Suppression of the Effect of Disturbance Applied to Control Systems*

(Application to the Design of Stabilized Platform)

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In the control systems designed on the basis of exact model matching theory, the output of the plant tracks the reference model output exactly but suffers from the errors due to continual disturbance. The effect of continual disturbance can be suppressed by inserting appropriate precompensator before the plant. The design technique is further extended to the case when the plant parameters are unknown. That is, an adaptive control system in which the effect of continual disturbance is suppressed is constructed. The theory presented here can be applied to the design of a stabilized platform which mounts a TV camera in a moving vehicle. In such case, the effect of disturbance must be suppressed thoroughly.

Key Words: Disturbance, Control System Design, Exact Model Matching, Adaptive Control, Stabilized Platform

1. Introduction

In this paper, the design method of control systems is illustrated in which the effect of disturbance applied to exact model matching or adaptive control systems is suppressed. A stabilized platform on which a TV camera is mounted on a moving vehicle or the active suspension of an automobile are practical examples of such control systems.

2. Basic Structure of Control Systems

Control system design methods have been studied for more than forty years. Formerly, they were studied on the basis of the classical control theory⁽¹⁾⁽²⁾, which could not yield more than trial-and-error techniques. With the advent of the modern control theory, control system design theory has advanced greatly^{(3)~(6)}. It is true that the systematization of modern control theory was stimulated by optimal control theory. This has made industrial engineers realize

that modern control theory is synonymous with optimal control theory and is not as useful in practice. The real nature of modern control theory, however, is not the same as optimal control theory, but it brought us the establishment of the concept of the state of dynamic systems and the recognition of the importance of state feedback for constructing control systems. Although the control theory employing state space representation is still being studied, the frequency domain technique (often called the polynomial algebra method), which is an extreme of the advanced modern control theory, is considered to be the most contemporary design method for control systems. The frequency domain technique tells us that the structure of control systems is not merely feedback, but is more sophisticated, as shown in Fig. 1. Pole placement, exact model matching and adaptive control have structures identical to that outlined in this figure⁽⁷⁾.

3. Exact Model Matching

Basically, the objective of control system design is to construct a closed loop system whose transfer function coincides with the prescribed reference model transfer function, subject to the pre-established

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condition of the plant to be controlled. This design philosophy gave rise to the concept of exact model matching. In order for exact model matching to be realized, two principal assumptions must be provided, as will be mentioned later. When these assumptions are not ensured, we are obliged to recede to the pole placement design; even so, a control system with satisfactory characteristics can be obtained.

The plant transfer function is assumed to be described by

$$t_d(s) = r(s)p^{-1}(s), \dots \quad (1)$$

where $\partial[p(s)] = n$, $p(s)$ is monic, $\partial[r(s)] = m$ with $0 \leq m \leq n-1$, and $r(s)$ and $p(s)$ are relatively prime. Let the reference model transfer function be,

$$t_d(s) = r_d(s)p_d^{-1}(s), \dots \quad (2)$$

where $\partial[p_d(s)] = n_d$ and $\partial[r_d(s)] = m_d$. Determination of the reference model is itself an extremely important problem, but its explanation is omitted here⁽⁷⁾⁽⁸⁾. The following two assumptions are needed:

(1) $r(s)$ is a stable polynomial; i. e., the plant is of minimal phase.

$$(2) \quad n_d - m_d \geq n - m \dots \quad (3)$$

Denote plant input, plant output, outer reference input and reference model output by $u(t)$, $y(t)$, $v(t)$ and $y_m(t)$, respectively. Let $\gamma(s)$ be any $n-m$ degree monic stable polynomial. Also, let the high frequency gain of the plant (i. e., the highest degree coefficient of $r(s)$) be k_r . Then, $(1/k_r)\gamma(s)r(s)$ must be an n degree monic stable polynomial. Let $q(s)$ be any $n-1$ degree monic stable polynomial; it is used as the characteristic polynomial of minimal order observer which yields the estimate of state $x(t)$ using $u(t)$ and $y(t)$. Since we use a frequency domain technique, we do not utilize the estimate of $x(t)$ explicitly.

The polynomial equation,

$$k(s)p(s) + h(s)r(s) = q(s)[p(s) - (1/k_r)\gamma(s)r(s)] \dots \quad (4)$$

has unique solutions $k(s)$ and $h(s)$ of degree $n-2$ and $n-1$, respectively. The control law,

$$u(s) = q^{-1}(s)k(s)u(s) + q^{-1}(s)h(s)y(s) + (1/k_r)\gamma(s)t_d(s)v(s), \dots \quad (5)$$

in which the solutions $k(s)$ and $h(s)$ of Eq. (4) are used, makes the closed loop transfer function from $v(s)$ to $y(s)$ match exactly with $t_d(s)$, as is desired⁽⁷⁾. The truth is that the relation between $v(s)$ and $y(s)$ becomes

$$y(s) = \frac{q(s)r(s)\gamma(s)}{q(s)r(s)\gamma(s)} t_d(s)v(s), \dots \quad (6)$$

which indicates the order of the closed loop system accurately. Since $q(s)$ and $\gamma(s)$ are taken as stable and $r(s)$ is stable by the assumption (1), the transfer function reduces to $t_d(s)$.

The structure of the control system derived above is identical to that shown in Fig. 1, which is called the exact model matching system. The block diagram of exact model matching system is shown in Fig. 2. An important relation to be used later is shown here. From Eq. (4), we obtain

$$k_r[q(s) - k(s)] = [k_r h(s) + q(s)\gamma(s)]r(s)p^{-1}(s) \dots \quad (7)$$

Since $r(s)$ and $p(s)$ are relatively prime, $k_r h(s) + q(s)\gamma(s)$ must be divided by $p(s)$. Noticing that the degree of $p(s)$ is n and the degree of $q(s)\gamma(s)$ is $n-1 + n-m (\geq n)$ while the degree of $k_r h(s)$ is at most $n-1$, we obtain the following results:

$$\left. \begin{aligned} h(s) &= -r_e(s)/k_r \\ k(s) &= q(s) - (1/k_r)\alpha(s)r(s) \end{aligned} \right\} \dots \quad (8)$$

where $\alpha(s)$ is the quotient of $q(s)\gamma(s)$ divided by $p(s)$ and $r_e(s)$ is its residue. Thus, the solutions of polynomial equation (4) are easily obtained.

4. Control System with Disturbance

Suppose that a disturbance, $w(t)$, is introduced to the exact model matching system described in the previous section. The motion of the plant is then described by

$$\left. \begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) + b_w w(t) \\ y(t) &= c^T x(t) \end{aligned} \right\} \dots \quad (9)$$

in terms of state space representation. Then, the frequency domain description of the plant will become

$$y(s) = r(s)p^{-1}(s)u(s) + r_w(s)p^{-1}(s)w(s) \dots \quad (10)$$

The disturbance is undetectable.

The following relation is obtained from Eq. (4).

$$\left. \begin{aligned} k(s)u(s) + h(s)r(s)p^{-1}(s)u(s) &= q(s)u(s) \\ -q(s)(1/k_r)\gamma(s)r(s)p^{-1}(s)u(s) &\dots \end{aligned} \right\} \quad (11)$$

However, since $r(s)p^{-1}(s)u(s) = y(s) - r_w(s)p^{-1}(s)w(s)$, the above equation can be rewritten as:

$$\left. \begin{aligned} k(s)u(s) + h(s)r_w(s)p^{-1}(s)w(s) &= q(s)u(s) \\ -q(s)(1/k_r)\gamma(s)y(s) & \\ + q(s)(1/k_r)\gamma(s)r_w(s)p^{-1}(s)w(s) &\dots \end{aligned} \right\} \quad (12)$$

From Eqs(5), (8) and (12), we obtain,

Fig. 1 Basic structure of control system

Fig. 2 Exact model matching system

$$y(s) = t_d(s)v(s) + \frac{\alpha(s)r_w(s)}{q(s)\gamma(s)}w(s) \dots \quad (13)$$

That is, the error $e_w(t)$ due to disturbance is evaluated as

$$e_w(s) = \frac{\alpha(s)r_w(s)}{q(s)\gamma(s)}w(s) \dots \quad (14)$$

Since the transfer function from $w(s)$ to $e_w(s)$ is a stable $\alpha(s)r_w(s)[q(s)\gamma(s)]^{-1}$, we can assert the following:

(1) If $w(t)$ is kept to zero for all time after any instant, $e_w(t)$ will become extinct in time.

(2) If $w(t)$ is continually applied to the plant, $e_w(t)$ cannot remain at zero.

(3) As long as $w(t)$ is bounded, $e_w(t)$ is also bounded.

5. Exact Model Matching with Disturbance-Effect Suppression

Various kinds of mathematical models can be applied to the behavior of disturbance. In this paper, the disturbance is assumed to consist of a DC component and q sinusoidal waves with known frequencies. It is to be noted that neither the amplitude nor phase of each sinusoidal wave must be known. The disturbance can be written as

$$w(t) = a_0 + \sum_{i=1}^q b_i \cos(\omega_i t + \phi_i), \quad (15)$$

where q and ω_i ($i=1,2,\dots,q$) are known, but a_0, b_i ($i=1,2,\dots,q$) and ϕ_i ($i=1,2,\dots,q$) are not assumed to be known. By using Laplace transform of Eq.(15), we obtain

$$w(s) = \frac{W_N(s)}{s \prod_{i=1}^q (s^2 + \omega_i^2)}, \quad (16)$$

where $W_N(s)$ is a polynomial of not more than $2q$ degrees with unknown coefficients. The denominator of $w(s)$ is a known polynomial and is denoted by $W_D(s)$. In order for the exact model matching system to be able to suppress the effect of disturbance, it must have one more freedom than the basic exact model matching system mentioned in Section 4. In order to elicit this freedom, we use a precompensator $t_c(s)$, as shown in Fig.3. The input to the precompensator is denoted by $\bar{u}(t)$, and we put $r_c(s)p_c^{-1}(s) = t_c(s)$, where $p_c(s)$ is taken as $W_D(s)$.

That is,

$$p_c(s) = s \prod_{i=1}^q (s^2 + \omega_i^2). \quad (17)$$

We postulate the following assumption on $p_c(s)$.

Fig. 3 Introduction of precompensator

(3) $p_c(s)$ and $r(s)$ are relatively prime.

The polynomial $r_c(s)$ is arbitrarily chosen as a $2q+1$ degree monic stable polynomial except that it is relatively prime with $p(s)p_c(s)$.

Then, the denominator $p(s)p_c(s)$ and numerator $r(s)r_c(s)$ of the augmented plant $r(s)r_c(s)[p(s)p_c(s)]^{-1}$ must be relatively prime. The exact model matching design procedure is executed for this augmented plant. Since the same k_r and $\gamma(s)$ as in Section 3 can be used, the polynomial equation is written as

$$k(s)p(s)p_c(s) + h(s)r(s)r_c(s) = q(s)[p(s)p_c(s) - (1/k_r)\gamma(s)r(s)r_c(s)], \quad (18)$$

where $q(s)$ is any $n+2q$ degree monic stable polynomial. This time, $k_r h(s) + q(s)\gamma(s)$ is divided by $p(s)p_c(s)$. We use the notation $\alpha(s)$ again for the quotient, the degree of which is $n-m-1$, as before. The control law is determined as

$$\bar{u}(s) = q^{-1}(s)k(s)\bar{u}(s) + q^{-1}(s)h(s)y(s) + (1/k_r)\gamma(s)t_d(s)v(s). \quad (19)$$

By a calculation similar to that in Section 4, we obtain

$$y(s) = t_d(s)v(s) + \frac{\alpha(s)r_w(s)W_N(s)}{q(s)\gamma(s)}, \quad (20)$$

from which it is concluded that the error due to disturbance is

$$e_w(s) = \frac{\alpha(s)r_w(s)W_N(s)}{q(s)\gamma(s)}. \quad (21)$$

The degrees of the numerator and denominator of $e_w(s)$ are $2n+2q-m-2$ and $2n+2q-m$, respectively, and $q(s)\gamma(s)$ is a stable polynomial.

Therefore,

$$\lim_{t \rightarrow \infty} e_w(t) = 0. \quad (22)$$

It has been thus clarified that a disturbance-effect free exact model matching system can be constructed without requirement of detecting $w(t)$, if it is known that the disturbance consists of a DC component and q sinusoidal waves with known frequencies. This means that a stabilized platform can be constructed.

6. Suppression of Disturbance Effect in Adaptive Control

The objective of adaptive control is to keep the closed loop characteristic the same as the desired constant characteristic while suppressing the disturbance effect, under the circumstance that polynomials $r(s), p(s)$ and $r_w(s)$ of the plant and a polynomial $W_D(s)$, which depends on the disturbance, are unknown.

First, we consider the adaptive control without disturbance. Then, the only unknowns are $r(s)$ and $p(s)$. Furthermore, let us return to the situation where the plant is known. Then, the control law (5) can be written as

$$u(t) = (1/k_r) \{ \theta^T \omega(t) + \gamma(p) t_d(p) v(t) \}, \quad (23)$$

where p is a differential operator, and θ and $\omega(t)$ are defined by

$$\theta = [k_r k_{n-2} \cdots k_r k_0 \ k_r h_{n-1} \cdots k_r h_0]^T \quad (24)$$

and

$$\omega(t) = \left[\frac{p^{n-2}}{q(p)} u(t) \cdots \frac{1}{q(p)} u(t) \frac{p^{n-1}}{q(p)} y(t) \cdots \frac{1}{q(p)} y(t) \right]^T, \quad (25)$$

respectively. In definition (24), k_i and h_i are the coefficients of $k(s)$ and $h(s)$, respectively, where

$$\begin{aligned} k(s) &= k_{n-2}s^{n-2} + \cdots + k_0 \\ h(s) &= h_{n-1}s^{n-1} + \cdots + h_0 \end{aligned} \quad (26)$$

Since $r(s)$ and $p(s)$ are unknown in the adaptive control problem, both k_r and θ are unknown. Then, by replacing k_r and θ by their respective estimates $\tilde{k}_r(t)$ and $\tilde{\theta}(t)$, we determine the control law as

$$u(t) = (1/\tilde{k}_r(t)) \{ \tilde{\theta}^T(t) \omega(t) + \gamma(p) t_d(p) v(t) \}. \quad (27)$$

The role of adaptive law is to update (i.e., to yield) $\tilde{k}_r(t)$ and $\tilde{\theta}(t)$ so that the signal $u(t)$ defined by Eq. (27) eventually converges to the signal $u(t)$ defined by Eq. (23). The adaptive law can be derived from the alternative plant dynamics representation;

$$y(t) = \Theta^T \Omega(t). \quad (28)$$

where

$$\Theta = [\theta^T, k_r]^T \quad (29)$$

and

$$\Omega(t) = \left[-\frac{1}{\gamma(p)} \omega^T(t), \frac{1}{\gamma(p)} u(t) \right]^T. \quad (30)$$

It is to be noticed that the estimates $\tilde{\theta}(t)$ and $\tilde{k}_r(t)$ to be used in the control law (27) are the estimates of Θ . Among several adaptive law possibilities, the following law is effective and used here.

$$\begin{aligned} \dot{\tilde{\theta}}(t) &= -\Gamma \frac{\Omega(t) \epsilon(t)}{c + \Omega^T(t) \Omega(t)} \\ \Gamma &= \Gamma^T > 0, \quad c > 0, \end{aligned} \quad (31)$$

where

$$\epsilon(t) = \tilde{\theta}^T(t) \Omega(t) - y(t). \quad (32)$$

We can prove that the combination of control law (27) and adaptive law (31) accomplishes adaptive control; that is, $y(t) \rightarrow y_m(t)$ as $t \rightarrow \infty$ and all signals within the control system are bounded, but the proof is omitted here.

Now, let us consider a disturbance-effect free adaptive control system under the postulation that frequency, amplitude and phase of each component of disturbance are unknown with the exception of the prior knowledge that the disturbance consists of a DC component and q sinusoidal waves. The precompensator is parted into two, as shown in Fig. 4, where λ is any positive number, $p_{co}(s)$ is a $2q$ degree polynomial of the form

$$p_{co}(s) = \prod_{i=1}^q (s^2 + \alpha_i^2), \quad (33)$$

with unknown constant α_i , and $r_{co}(s)$ is any $2q$ degree

monic stable polynomial. Again, we suppose that the plant as well as the disturbance is completely known. Then, the control law must have the following form:

$$\begin{aligned} \bar{u}(s) &= q^{-1}(s) k(s) \bar{u}(s) + q^{-1}(s) h(s) y(s) \\ &\quad + (1/k_r) \gamma(s) t_d(s) v(s), \end{aligned} \quad (34)$$

which is rewritten in terms of $\bar{u}(t)$ (see Fig. 4) as follows:

$$\begin{aligned} \bar{u}(s) &= \frac{1}{k_r} \left\{ \frac{-k_r \bar{p}_{co}(s) q(s) + k_r k(s) p_{co}(s)}{q(s) r_{co}(s)} \bar{u}(s) \right. \\ &\quad \left. + \frac{k_r h(s)}{q(s)} y(s) + \gamma(s) t_d(s) v(s) \right\}, \end{aligned} \quad (35)$$

where $\bar{p}_{co}(s) = p_{co}(s) - r_{co}(s)$, which is of degree $2q - 1$. This control law can be represented in the form of Eq. (24). That is,

$$\bar{u}(t) = (1/k_r) \{ \theta^T \omega(t) + \gamma(p) t_d(p) v(t) \}, \quad (36)$$

where both θ and $\omega(t)$ are $2n + 6q + 1$ vectors. If $\bar{u}(t)$ is determined, we can determine $u(t)$ as

$$u(t) = \int \bar{u}(\tau) d\tau + \bar{u}(t). \quad (37)$$

Since $r(s)$, $p(s)$ and $p_{co}(s)$ are unknown, both k_r and θ in the control law (36) are unknown. Then, we change the control law (36) by using estimates $\tilde{k}_r(t)$ and $\tilde{\theta}(t)$ as follows:

$$\bar{u}(t) = \{1/\tilde{k}_r(t)\} \cdot \{ \tilde{\theta}^T(t) \omega(t) + \gamma(p) t_d(p) v(t) \}. \quad (38)$$

The alternative plant dynamics representation is derived as illustrated below. From Eq. (18) and the relation $k_r h(s) + q(s) \gamma(s) = \alpha(s) p(s) p_c(s)$, we obtain

$$\begin{aligned} \frac{k(s)}{q(s) \gamma(s)} \bar{u}(s) + \frac{h(s)}{q(s) \gamma(s)} y(s) &= \frac{1}{\gamma(s)} \bar{u}(s) \\ &\quad - \frac{1}{k_r} y(s) + (1/k_r) \frac{\alpha(s) p_c(s) r_w(s)}{q(s) \gamma(s)} w(s). \end{aligned} \quad (39)$$

Since $p_c(s)$ converges to $W_p(s)$ as the function of adaptive control progresses, the time function corresponding to the third term on the right-hand side of Eq. (39) converges to zero. Therefore, we can neglect this term. Then, the alternative plant dynamics representation will be obtained as

$$\begin{aligned} y(t) &= -\frac{k_r \bar{p}_{co}(p) q(p) + k_r k(p) p_{co}(p)}{q(p) r_{co}(p)} \frac{1}{\gamma(p)} \bar{u}(t) \\ &\quad - \frac{k_r h(p)}{q(p) \gamma(p)} y(t) + k_r \frac{1}{\gamma(p)} \bar{u}(t). \end{aligned} \quad (40)$$

By using θ and $\omega(t)$ already defined, Eq. (40) can be rewritten as

$$y(t) = [\theta^T, k_r] \begin{bmatrix} -\gamma^{-1}(p) \omega(t) \\ \gamma^{-1}(p) u(t) \end{bmatrix}, \quad (41)$$

from which an adaptive law which yields $\tilde{\theta}(t)$ and $\tilde{k}_r(t)$ is derived. Thus, the disturbance-effect free adaptive control system has been obtained. This

Fig. 4 Partition of precompensator

means that adaptive stabilized platform can be constructed.

7. Simulation

7.1. Exact model matching

The motion of a plant is assumed to be described by

$$y(s) = \frac{10}{s^2 + 4s + 3} u(s) + \frac{1}{s^2 + 4s + 3} w(s).$$

The reference model is assumed as

$$t_d(s) = 4/(s^2 + 2s + 4).$$

We arbitrarily choose $\gamma(s)$ as $s^2 + 6s + 8$. It is assumed that only a stepwise disturbance is applied to the plant. Then, the order of augmented plant is three, and hence the degree of $q(s)$ is two. We arbitrarily choose $q(s)$ as $s^2 + 8s + 15$. The resulting control system is shown in Fig. 5. Figure 6 shows the time function $e_w(t)$ when $v(t)$ is assumed as 0 and $w(t)$ as 5. It is seen that $e_w(t)$ converges to zero, and the disturbance effect is suppressed completely.

7.2. Adaptive control

The motion of a plant is assumed to be described by

$$y(s) = \frac{10}{s+1} u(s) + \frac{1}{s+1} w(s).$$

The reference model is assumed as

$$t_d(s) = 4/(s+4).$$

We arbitrarily choose $\gamma(s)$ as $s+3$. It is assumed that

Fig. 5 Disturbance-effect free exact model matching system

the disturbance is the sum of a step function and one sinusoidal wave. We arbitrarily choose $q(s)$ as $(s+3)^3$, and $r_{cc}(s)$ as $(s+3)^2$. The block diagram of the resulting adaptive control system is shown in Fig. 7. Figure 8 shows the response $y(t)$ when $v(t)$ is assumed as $\sin 0.5t$ and $w(t)$ as $0.125 + 0.125 \sin 0.75t$. It is seen that $y(t)$ effectively tracks $y_m(t)$ as time progresses; that is, the disturbance effect is completely suppressed.

8. Conclusions

It was clarified that the disturbance-effect free exact model matching system can be constructed if the frequency of each sinusoidal component of disturbance is known beforehand, even if the magnitude of the DC component or amplitude of each sinusoidal component is not known. The design technique for exact model matching was extended to adaptive control, where only the number of sinusoidal waves of disturbance is needed for constructing the control system. Thus, the design theory for stabilized platform, a typical control application which needs disturbance-effect suppression, has been established.

Fig. 6 Error response when suffering from disturbance

Fig. 7 Block diagram of adaptive control system

Fig. 8 Response of adaptive control system when suffering from step + sinusoidal wave disturbance

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