An Approximate Model with an Infinite Number of Vehicles for Analysis of Coupled Vibrations Between Railway Vehicle Wheels and Rail in the Vertical Direction*

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This paper presents an approximate model with an infinite number of vehicles in which an infinite number of wheels are arranged at intervals on the head of an infinitely long and straight rail installed on a distributed spring in the vertical direction. This model is very effective for the analysis of the coupled vibrations between railway vehicle wheels and a rail in the case of a large number of wheels. Using the analytical model, the contribution of rail to the coupled vibrations was examined for each section partitioned by the wheels of one Shinkansen vehicle. As a result, it became clear that rail vibration between truck wheels is dominant in the frequency region below 2 kHz and that of the rail near a coupler between vehicles is also significant in the frequency region above 1.5 kHz.

Key Words: Vibration of Continuous System, Railway, Rail and Wheels, Random Vibration, Coupled Vibration

1. Introduction

It is the rolling noise attributable to the irregularity of the surfaces of contact between wheels and rail that is the most serious cause of noise in the Shinkansen. There have been a number of experimental investigations into the rolling noise. However, the authors have found no investigations which deal with the estimation of the contribution of both wheels and rail to the coupled vibrations, taking the interaction between wheels and rail into account. For the purpose of the theoretical explanation of the rolling noise, they have analyzed separately the coupled vibrations through the points of elastic contact between wheels and rail in the vertical and the horizontal directions in previous reports^{(1)~(3)}. Moreover, they examined the characteristics of the coupled vibrations for the systems with one and two wheels on the rail and the

interference between wheels through the points of contact.

In this report, the vertical coupled vibration between wheels and rail is analyzed, keeping the actual train composition in mind. However, the analytical treatment becomes complicated and the computing time is remarkably increased with the increasing number of wheels if the analytical method described in the previous report⁽¹⁾ is applied directly to this problem. Therefore, the authors present an approximate model in which an infinite number of wheels are arranged at regular intervals on the head of an infinitely long and straight rail installed on a distributed spring in the vertical direction.

The concept of periodic arrangement has often been used for the vibrational systems. For example, we can refer to analyses of structures with a certain periodicity such as a beam supported by springs arranged at regular intervals^{(4)~(7)}. There is also the analysis of the system in which an infinite number of vehicles travel at regular intervals continuously on the guideway which is regarded as an assembly of beams of equal length simply supported at both ends⁽⁸⁾. In this case, the periodicity is introduced into not only the

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guideway but also the arrangement of the vehicles. The condition of the periodicity is introduced in these analyses after the beams have been partitioned into the elements with finite length. Therefore, the analytical treatment generally becomes complicated when the coupling between beams and wheels or vehicles is considered.

The approximate model presented in this report is based on the method of Fourier complex transformation and the superposition of the solutions. This method has the advantage that a solution is easily obtained by some changes of the analytical results in the previous reports⁽¹⁾, and the calculation time can be greatly reduced. The effectiveness of this approximate model is confirmed by the numerical computation applied to the Shinkansen and the contribution to the total vibration of the rail is examined for each section partitioned by the wheels of one Shinkansen vehicle.

2. Approximate Model with an Infinite Number of Vehicles

The rail is infinitely long and it is modelled by straight Timoshenko beam installed on a distributed spring in the vertical direction whose spring constant is k_z . Some assumptions used in the present analysis are the same as in the previous reports⁽¹⁾.

We consider the system in which an infinite number of wheels are arranged with a regular periodicity on the head of the rail. They travel in the longitudinal direction, that is, the x -direction at a constant speed v . Namely, we introduce an approximate model with an infinite number of vehicles as shown in Fig. 1. In the following analysis, the moving coordinate $\bar{x} = x - vt$ (t is time) is divided into an infinite number of sections of finite length l . We put the coordinate of a point ξ away from the left hand end of the N -th section as $\bar{x}^N = Nl + \xi$ ($0 \leq \xi \leq l$), where $N = 0, \pm 1, \dots, \pm \infty$, and l corresponds, for example, to the length of a Shinkansen vehicle and so on.

2.1 Response of rail to shearing forces arranged periodically

First, the results obtained in the previous report⁽¹⁾ are summarized. When a shearing force $P(t) = \bar{P} \exp$

Fig. 1 Arrangement of the wheels (dimension in m)

$(j\omega t)$ acts at $\bar{x} = \bar{x}_s$, the displacement amplitude of the rail $\bar{z}_r$ at $\bar{x} = \bar{x}_r$ is represented by

$$\bar{z}_r = j\Gamma \bar{P} f(\bar{X}_r - \bar{X}_s, A) \quad (1 \cdot a)$$

where $j^2 = -1$, ω is a circular frequency of the shearing force,

$$\begin{aligned} \text{Im}(\alpha_1), \text{Im}(\alpha_2) > 0; \text{Im}(\alpha_3), \text{Im}(\alpha_4) < 0, \\ \bar{X}_r = \beta \bar{x}_r, \bar{X}_s = \beta \bar{x}_s, \Gamma = 4\beta/k_z, \beta^4 = k_z/4EI, \end{aligned}$$

$$f(\bar{X}, A) = \begin{cases} \sum_{i=1}^2 A_i \exp(j\alpha_i \bar{X}) : \bar{X} \geq 0 \\ -\sum_{i=3}^4 A_i \exp(j\alpha_i \bar{X}) : \bar{X} \leq 0 \end{cases} \quad (1 \cdot b)$$

and EI is the flexural rigidity of the rail. α_i ($i=1, \dots, 4$) are roots of $\Delta=0$ of Eq. (5) and A_i are complex constants of Eq. (6) in the previous report⁽¹⁾.

Using the results mentioned above, we consider the system in which a shearing force in each section acts at regular intervals as shown in Fig. 2(a). Every shearing force is of equal amplitude, but has the phase difference (time difference) of $T=l/v$ per section. The phase difference is considered because of the fact that the wheels pass at certain time intervals on the rail with the sinusoidal roughness in the vertical direction. We put the shearing force acting in the K -th section ($K=0, \pm 1, \dots, \pm \infty$) as

$$P^K(t) = \bar{P} \exp[j\omega(t + KT)]. \quad (2)$$

As shown in Fig. 2(a), the acting force $P^K(t)$ is acting at $\bar{x}_s^K = Kl + \xi_s$, and the response point of the rail in the N -th section is at $\bar{x}_r^N = Nl + \xi_r$, where ξ_s ($0 \leq \xi_s \leq l$) and ξ_r ($0 \leq \xi_r \leq l$) are the local coordinates defined in each section.

We describe the displacement of the rail at $\bar{x}_r^N$ as $\bar{z}_r^N = \bar{z}_r^N \exp(j\omega t)$. The amplitude $\bar{z}_r^N$ at $\bar{x}_r^N$ is represented by the superposition of the displacements at the response point to each shearing force. From Eq. (1), we obtain

$$\bar{z}_r^N = j\Gamma \bar{P} \sum_{K=-\infty}^{\infty} \exp(jK\omega T) f(\bar{X}_r^N - \bar{X}_s^K, A) \quad (3 \cdot a)$$

where

$$\begin{aligned} \bar{X}_r^N = \beta \bar{x}_r^N = Nl + X_r, \bar{X}_s^K = \beta \bar{x}_s^K = Kl + X_s \\ X_r = \beta \xi_r, X_s = \beta \xi_s, L = \beta l \end{aligned} \quad (3 \cdot b)$$

As $|X_r - X_s| \leq L$, Eq. (3) is calculated as follows:

(1) In the case of $X_r \geq X_s$. We divide the summation range $(-\infty, \infty)$ into two parts, that is, $(-\infty, N)$ and $(N+1, \infty)$, and rewrite K to $-K$ for the former range. The result is as follows:

$$\begin{aligned} \bar{z}_r^N = j\Gamma \bar{P} \left\{ \sum_{K=-N}^{\infty} \exp(-jK\omega T) f(Nl + KL \right. \\ \left. + X_r - X_s, A) + \sum_{K=N+1}^{\infty} \exp(jK\omega T) f(Nl - KL \right. \\ \left. + X_r - X_s, A) \right\} \quad (4) \end{aligned}$$

It follows from Eq. (1·b) that the first and second terms on the right hand side of Eq. (4) are an infinite geometrical progressions with the equal ratios of $\exp[\pm j(\alpha_i L - \omega T)]$, where the dual sign means plus for

the first term and minus for the second one. On the other hand, the primary argument of the function f is positive and negative for the first and the second terms respectively. Therefore, Eq. (4) converges because the signs of $\text{Im}(\alpha_i)$ in the function f are also plus and minus for the first and the second terms respectively, and the absolute values of two equal ratios are both less than unity. The result is shown as

$$\bar{z}_r^N = j\Gamma\bar{P} \exp(jN\omega T) \cdot f_+(X_r - X_s, A) \quad (5)$$

(2) In the case of $X_r \leq X_s$. After the summation range is divided into two parts of $(-\infty, N-1)$ and (N, ∞) , we can calculate Eq. (3) in the same way as described above.

$$\bar{z}_r^N = j\Gamma\bar{P} \exp(jN\omega T) \cdot f_-(X_r - X_s, A) \quad (6)$$

The functions f_+ and f_- in Eqs. (5) and (6) are defined by

$$f_{\pm}(X, A) = \pm \sum_{i=1}^4 \frac{A_i \exp(j\alpha_i X)}{1 - \exp[\pm j(\alpha_i L - \omega T)]} \quad (7)$$

where the dual signs appear in the same order.

(3) Continuity at $X_r = X_s$. We can demonstrate that the displacement of the rail is continuous at the acting points of the shearing forces by subtracting Eq. (5) from Eq. (6) and taking the relations $X_r = X_s$ and $A_1 + A_2 + A_3 + A_4 = 0^{(1)}$ into account.

However, the derivatives of the displacement are not continuous at the acting points of the shearing forces if the rail is modeled by the Timoshenko beam.

(4) Continuity of displacement at boundaries of each section. The amplitude of the rail $\bar{z}_L^N$ at the right end of the N -th section is obtained by substituting $X_r = L$ into Eq. (5).

$$\bar{z}_L^N = j\Gamma\bar{P} \exp(jN\omega T) \cdot f_+(L - X_s, A) \quad (8)$$

The amplitude of the rail $\bar{z}_0^{N+1}$ at the left end of the $(N+1)$ -th section is obtained by substituting $X_r = 0$ into Eq. (6).

$$\bar{z}_0^{N+1} = j\Gamma\bar{P} \exp[j(N+1)\omega T] \cdot f_-(-X_s, A) = \bar{z}_L^N \quad (9)$$

Hence, it turns out that the displacement is continuous at the boundaries of each section.

From the results obtained above, when shearing force $P^K(t) = \bar{P} \exp[j\omega(t + KT)]$ acts at the point X_s , represented by the local coordinate defined in each section ($K=0, \pm 1, \dots, \pm \infty$), the displacement amplitude of the rail at the local point X_r in the N -th section is summarized as follows:

$$\bar{z}_r^N = j\Gamma\bar{P} \exp(jN\omega T) \cdot \bar{f}(X_r - X_s, A) \quad (10)$$

where

$$\bar{f}(X, A) = \begin{cases} f_+(X, A) & 0 \leq X \leq L \\ f_-(X, A) & -L \leq X \leq 0 \end{cases} \quad (11)$$

It follows from Eq. (10) that the displacement response in the N -th section is obtained from the product of that in the 0-th section and $\exp(jN\omega T)$. Namely, the amplitude responses of the displacement in respective sections are equal to one another. Thus,

we need only pay attention to the 0-th section in analysis and the superscript 0 (meaning section number) is omitted in the following. As a result, the response of the system shown in Fig. 2(a) is obtained only by replacing the function $f(\bar{X}, A)$ in Eq. (1) by the function $\bar{f}(X, A)$ in Eq. (11).

Next, we consider the case that more than one shearing force is applied to each section. In this case, the resulting response is represented by superimposing the responses to every shearing force acting in each section mentioned above. As shown in Fig. 2(b), the magnitudes of n shearing forces in each section are not necessarily equal to one another. The response point is located at ξ (in the following, the dimensionless local coordinate $X = \beta\xi$ is used to specify a response point, where $0 \leq X \leq L$). The position of the k -th shearing force in each section is defined as $X_k = \beta\xi_k$ in the local coordinate system, where $0 \leq X_k \leq L$ and $k=1, \dots, n$. We write the shearing force as $P_k(t) = \bar{P}_k \exp(j\omega t)$. Then the amplitude response of the rail in the 0-th section is given by the superposition of Eq. (10) as

$$\bar{z}(X) = j\Gamma \sum_{k=1}^n \bar{P}_k \bar{f}(X - X_k, A) \quad (12)$$

By substituting $X = X_1, \dots, X_n$ into Eq. (12), the

(a) A shearing force per section applied to the rail

(b) n shearing forces acting in the 0-th section

(c) Wheels-rail system in the 0-th section

Fig. 2 Approximate model with an infinite number of vehicles

amplitudes at the applied points of the shearing forces may be determined. These are represented in the matrix form as

$$\mathbf{d} = \mathbf{G}(\omega) \mathbf{P} \quad (13)$$

where $r, s = 1, \dots, n$,

$$\mathbf{d} = {}^t(\bar{z}_1, \dots, \bar{z}_n), \quad \mathbf{P} = {}^t(\bar{P}_1, \dots, \bar{P}_n), \quad \left. \begin{aligned} \mathbf{G}(\omega) &= \begin{bmatrix} \Phi_{11}(\omega) & \dots & \Phi_{1n}(\omega) \\ \vdots & & \vdots \\ \Phi_{n1}(\omega) & \dots & \Phi_{nn}(\omega) \end{bmatrix} \\ \Phi_{rs}(\omega) &= j\Gamma \bar{f}(X_r - X_s, A) \end{aligned} \right\} \quad (14)$$

The element $\Phi_{rs}(\omega)$ of the matrix $\mathbf{G}(\omega)$ represents the displacement amplitude at the r -th acting point when a shearing force of unit magnitude is applied to the s -th one. It is the so-called dynamic influence coefficient among the acting points defined in each section. Symbol t denotes the transposition.

2.2 In the case that more than one wheel per section travels on the rail

As shown in Fig. 2(c), the wheels are coupled with the rail through the contact elastic springs in the representative section, that is, the 0-th section. The mass of the k -th wheel is m_k , the spring constant of the contact elastic spring is k_{ck} and the local coordinate of the contact point is X_k ($X_k = \beta \xi_k$). Namely, contact points exist between wheels and rail at the acting points of the shearing forces described in section 2.1. The k -th wheel is assumed to be supported by the supporting part with spring constant k_{sk} and damping coefficient c_{sk} respectively. The wheels are rigid except in the neighborhood of the contact points. We consider the periodic vibration system in which such an arrangement of the wheels continues infinitely to both sides repeatedly. The exciting source is assumed here to be the sinusoidal roughness $a \exp(j2\pi x/\lambda)$ (where a is the amplitude and λ is the wavelength of the roughness), that is, the irregularity of the railhead in the vertical direction. The circular frequency of the corresponding shearing force due to the irregularity becomes $\omega = 2\pi v/\lambda$.

The contact force $P_k(t)$ between the k -th wheel and the rail is given from the linear elastic contact theory by

$$P_k(t) = k_{ck}[z_k + a \exp\{j\omega(t + T_k)\} - w_k] \quad (15)$$

where z_k and w_k are the displacements of the rail and the wheel at the k -th contact point respectively and $T_k = \xi_k/v$.

The equation of motion of the k -th wheel is given by

$$m_k \ddot{w}_k + c_{sk} \dot{w}_k + k_{sk} w_k = P_k(t) \quad (16)$$

We write the following expression in order to consider the steady state vibrations:

$$[P_k(t), z_k(t), w_k(t)] = [\bar{P}_k, \bar{z}_k, \bar{w}_k] \exp(j\omega t) \quad (17)$$

Substituting Eq. (17) into Eq. (15) yields

$$\bar{P}_k = k_{ck}[\bar{z}_k + a \exp(j\omega T_k) - \bar{w}_k] \quad (18)$$

Substituting Eq. (17) into Eq. (16) yields

$$\bar{w}_k = u_k(\omega) \bar{P}_k \quad (19a)$$

where $u_k(\omega)$ is the dynamic influence coefficient for the k -th wheel denoted by

$$u_k(\omega) = 1/(k_{sk} + jc_{sk}\omega - m_k\omega^2) \quad (19b)$$

From Eqs. (18), (19) and (13), the equation with respect to the contact spring force $\bar{P}_k$ is given in the matrix form by

$$[\mathbf{I} - \mathbf{K}_c\{\mathbf{G}(\omega) - \mathbf{U}(\omega)\}]\mathbf{P} = \mathbf{K}_c \mathbf{F} \quad (20a)$$

where $\mathbf{I}$ is a unit matrix of order n , and

$$\begin{aligned} \mathbf{P} &= {}^t(\bar{P}_1, \dots, \bar{P}_n), \\ \mathbf{F} &= {}^t[a \exp(j\omega T_1), \dots, a \exp(j\omega T_n)] \\ \mathbf{K}_c &= \text{diag}(k_{c1}, \dots, k_{cn}), \\ \mathbf{U}(\omega) &= \text{diag}[u_1(\omega), \dots, u_n(\omega)] \end{aligned} \quad (20b)$$

Moreover, $\mathbf{G}(\omega)$ is the dynamic influence coefficient matrix defined by Eq. (14).

If $\bar{P}_k$ is obtained from Eq. (20), then the displacement amplitude of the rail is obtained from Eq. (12). Here, $\bar{P}_k$ in Eq. (12) is replaced by the solution of Eq. (20); namely, the amplitude of the contact spring force. The displacement amplitudes of the wheels are also calculated by Eq. (19).

Even if the wheel system has different values of m_k , c_{sk} and k_{sk} in one section, the displacement of the rail is continuous at the boundaries of each section, because the response of the rail is given by the superposition of the solutions obtained in section 2.1 as long as the construction of the system maintains a certain periodicity.

3. Numerical Computational Results and Discussion

(1) First, we deal with the case of a Shinkansen train made up of 6 vehicles, using the analytical method of the previous report⁽²⁾. In reference (2), the coupled vibration between a rail and more than one wheel travelling on it was not treated by adopting the present approximate model but by superimposing merely the solutions of reference (1) for the response of rail. The result was led to the simultaneous equations of n unknowns with respect to the contact spring forces, where n (=24) is the total number of wheels on the rail.

The 60 kg rail and the standard wheel of the Shinkansen are adopted as a calculation model. The wheels are all the same and the static loads are also equal to one another ($7.84 \times 10^4 \text{ N}$). The length of a vehicle and the arrangement of the wheels are based on the actual dimensions of the Shinkansen as shown in Fig. 1. The mass of rail per unit length is 60.8 kg/m. The values of the other parameters used in this report are as follows:

$$\begin{aligned}
 m_k &= 356 \text{ kg}, & k_{ck} &= 1.47 \times 10^9 \text{ N/m} \\
 k_z &= 6 \times 10^7 \text{ N/m}^2, & EI &= 6.36 \times 10^6 \text{ N} \cdot \text{m}^2 \\
 v &= 210 \text{ km/h}, & \mu &= 7.5
 \end{aligned}$$

where μ is the nondimensional parameter defined in the previous report⁽¹⁾. In calculation, we let $k_{sk}=0$ and $c_{sk}=0$ for simplicity.

Figures 3(a) and (b) indicate the envelopes of velocity along the rail at the frequencies 986 Hz and 1 141 Hz respectively, where $\Omega = \omega/\omega_0$, $\omega_0 (= \sqrt{k_z/\rho A})$ is the characteristic circular frequency and ρA is the mass per unit length of the rail ($=60.8 \text{ kg/m}$). The train travels toward the right hand side, where the first vehicle is located at the end of the train. The following became clear :

(a) The vibrations of the rail on the left and the right hand sides of the train decrease monotonously with distance from the train.

(b) The vibration characteristics of the rail under the intermediate vehicles, i. e. the second to the fifth, are almost the same. That is, it is recognized that the vibration characteristics of the rail under the intermediate vehicles are periodic, the period of the length per vehicle.

(c) The vibration levels of the rail under the intermediate vehicles are the highest overall.

The results mentioned above hold true regardless of the frequencies. If the train is made up of 16 vehicles as in the usual Shinkansen, then the regions exhibiting the periodic vibration characteristics of the rail become more widespread.

(2) Figures 4(a) and 4(b) indicate similar

Fig. 3 Velocity distributions of the rail in the longitudinal direction when the train is composed of six vehicles

computational results with Figs.3(a) and 3(b) obtained from the approximate model with an infinite number of vehicles. The values of parameters, the length per vehicle and the arrangement of the wheels ($n=4$) are all as in Figs.3(a) and 3(b). The velocity distributions along the rail under about two vehicles are shown in the figures. The vibration characteristics of the rail under the intermediate vehicles shown in Figs.3(a) and 3(b) coincide well with those in Figs.4(a) and 4(b) obtained from the approximate model respectively.

Equation (20) is the simultaneous equations of n unknowns. If we use the analysis described in the previous report⁽²⁾, in the case of the train made up of 16 vehicles, for example, then we must solve the simultaneous equations of $16 \times 4 = 64$ unknowns as there are four wheels per vehicle. On the contrary, when the approximate model with an infinite number of vehicles is adopted, the simultaneous equations to be solved have only 4 unknowns and so the calculation time may be remarkably reduced.

Thus, when dealing with the vibration and the noise problems occurring in railway vehicles, this model has the great advantage of extracting analytically the periodic vibration characteristics in the region of the intermediate vehicles which may give the most important information.

The computational results from the approximate model with an infinite number of vehicles are shown below.

(3) Figure 5 shows the displacement responses w_1, w_2 of the wheels and z_1, z_2 of the rail at the points of contact 1 and 2 indicated in the figure. Figure 6 indicates the displacement responses of the rail at the central points c_1, c_2 and c_3 between wheels. The results

Fig. 4 Velocity distributions of the rail in the longitudinal direction using an approximate model with an infinite number of vehicles

are summarized as follows:

(a) In the frequency region above 100 Hz, the vibration level of the rail is much higher than that of the wheels. The maximum vibration levels at the points of contact 1 and 2 of the rail are almost same as each other. Roughly speaking, the vibration characteristics of the rail have a vague peak in the vicinity of 1 kHz.

(b) The vibration levels of the rail between wheels become higher in the order, c_2, c_1, c_3 . The vibration level of c_3 is remarkably low.

(c) The beats are clearly recognizable in c_1 and c_2 . The frequency intervals are in inversely proportional to the distance between wheels so that the frequency interval of c_2 is twice that of c_1 .

(d) The maxima of the beats in c_1 and c_2 are shifted to each other and the maxima and the minima of the beats in c_2 roughly correspond to the minima in c_1 .

(e) In the lower frequency region, the vibration levels of the rail are higher at the points of contact than in the region between wheels. In the frequency range from 0.5 to 2 kHz, the average vibration levels at positions 1, 2, c_1 and c_2 are almost equal. The beats are more violent in the order, c_2, c_1 and the points of contact.

(4) As an estimate relative to the noise level, we compute the integration value $\mathcal{Q}^2 J$ with respect to the square of the absolute value of the velocity in the vertical direction, where J is the integral of $|\bar{z}(X)/a|^2$ with respect to X .

In Figs. 7(a) to 7(e) the results are shown for the respective regions illustrated in the figures. Figure 7(a) show the integration value over a vehicle length and Fig. 7(b) to 7(e) show values for each region partitioned by the wheels. So the integration ranges in Figs. 7(a) to 7(e) are different. The results are

Fig. 5 Displacement responses at contact points between wheels and rail

summarized as follows:

(a) Comparing Fig. 7(a) with Figs. 7(b) and 7(c), the pattern of Fig. 7(a) resembles that of Fig. 7(c). From the comparison between Figs. 7(b) and 7(c), the latter has a higher integration level on average than the former in spite of the shorter integral range. Especially in the vicinity of 1 kHz, the latter is higher by about 10 dB than the former. As the result, the integration level is high in the regions near the couplers between vehicles and the trucks.

The integration range of Fig. 7(c) is further divided into two parts, that is, Figs. 7(d) and 7(e), and the results are compared with Fig. 7(a).

(b) The peaks of the beats denoted by $a, b, c, \dots$ and $c', d', \dots$ in Fig. 7(a) correspond to those of similar notation in Figs. 7(e) and 7(d).

(c) In Fig. 7(a), beats corresponding to Fig. 7(e) can be recognized in the frequency region below 1 kHz. Beats corresponding to Fig. 7(d) appear in the vicinity of 1 kHz, and they become dominant in the frequency region above 1.5 kHz.

(d) As a result, the vibration of the rail between wheels in a truck is dominant in the frequency region below 2 kHz and that of the rail near the couplers between vehicles is also important in the frequency region above 1.5 kHz.

(5) The peaks b and c' of the beats shown in Fig. 7(a) correspond to the velocity distributions of Figs. 4(a) and 4(b) for the approximate model respectively. At points b and c' , the vibrations of the rail between wheels in a truck and in the vicinity of the couplers between vehicles are dominant.

(6) Next, we compute the integral estimation when some of the parameters of the Shinkansen

Fig. 6 Displacement responses of the rail between wheels

model are tentatively changed.

(a) Change in distance between truck wheels. It became clear from the computational results mentioned above that the vibrations of the rail in the vicinity of the trucks have significant effects upon the total vibration. Here, we change the distance between

truck wheels. In Figs. 8 and 9, the integral estimations are indicated for the case when the distance between truck wheels is exchanged from 2.5 m (Fig. 1) to 0.8×2.5 m and 1.2×2.5 m respectively (the central positions of the trucks do not change). By comparison of Figs. 7(a), 8 and 9, it became clear that the narrower the distance between truck wheels becomes, the lon-

(a) Region over a vehicle

(b) Central region of vehicle

(c) Regions near couplers

(d) Region under coupler between vehicles

(e) Region between truck wheels

Fig. 7 Comparison of integral estimates in the region of a vehicle

Fig. 8 Change of distance between wheels in a truck

Fig. 9 Change of distance between wheels in a truck

Fig. 10 Change of traveling speed (300 km/h)

ger the frequency interval of the beats becomes and the more violent the fluctuation becomes, and that the integration level does not change as much though the location of the beats shifts with the distance between truck wheels.

(b) Change in the traveling speed. In the near future, a higher speed Shinkansen is planned. Here, we examine the effect of increasing the traveling speed upon the integration level. Figure 10 shows the integration level of the traveling speed at 300 km/h. Figure 11 shows the result for the case of 300 km/h traveling speed, 6.37×10^4 N of static load to a wheel which is planned in the near future for the Shinkansen, and 70 kg/m rail whose cross section is similar to the 60 kg/m rail. Comparing Fig. 7(a) with Fig. 10, there is no significant difference between them in integration levels, but the period of the beats is longer just a little in Fig. 10 than in Fig. 7(a). On the contrary, the integration level in Fig. 11 is lower by about 3 to 4 dB than that in Fig. 7(a). This is caused by reducing the static load and increasing the mass of the rail. Hence, these measures seem to be effective as far as Fig. 11 is concerned.

Incidentally, the exciting source due to the roughness of the railhead may be a random process with the following power spectrum density:

$$S(\omega) = C v^{s-1} / \omega^s \quad (21)$$

where C and s are both constants. The measured values of s are 2 to 4⁽⁹⁾⁽¹⁰⁾⁽¹¹⁾. The exciting sources, that is, the forced displacements at the contact points between wheels and rail, exhibit phase differences, but the power spectrum densities are identical. Therefore, the power spectrum density for the velocity response of the rail at the k -th contact point is expressed as⁽¹⁾

$$S_k(\omega) = \omega^2 |\bar{z}_k|^2 S(\omega) = C v^{s-1} |\bar{z}_k|^2 / \omega^{s-2} \quad (22)$$

Substituting $\bar{z}_k$, obtained from Eq. (12), into Eq. (22) yields the velocity power spectrum density of the rail at the k -th contact point. The integration level of the random vibration is also estimated by the following power spectrum density $S_f(\Omega)$

$$S_f(\Omega) = C' v^{s-1} \Omega^2 J / \Omega^s \quad (23)$$

where C' is constant. From Eq. (23), $S_f(\Omega)$ is propor-

Fig. 11 Changes of traveling speed, static load to a wheel and mass of rail

tional to v^{s-1} . When the traveling speed is increased to 300 km/h, $S_f(\Omega)$ may increase by 3 dB and 9.3 dB for $s=2$ and 4 respectively. Therefore, even if reduction of the static load to a wheel and the scaling up of the rail are performed to the degree assumed here, the increase of the vibration and noise due to increased speed may not be suppressed.

4. Conclusions

Assuming the periodic arrangement of the vehicles, for example, a period of a vehicle length, the authors presented a convenient analytical model for the coupled vibration analysis between wheels and rail. They examined, by numerical computation, how the wheels and the rail contribute to the vertical vibrations of the Shinkansen. The results are summarized as follows:

(1) The vibration characteristics of the rail under the intermediate vehicles have a periodicity when the train comprises many vehicles. Moreover, the vibration levels are higher in the regions of the rail under the intermediate vehicles than under the first and the last ones. Therefore, the approximate model with an infinite number of vehicles in which periodicity of the wheel arrangement is assumed is very effective for the vibration analysis when the train comprises many vehicles.

(2) The vibration of the rail between wheels in a truck is dominant in the frequency region below 2 kHz and that of the rail near a coupler between vehicles is also significant in the frequency region above 1.5 kHz.

References

- (1) Sueoka, A., Ayabe, T., Tamura, H. and Yamamoto, N., Coupled Vibrations between Railway Vehicle Wheels and the Rail (3rd Report, Analysis of Vertical Steady-state Vibrations), Bull. JSME, Vol. 29, No. 258 (1986), p. 4344.
- (2) Sueoka, A., Ayabe, T., Gondo, H. and Tamura, H., Coupled Vibrations between Railway Vehicle Wheels and a Rail (4th Report, Analysis of Coupled Vertical Vibrations between Wheels and a Rail) Trans. Jpn. Soc. Mech. Eng. (in Japanese),

- Vol. 53, No. 485, C (1987), p. 82.
- (3) Ayabe, T., Sueoka, A. and Tamura, H., Coupled Vibrations between Railway Vehicle Wheels and a Rail (Coupled Vibration Characteristics in the Horizontal Direction Taking Creep Forces into Account), JSME Int. J., Ser. III, Vol. 31, No. 1 (1988), p. 75.
 - (4) Munjal, M. L. and Heckl, M., Response of a Periodically Supported Infinite Beam to a Harmonic Transverse Point Force, J. Acustica, Vol. 50 (1982), p. 230.
 - (5) Moritoh, Y., On the Solution of Flexural Vibration of a Bar Supported by Uniformly Spaced Springs and Its Applications to Rail Dynamics, Railway Technical Research Report (in Japanese), No. 1118 (1979), p. 1.
 - (6) Jezequel, L., Response of Periodic Systems to a Moving Load J. Appl. Mech., Vol. 48, No. 3 (1981), p. 613.
 - (7) Smith, C. C. and Wormley, D. N., Response of Continuous Periodically Supported Guideway Beams to Traveling Vehicle Loads, Trans. ASME, Ser. G, Vol. 97, No. 1 (1975), p. 21.
 - (8) Doran, A. L. and Mingori, D. L., Periodic Motion of Vehicles on Flexible Guideways, Trans. ASME, Ser. G, Vol. 99, No. 2 (1977), p. 268.
 - (9) Matsui, N., Power Spectral Density of Track Irregularities, JSME P-SC64 (in Japanese), No. 335 (1984), p. 190.
 - (10) Sato, K., Theoretical Analysis of High Frequency Vibrations of Track, Tech. Note of Railway Engineering Research, (in Japanese), Vol. 34, No. 4 (1977), p. 26.
 - (11) Bender E. K. and Remington P. J., Note of Railway Engineering Research, J. Sound Vib., Vol. 37, No. 3 (1974), p. 328.
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