagram whose
\( p_1 - p_i \) curves shift upward is obtained.

4. Width of an Unstable Vibration of \( p_1 - p_i \equiv \Omega \)
Mode

4.1 Theoretical analysis

From the analysis in chapter 3, it follows that, in
the system with no damping, two of the natural
frequencies \( p_i \) become complex numbers near the
frequency \( \Omega = \Omega \) satisfying the relationship \( p_1 - p_2 = \Omega \)
\( (i=1, 2, j=3, 4) \), and that the solution for a natural vibration
with a constant amplitude given by Eq. (6) does not
exist near such frequency \( \Omega \). The above-mentioned
method of analysis is available for the purpose of
finding the value of \( \Omega \) where an unstable vibration
occurs in a no damping system. In a practical system
with damping, however, this method is useless to
investigate the width of an unstable vibration, the
presence of a steady-state oscillation, and the time
history of the amplitude of an unstable vibration.
Accordingly, in this chapter, we analyze the
parametrically excited oscillation by another method
although the frequency \( \Omega \), where the method is applicable,
is limited near \( \Omega \).

Near \( \Omega = \Omega \), we assume the approximate solution
for Eq. (5) with an accuracy of \( O(e) \) as follows:
\[
\begin{align*}
\frac{dx}{t} &= \sum_{k} \left[ R_k \cos \theta_k + e^{i \omega_k t} (a_k \cos \theta_k + b_k \sin \theta_k) \right] \\
\frac{dy}{t} &= \sum_{k} \left[ R_k \sin \theta_k + e^{i \omega_k t} (a_k \sin \theta_k - b_k \cos \theta_k) \right]
\end{align*}
\]

where \( \theta_i = \omega_i t + \delta_i \) and \( \theta_j = \omega_j t + \delta_j \). In Eq. (10), \( \sum_{k=1}^{N} \) means the summation of the terms for \( k = i \) and \( j \).

The amplitude and the phase angles on Eq. (10) are assumed to have the magnitude of \( O(\varepsilon^0) \) and varying slowly with time. As the frequencies \( \theta_i \) and \( \theta_j \) are considered to be proportional to \( \Omega \) with an accuracy of \( O(\varepsilon) \), we get the relations \( \delta_i = \omega_i \) and \( \delta_j = \omega_j \) with the same accuracy, where

\[
\omega_i = (p_i/Q_i) \Omega, \quad \omega_j = (p_j/Q_j) \Omega
\]

Next, with an accuracy of \( O(\varepsilon) \), the following relation holds:

\[
\delta_i - \delta_j = \Omega
\]

From Eqs. (11) and (12), we obtain the following equation:

\[
\delta_i - \delta_j = 0
\]

Substituting Eq. (10) into Eq. (5), and equating the coefficients of the phases \( \theta_i \) and \( \theta_j \) on both sides, respectively, we get the ratio of the amplitudes \( R_i \) and \( R_j \) as follows:

\[
\frac{R_i}{R_j} = \frac{H(\omega)}{G(\omega)} (k = i, j)
\]

Since the relations \( H(\omega) = H(p) \) and \( G(\omega) = G(p) \) hold with an accuracy of \( O(\varepsilon) \), it follows from Eq. (14) that the amplitude ratio is constant. Designating this ratio by \( R_i \), we obtain

\[
R_i = x_i R_i, \quad R_j = x_j R_j
\]

Next, equating the coefficients of the phases \( \theta_i \) and \( \theta_j \), with an accuracy of \( O(\varepsilon) \), respectively, we obtain the following equations:

\[
\begin{align*}
M_i R_i \delta_i &= \sigma_i R_i + \Delta_i R_i \cos \phi \\
M_i R_i \delta_j &= -c_i \omega_i R_i + \Delta_i R_i \sin \phi \\
M_j R_i \delta_j &= \sigma_j R_j + \Delta_j R_j \cos \phi \\
M_j R_i \delta_j &= -c_i \omega_i R_i - \Delta_j R_i \sin \phi
\end{align*}
\]

where

\[
\begin{align*}
\sigma_i &= 2(2a_i + x_i^2) - i_p \omega x_i^2 \\
c_i &= 2(\omega_i + x_i x_0 c_0 + x_i^2 c_2) \\
\sigma_j &= 2[H(\omega_i)G(\omega_i) - \gamma_i]G(\omega_i) \\
\Delta_i &= \Delta_i + (x_i + x_j) \gamma_i \Delta_i + x_i x_j \Delta_i \\
(\Delta_i = \Delta_i, \Delta_j = \Delta_j)
\end{align*}
\]

When the amplitudes \( R_i \) and \( R_j \) are not zero, the first and third equations in Eq. (16) reduce to

\[
\begin{align*}
\phi &= \frac{1}{M_i} \sigma_i + \Delta_i \left( \frac{R_i}{R_j} \right) \cos \phi \\
\frac{1}{M_j} \sigma_i + \Delta_i \left( \frac{R_i}{R_j} \right) \cos \phi
\end{align*}
\]

We can determine the unknown variables \( R_i, R_j \) and \( \phi \) from the second and fourth equations of Eq. (16) and Eq. (18). The steady-state solutions can be obtained by putting \( R_i = R_j = \phi = 0 \). As the result, we see that there exists no steady-state solution with a nonzero amplitude, and that the steady-state solutions of \( R_i = 0 \) and \( R_j = 0 \) exist. Since the phase angles of the solutions \( R_i = 0 \) and \( R_j = 0 \) are indeterminate, we cannot discuss the stability of these solutions by Eq. (16). So, using the transformations

\[
u_a = R_a \cos \delta_a, \quad \nu_b = R_a \sin \delta_a (k = i, j)
\]

we change Eq. (16) into the equation for the variables \( \nu_a \) and \( \nu_b \). As a result, we get the following equation:

\[
\begin{align*}
M_i \dot{\nu}_i &= -c_i \omega_i \nu_i + \sigma_i \nu_i - \Delta_i \nu_i \\
M_j \dot{\nu}_j &= \sigma_j \nu_j - c_i \omega_i \nu_j + \Delta_j \nu_j \\
M_j \dot{\nu}_j &= -c_i \omega_i \nu_j - \sigma_j \nu_j - \Delta_j \nu_j \\
M_j \dot{\nu}_j &= \sigma_j \nu_j - c_i \omega_i \nu_j + \Delta_j \nu_j
\end{align*}
\]

We introduce the small deviations \( \xi_i, \eta_i, \xi_j, \) and \( \eta_j \) from the steady-state solutions \( \nu_i = \nu_i = \nu_j = 0 \), respectively. Substituting the equations

\[
u_i = \xi_i, \quad \nu_j = \eta_i, \quad \nu_j = \xi_j, \quad \nu_j = \eta_j
\]

into Eq. (20), and neglecting the terms smaller than \( O(\varepsilon) \), we get the following differential equations:

\[
\begin{align*}
M_i \dot{\xi}_i &= -c_i \omega_i \xi_i - \sigma_i \xi_i + \Delta_i \xi_i \\
M_j \dot{\eta}_i &= \sigma_i \xi_i - c_i \omega_i \eta_i + \Delta_j \eta_i \\
M_j \dot{\xi}_j &= -c_i \omega_i \xi_j - \sigma_j \xi_j - \Delta_j \xi_j \\
M_j \dot{\eta}_j &= \sigma_j \xi_j - c_i \omega_i \eta_j + \Delta_j \xi_j
\end{align*}
\]

The characteristic equation can be obtained from Eq. (22). By applying Routh–Hurwitz’s stability criterion to this characteristic equation, we can get the stability condition. For example, the stability condition for the case of no damping \( c_i = c_j = 0 \) is as follows:

\[
(\sigma_i M_i + \sigma_j M_j)^2 + 2(\Delta_i M_i)(\Delta_j M_j) > 0
\]

If Eq. (23) does not hold, the solution with zero amplitude becomes unstable and an unstable vibration appears.

4.2 Results of numerical calculations

We can determine the width of an unstable region by the method discussed in section 4.1. In the present section, the width is considered in a three-dimensional diagram whose axes represent \( \Delta \) relating to the axial force \( F_a \), the frequency \( \Omega \) of the axial force, and the rotating speed \( \omega \) of the shaft. Figures 4 and 5 are the cross sectional views of this three-dimensional diagram and represent the change of the unstable region, when \( \Delta \) and \( \omega \) are constant for the system with damping. In these figures, the shaded zones represent the unstable regions, and the others the stable regions. Also, the symbols \( \circ \) and \( \bullet \) represent the results of numerical simulations that will be described in section 4.3. From Figs.3, 4, and 5, the following characteristics are clarified as to the occurrence of the unstable vibration:

(1) In the rotating shaft system, only the parametrically excited oscillations of summed-and-differentiable types appear, that is, \( p_i - p_j = \Omega T (i = 1, 2, j = 3, 4) \) modes when the shaft is rotating \( \omega \neq 0 \). (2) As the rotating speed \( \omega \) decreases, these parametrically excited oscillations converge to the corre-
sponding oscillations which occur in a column subjected to a periodic axial force. Since \( p_1 = |p_1| \) and \( p_2 = |p_2| \) at \( \omega = 0 \) in Fig. 1, the oscillations of \( p_1 - p_1 \approx \Omega \) and \( p_2 - p_2 \approx \Omega \) modes converge to those of \( 2p_1 \approx \Omega \) and \( 2p_2 \approx \Omega \) modes, respectively, which are subharmonic types with a single mode. Also, the oscillations of \( p_1 - p_1 \approx \Omega \) and \( p_2 - p_2 \approx \Omega \) modes converge to that of \( p_1 + p_1 \approx \Omega \) mode with the summed type, but the oscillation of \( p_1 + p_2 \approx \Omega \) mode vanishes in Fig. 4 due to damping. (3) The width of the unstable region of \( p_2 - p_2 \approx \Omega \) mode is not so influenced by the rotating speed \( \omega \). The oscillation of \( p_1 - p_1 \approx \Omega \) mode is not likely to appear in the high-speed region of \( \omega \), while the oscillations of \( p_1 - p_1 \approx \Omega \) and \( p_2 - p_2 \approx \Omega \) modes become liable to occur as \( \omega \) increases (see Fig. 4). (4) From Fig. 5 it follows that the unstable region becomes wide proportionally to the axial force \( F_a \).

4.3 Numerical simulations

In order to ascertain the approximate analysis mentioned above, we investigate the appearance of unstable vibrations by integrating Eq. (5) numerically. As an example, we show the waveform of the unstable vibration of \( p_2 - p_2 \approx \Omega \) mode in Fig. 6. From this figure we see that the two vibratory components with the frequencies close to the natural frequencies \( p_1 \) and \( p_2 \) grow gradually with time. We plot these results.
of numerical integration by symbol for the damped vibration and by symbol for the unstable vibration in Fig. 4. These results of the simulation are in good agreement with the stability boundaries obtained in section 4.2.

4.4 The effects of the position and dimension of the rotor on the width of the unstable region

We investigate the effects of the mounting position of the rotor, that is, the ratio to shown in Fig. 1, and the rotor dimension. For example, we show the results for to in the case with damping in Figs. 7(a) and 7(b), respectively. From Fig. 7 we see that the width of the unstable region varies markedly with the mounting position, that is, the ratio . However, this variation is not monotonous. For example, Fig. 8 shows the effect of the rotor position on each unstable region at . The variations of the width of the unstable region are also different from each other depending on the modes of oscillation.

The width of the unstable region of when , which corresponds to and , is extremely small in Fig. 7. This width becomes zero depending on the mounting position of the rotor, as shown in Fig. 8. This width may also vanish depending on the rotor dimension. Figure 9 shows the variation of the width of the unstable region when changing of the rotor diameter while keeping the rotor mass constant (kg), when . The width of the unstable region becomes zero only for the mode when the value of takes a certain value. Figure 7 is an example where the width of the unstable region of is extremely small in Figs. 8 and 9.

5. Conclusions

The conclusions about the parametrically excited oscillations of a rotating shaft subjected to a periodic axial force are summarized as follows:

Fig. 7 Effects of the mounting position of the rotor

Fig. 8 The widths of unstable regions versus the mounting position of the rotor (a case in which the shaft length is constant and )

Fig. 9 The widths of unstable regions versus the rotor dimension (a case in which the rotor mass is constant and )
(1) The terms of parametric excitation in the equation of motion have a different constitution from those in the unsymmetrical shaft system and the unsymmetrical rotor system.

(2) The modes of the parametrically excited oscillations, which appear in the rotating state of the shaft, are different from those in the column system subjected to a periodic axial force.

(3) When the shaft is rotating, only the oscillations of the summed-and-differential type of $\Omega$ modes occur, and the oscillations of the summed-and-differential type of $p_i + p_i \Omega$ modes and the subharmonic type of $2p_i \Omega$ modes, which appear in the column system, do not occur.

(4) The natural frequencies of the system vary with the rotating speed $\omega$ of the shaft. So, as $\omega$ approaches zero, the oscillations of $p_i - p_i \Omega$ modes converge to those of $2p_i \Omega$ or $p_i + p_i \Omega$ mode appearing in the column system.

(5) The effect of the rotating speed $\omega$ on the width of the unstable region is different depending on the modes of the oscillations. According to the modes, the variations of the width are classified into three groups. They are that the width becomes wider as $\omega$ increases, that it becomes narrower, and that it is almost constant. Therefore, the mode of the parametrically excited oscillation which is liable to occur is different according to the rotating speed $\omega$.

(6) The width of the unstable region depends on the dimension and the mounting position of the rotor.

References


