Instability Analysis of Groove-Compensated Journal Air Bearings*

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The whirl instability of the groove compensated journal air bearing, which had not been analyzed before due to the complicated bearing shape, is analyzed in this study. The finite element method is used to obtain the numerical solution of the Reynolds equation and the computer simulation method is used to obtain the accurate motion of the shaft within the bearing. Concretely, the locus of the shaft center after a step displacement disturbance is determined through the simulation. Based on the simulated shaft loci, the effects of various design parameters on the stability of the bearings are discussed.

Key Words: Tribology, Lubrication, Air Bearing, Groove Compensation, Whirl Instability, FEM, Simulation

1. Introduction

The groove-compensated journal air bearing is shown in Fig. 1. In the lubrication surface of this bearing, small grooves which are about 10 micrometers deep, are machined to compensate for the air flow. For this bearing, the optimal compensation for high stiffness can be realized by the adjustment of the groove depth, and the bearing is suitable for the high-precision machine tools or high-precision instruments with small bearing clearance.

In order to design the groove-compensated bearing, it is necessary to know the load carrying capacity and the stability of the bearing. So far, the static characteristics have been investigated by the authors with a finite element method⁽¹⁾, and the dynamic properties of the groove-compensated bearing have also been reported^{(2),(3)}. However, in most of the studies about this bearing, the rotational speed has been

ignored or been assumed to be very low. In particular, the whirl instability of the bearing has not been discussed at all because of its sophisticated shape. Therefore, the purpose of this study is to develop a computer simulation program to investigate the whirl instability of the groove-compensated bearing and to give guidance for the design of the high-stability bearing.

Moreover, compared with most methods used to carry out the studies on whirl instability, the assumptions concerning the bearing shape and the bearing

Fig. 1 Configuration of groove-compensated journal gas bearing

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working conditions are kept to a minimum in the present method, namely, the finite element simulation method. Thus with the present method, it is possible to discuss in detail of the effects of all the bearing specific dimensions and working conditions on the whirl stability of the bearing.

2. Nomenclature

- bg, Bg : width of compensation grooves
 $Bg = nb g / (D\pi)$
 D : bearing diameter
 e : eccentricity
 h : clearance
 h_0 : mean clearance
 $H = h/h_0$
 hg : depth of compensation grooves
 $Hg = hg/h_0$
 L : bearing length
 lg, Lg : length of compensation grooves
 $Lg = 2lg / (L - b_f)$
 n : number of the compensation grooves
 p : pressure
 $P = p/p_a$
 p_a : ambient pressure
 p_s : supply pressure
 $p_s = p_s/p_a$
 R : bearing radius
 t : time
 x : coordinate in lubrication surface
 $X = x/R$
 y : coordinate in lubrication surface
 $Y = 2y/L$
 ω_1 : rotational speed
 ω_2 : natural frequency
 $\varepsilon = e/h_0$: eccentricity ratio
 μ : viscosity of air
 Λ : bearing number $= \frac{6\mu\omega_1}{p_a} \left(\frac{R}{h_0} \right)^2$
 σ : squeeze number $= \frac{12\mu\omega_2}{p_a} \left(\frac{R}{h_0} \right)^2$

3. Method of Simulation

The air flow in the bearing clearance is governed by the following Reynolds equation.

$$\frac{\partial}{\partial X} \left(H^3 P \frac{\partial P}{\partial X} \right) + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial Y} \left(H^3 P \frac{\partial P}{\partial Y} \right) = \Lambda \frac{\partial(PH)}{\partial X} + \sigma \frac{\partial(PH)}{\partial T} \quad (1)$$

This is a nonlinear, partial differential equation. Apparently, the exact solution of the Reynolds equation can't be obtained, but a numerical method, i.e., a finite element method with the application of the Galerkin Weighted Residual Method, has been proposed by the authors and the numerical solution

has been successfully obtained^{(1),(2)}.

The dynamic equation which governs the motion of the shaft supported by the bearing is,

$$m \frac{d^2 e}{dt^2} = F \quad (2)$$

where, F is the force acting on the shaft (if no external force exists, then it is the resistance of the lubrication film), m is the mass of the shaft within the bearing, e is the coordinate expressing the motion of the shaft center and t is time.

The motion of the shaft supported by the bearing is determined by both the dynamic equation (2) and the Reynolds equation (1). Therefore, the motion of the shaft supported by the bearing could be simulated if Eq. (1) and Eq. (2) could be solved within the time domain⁽⁴⁾. In this study, the dynamic equation and Reynolds equation are solved, sampling by sampling. By the simulation method, the motion of the shaft within the bearing, i.e., the locus of the shaft center, is obtained. Moreover, the validity of our computational method has already been confirmed⁽⁵⁾.

Concretely, the step response of the shaft is simulated in this study; the motion responding to a step displacement is calculated. The locus of the shaft center after a step displacement is obtained and employed for the discussion of the bearing stability.

The procedure of the simulating computation is as follows.

- (1) Specify the boundary conditions and calculate the resistant force acting on the shaft using the Reynolds equation.
- (2) Apply a small displacement as a step disturbance to the shaft.
- (3) Give the initial conditions, and calculate the pressure distribution and resistant force using the Reynolds equation.
- (4) Remove the force used to give the step disturbance in procedure 2).
- (5) Calculate the shaft displacement and velocity in the next sampling, caused by the difference between the resistant and acting forces, using the dynamic equation.
- (6) Calculate the pressure distribution and resistance in the next sampling using the Reynolds equation.
- (7) Repeat procedures 5) and 6) until the simulated motion is sufficient to analyze the bearing dynamics. The flow chart of this procedure is shown in Fig. 2.

In addition, the compensation grooves can be machined on either the bearing surface or the shaft surface. Although the grooves on the shaft surface are easy to machine, it is assumed that all the grooves are machined on the bearing in this study.

4. Analyzed Examples of the Whirl Phenomena

As with the other kinds of journal air bearings, the whirl instability happens when the rotational speed of the groove compensated journal bearing increases, and the maximum rotational speed is then limited. If we insert the expression of σ and Λ into Eq. (1), then we have,

$$\frac{\partial}{\partial X} \left(H^3 P \frac{\partial P}{\partial X} \right) + \left(\frac{D}{L} \right) \frac{\partial}{\partial Y} \left(H^3 P \frac{\partial P}{\partial Y} \right) = \Lambda \left\{ \frac{\partial(PH)}{\partial X} + 2 \frac{\omega_2}{\omega_1} \frac{\partial(PH)}{\partial T} \right\} \quad (3)$$

where ω_1 is the rotational speed and ω_2 is the natural frequency of the bearing. From the results obtained by many researchers (e.g. refs.⁽⁶⁻¹⁰⁾), it is known that the ω ratio in Eq. (3) is a most important factor affecting the bearing stability. The critical value of the ratio is about 0.5; when the ω ratio becomes less than 0.5, the bearing will tend to be unstable.

In order to keep the variation of the ω ratio to a minimum in our stability discussions, it is assumed that no external force is acting on the normally fitted shaft, i.e., the shaft has no eccentricity in equilibrium state. Moreover, the step displacement applied to the shaft is 14.14% of the mean clearance (10% in both x and y direction).

In Fig. 3, the variation of the locus with respect to the change of the bearing number is shown, where the axis in the radial direction indicates the eccentricity ratio. Apparently, the returning speed to the equilibrium point becomes lower as the rotational speed increases, i.e., the instability of the bearing increases with the rotational speed.

As mentioned above, the ω ratio is a most important factor for the bearing stability. The examples in Fig. 4 show the effect of the ω ratio on the locus of the shaft center. In this figure, an unstable bearing becomes stable when the shaft is made lighter. This means that the lighter shaft leads to a larger ω ratio and a higher stability.

Fig. 2 Flow chart of step response simulation

Fig. 3 Loci of shaft center versus rotational speed

From Figs. 3 and 4, it is clear that the results obtained by our simulation method are in accordance with the usually observed whirl phenomena. The effects of the specific design parameters on the bearing stability will be discussed in detail.

Figure 5 is an example of the groove width ratio changing from 0.1 to 0.3. The stiffness of the bearing becomes slightly lower when the groove is wider. Consequently, the ω ratio becomes smaller and the bearing stability is slightly decreased.

In Fig. 6, the rotational speed becomes slower than that of Fig. 5, and the bearings become stable. Moreover, to minimize the variation in the ω ratio, the depth of the grooves is also changed with the groove width. That is, the grooves become deeper as they grow narrower and the actual values of the groove depths are 0.0135 mm, 0.0165 mm and 0.0215 mm. The loci in the figure are almost the same. Therefore, from the results of Figs. 5 and 6, it is known that the width and the depth of the groove have very little

effect on the stability of the bearings if they have similar ω ratios.

The relationship between the stability and the number of the grooves is shown in Fig. 7. In this example, the stiffness and the ω ratio are almost unchanged, but a slight difference in the locus can be seen. That is, the increase in the number of grooves slightly improves the stability of the bearings. The reason why the increasing number of grooves can improve the stability is that the aerodynamic effect caused by the rotation is thereby decreased.

The relationship between the bearing stability and the supply pressure is shown in Fig. 8. To decrease the variation in the ω ratio, the mass of the shaft is changed. Because the aerodynamic effect is decreased by the higher supply pressure⁽²⁾, the bearing with the higher supply pressure has a higher stability; even the ω ratio is actually smaller.

Figure 9 shows the influence of the slenderness ratio on the bearing stability. In this figure, the mass

Fig. 4 Loci of shaft center versus mass of shaft

Fig. 5 Loci of shaft center versus width of the groove

Fig. 6 Loci of shaft center versus variations of groove width and groove depth

Fig. 7 Loci of shaft center versus the number of the grooves

Fig. 8 Loci of shaft center versus supply pressure

Fig. 9 Loci of shaft center versus slenderness ratio

of the shaft is also changed to keep the variation of the ω ratio to a minimum.

Apparently, a decrease in the slenderness ratio leads to a much higher stability of the bearing. The reason for this is that the aerodynamic effect of the rotation is largely increased when the bearing length becomes longer.

5. Principles for High-stability Design

Based on the discussions carried out above, the following design principles for higher bearing stability are proposed.

(1) Make the bearing stiffness higher and the shaft lighter, so that the ω ratio becomes larger and the stability can become higher.

(2) The groove width and depth should be determined by the optimal combinations to get the higher stiffness and larger ω ratio.

(3) The supply pressure should be made higher to decrease the effect of rotational speed, and then improve the stability.

(4) The slenderness ratio should be designed smaller to increase the stability.

6. Conclusions

By means of the FEM simulation method, the motion of the shaft within the groove-compensated air bearing is successfully analyzed. From the results, the effects of all the design elements on the whirl instability of the groove-compensated air bearings, which can't be investigated by the traditionally used analyzing method, are clarified. And finally, the principles for higher stability bearing are proposed.

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