Experimental Identification of Mechanical Structure with Characteristic Matrices*

(Proposition of Curve Fitting in the Frequency Domain and MCK Identification Using Curve Fitting)

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It is not always possible to obtain characteristic matrices from modal parameters obtained by experimental modal analyses, such as curve fitting. If the degrees of freedom of the mathematical model are different from those of the structure identified, which occurs in almost all cases, contradiction occurs between the modal parameters when transformed to physical parameters. This paper proposes certain constraints that modal parameters must satisfy in order to be transformed to mass, stiffness and damping matrices without contradiction. A nonlinear optimization technique is used to solve these constraints. A new multireference curve fitting method in the frequency domain is also proposed. The number of unknown parameters depend only on the number of degrees of freedom identified, regardless of the number of response points. This method is effective when there are a number of response points. Some experimental models are examined to validate this method.

Key Words: System Identification, Frequency Response Function, Characteristic Matrices, Curve Fitting, Optimization, Modal Parameter, Physical Parameter

1. Introduction

It is very difficult to identify exactly the dynamic behavior of a complicated structure with theoretical or numerical methods, such as the finite element method. It is necessary to develop the experimental modal analysis to confirm the validity of the finite element models of structures. Experimental modal analysis techniques have been used extensively for characterizing the dynamic properties of a structure, thanks to the development of the digital Fourier transform analyzer. There are two methods used in the experimental modal analysis technique, the curve fitting method and the characteristic matrix identification. The former uses modal parameters (natural frequency, natural mode etc.), and the latter uses physical parameters (mass, stiffness, damping matrix etc.). Curve fitting⁽¹⁾⁻⁽³⁾ is used more extensively than characteristic matrix identification because of its simple algorithms, good convergence and quick

computer time. The ultimate objective of the experimental modal analysis is, however, to identify the physical parameters so that the vibration problems can be effectively solved, or eliminated by combining the result to the finite element method or structural dynamic modification. There have been a few previous attempts⁽⁴⁾⁻⁽⁷⁾ to identify the physical mass, stiffness and damping parameters. However, the methods used have generally been extremely complex or have given such a poor result thus, the method cannot be applied to more complicated structures.

In a previous paper⁽⁸⁾, it was suggested that the mass, stiffness and damping matrices can be identified with single-point excited data or arbitrary multipoint excited data using the nonlinear optimization technique. However, the major fault of this method is that it requires too much computing time.

This paper proposes a new method of identifying the mass, stiffness and damping matrices with decreased computer time and good convergence. The mass stiffness and damping matrices are identified by coordinate transformation from modal parameters of curve fitting to physical parameters. If the degrees of freedom of a mathematical model are different from

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those of the structure identified, unfortunately, as occurs in almost all cases, contradiction is encountered between modal parameters when transformed to physical parameters. This paper presents the constraints that the modal parameters must satisfy in order to be transformed to the characteristic matrix without contradiction. A nonlinear optimization technique is used to identify the mass, stiffness and damping matrix from the modal parameters.

This paper also proposes a new curve fitting method in the frequency domain with quick and accurate convergence. Linear parameters (residue, residual terms) and nonlinear parameters (damping factor, natural frequency) exist in the curve fitting of the frequency domain. Only the nonlinear parameters are dealt with as unknown parameters by considering the linear parameters as a function of the nonlinear parameters. Hence, the number of unknown parameters can be reduced, which is effective in multireference curve fitting.

The proposed method is tested according to the experimental data obtained from the bar, cylinder block and crankshaft of an automobile. The data shows good agreement thus validation of these methods is obtained.

2. Relation Between Physical and Modal Parameters

Consider the equations of motion of n degrees of freedom system

$$\begin{bmatrix} C & M \\ M & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \quad (1)$$

which can be written in the $2n$, 1st order differential equations

$$[D]\{\dot{y}\} + [E]\{y\} = \{P\} \quad (2)$$

where $[M]$, $[C]$ and $[K]$ are the $(n \times n)$ mass, damping and stiffness matrices respectively. $\{x\}$ is a resulting displacement vector, and $\{f\}$ is an applied force vector, while the dots denote differentiation with respect to time. The solution of the eigenvalue problem yields a set of $2n$ eigenvalues λ_r , λ_r^* ($r=1 \sim n$, * means conjugate), and modal matrix

$$[\Psi] = \begin{bmatrix} \phi & \phi^* \\ \phi\lambda & \phi^*\lambda^* \end{bmatrix} \quad (3)$$

where the diagonal matrix $[\Lambda]$, $[\Lambda^*]$ gives the eigenvalue λ , λ^* . The orthogonality relation of the eigenvector gives

$$\begin{aligned} \{\Psi_p\}^T [D] \{\Psi_q\} &= d_p \delta_{pq} \\ \{\Psi_p\}^T [E] \{\Psi_q\} &= e_p \delta_{pq} \end{aligned} \quad (4)$$

where δ_{pq} is the Kronecker delta. Letting the complex r th eigenvalue be

$$\lambda_r = -\frac{e_r}{d_r} = -\sigma_r + j\omega_{dr} \quad (5)$$

the frequency response function (compliance) yields

$$G_{ji}(\omega) = \sum_{r=1}^n \left\{ \frac{\phi_{ir}\phi_{jr}}{d_r(j\omega - \lambda_r)} + \frac{\phi_{ir}^*\phi_{jr}^*}{d_r^*(j\omega - \lambda_r^*)} \right\} \quad (6)$$

where i is the excitation point, j is the response point, and ϕ_{ir} and ϕ_{jr} are the i th and j th component of the r th eigenvector. Introducing the residues U and V as

$$\frac{\phi_{ir}\phi_{jr}}{d_r} = U_r + jV_r \quad (r=1 \sim n) \quad (7)$$

Eq.(6) yields⁽⁹⁾

$$G_{ji}(\omega) = \sum_{r=1}^n \left\{ \frac{U_r + jV_r}{j(\omega - \omega_{dr}) + \sigma_r} + \frac{U_r - jV_r}{j(\omega + \omega_{dr}) + \sigma_r} \right\} \quad (8)$$

3. Proposed New Curve Fitting Method

Let us assume that the frequency response functions (compliance) were obtained experimentally in multi degrees of freedom system. The real and imaginary parts of the frequency response functions are A_{kj}^i and A_{lj}^i , where $j(j=1 \sim m)$ is a response point and ω_i is a frequency of excitation. In the curve fitting in the frequency domain with n degrees of freedom, the $2n + 2nm$ modal parameters, such as σ_r , ω_{dr} , U_{rj} , V_{rj} ($r=1 \sim n$, $j=1 \sim m$), must be identified so that a weighted residual sum of squares, S , must be minimized.

$$S = \sum_{i=1}^p \sum_{j=1}^m \{ W_{kj}^i (G_{kj}^i - A_{kj}^i)^2 + W_{lj}^i (G_{lj}^i - A_{lj}^i)^2 \} \quad (9)$$

where σ_r and ω_{dr} are the modal damping factor and the damped natural frequency, and U_{rj} and V_{rj} are the r th row of real and imaginary parts of residue with respect to point j . W_{kj}^i and W_{lj}^i are the weighting functions which can be given the inverse of variance in the case that noise has a Gaussian distribution. G_{kj}^i and G_{lj}^i , which are the real and imaginary parts of the frequency response function identified, can be written as follows from Eq.(8).

$$\begin{aligned} G_{kj}^i &= \sum_{r=1}^n \left\{ \frac{U_{rj}\sigma_r + V_{rj}(\omega_i - \omega_{dr})}{a_r} \right. \\ &\quad \left. + \frac{U_{rj}\sigma_r - V_{rj}(\omega_i + \omega_{dr})}{b_r} \right\} \\ G_{lj}^i &= \sum_{r=1}^n \left\{ \frac{-U_{rj}(\omega_i - \omega_{dr}) + V_{rj}\sigma_r}{a_r} \right. \\ &\quad \left. - \frac{U_{rj}(\omega_i + \omega_{dr}) + V_{rj}\sigma_r}{b_r} \right\} \end{aligned} \quad (10)$$

where

$$\begin{aligned} a_r &= \sigma_r^2 + (\omega_i - \omega_{dr})^2 \\ b_r &= \sigma_r^2 + (\omega_i + \omega_{dr})^2 \end{aligned}$$

Even though there are a number of methods identifying the modal parameters which minimize Eq.(9), either much computer time is needed or bad convergence is acquired because of a large number of unknown parameters. Hence this paper proposes a new approach method as follows.

By partially differentiating Eq.(9) with U_{rj} , V_{rj} ,

and equating to zero, we obtain

$$\begin{aligned} \sum_{i=1}^p \left\{ W_{Rj}^i (G_{Rj}^i - A_{Rj}^i) \frac{\partial G_{Rj}^i}{\partial U_{rj}} + W_{Lj}^i (G_{Lj}^i - A_{Lj}^i) \frac{\partial G_{Lj}^i}{\partial U_{rj}} \right\} &= 0 \\ \sum_{i=1}^p \left\{ W_{Rj}^i (G_{Rj}^i - A_{Rj}^i) \frac{\partial G_{Rj}^i}{\partial V_{rj}} + W_{Lj}^i (G_{Lj}^i - A_{Lj}^i) \frac{\partial G_{Lj}^i}{\partial V_{rj}} \right\} &= 0 \end{aligned} \quad (11)$$

There are nonlinear parameters (σ_r, ω_{dr}) and linear parameters (U_{rj}, V_{rj}). If the nonlinear parameters are given, we obtain linear parameters directly without iteration by solving the simultaneous linear equations given by Eq.(11). Hence we can assume that the compliances G_{Rj}^i and G_{Lj}^i in Eq.(10) are functions of σ_r and ω_{dr} only by considering U_{rj} and V_{rj} as functions of σ_r and ω_{dr} . Letting γ_r ($r=1 \sim 2n$) represent σ_r and ω_{dr} ($r=1 \sim n$), we can introduce H_{Rj}^i and H_{Lj}^i in replace of G_{Rj}^i and G_{Lj}^i , as Eq.(12).

$$\frac{\partial H_{Rj}^i}{\partial \gamma_k} = \frac{\partial G_{Rj}^i}{\partial \gamma_k} + \sum_{r=1}^n \left\{ \frac{\partial G_{Rj}^i}{\partial U_{rj}} \frac{\partial U_{rj}}{\partial \gamma_k} + \frac{\partial G_{Rj}^i}{\partial V_{rj}} \frac{\partial V_{rj}}{\partial \gamma_k} \right\} \quad (12)$$

H_{Lj}^i can also be expressed the same as H_{Rj}^i . By partially differentiating eq.(11) with γ_k , we have

$$\sum_{i=1}^p \left\{ W_{Rj}^i \frac{\partial G_{Rj}^i}{\partial U_{rj}} \frac{\partial H_{Rj}^i}{\partial \gamma_k} + W_{Lj}^i \frac{\partial G_{Lj}^i}{\partial U_{rj}} \frac{\partial H_{Lj}^i}{\partial \gamma_k} \right\} = 0 \quad (13)$$

$\partial U_{rj}/\partial \gamma_k$ and $\partial V_{rj}/\partial \gamma_k$ can be obtained by solving the simultaneous linear equation given by Eq.(13). When we obtain the sensitivity of the frequency response function with respect to γ_k , the modal parameters which minimize the residual sum of squares in Eq.(9) can be obtained using any iterative method. If the Gauss-Newton method is used, the modified values of the modal parameters $\{\gamma\}_{\text{new}} = \{\gamma\}_{\text{old}} + \{\delta\gamma\}$ can be obtained by solving the simultaneous linear equation

$$[a]\{\delta\gamma\} = \{\beta\} \quad (14)$$

where

$$\begin{aligned} a_{kl} &= \sum_{i=1}^p \sum_{j=1}^m \left\{ W_{Rj}^i \frac{\partial H_{Rj}^i}{\partial \gamma_k} \frac{\partial H_{Rj}^i}{\partial \gamma_l} + W_{Lj}^i \frac{\partial H_{Lj}^i}{\partial \gamma_k} \frac{\partial H_{Lj}^i}{\partial \gamma_l} \right\} \\ \beta_k &= \sum_{i=1}^p \sum_{j=1}^m \left\{ W_{Rj}^i (A_{Rj}^i - G_{Rj}^i) \frac{\partial H_{Rj}^i}{\partial \gamma_k} \right. \\ &\quad \left. + W_{Lj}^i (A_{Lj}^i - G_{Lj}^i) \frac{\partial H_{Lj}^i}{\partial \gamma_k} \right\} \end{aligned}$$

Here, $[a]$ is $2n \times 2n$ matrix, $\{\beta\}$ is $2n$ column vector and Eq.(14) must be iterated until Eq.(9) has extreme value. It may be efficient using an other least square estimation algorithm^{(10),(11)} to accelerate the convergence, such as a modified Marquardt method or a Hybrid method which modifies the Gauss-Newton method. Residual mass and residual stiffness are not considered in this paper in order to transform to mass,

Fig. 1 Model with 5 degrees of freedom

stiffness and damping matrices. However, residual terms can be considered in the same way. The residual terms are usually linear parameters which can be included in Eq.(11).

The partial differential iterative method⁽⁹⁾ uses Eq.(14) without distinction between linear and nonlinear parameters. However, there are a number of response points in which the number of unknown parameters, $2n+2nm$, increases rapidly. Computer time is proportional to about m , where n is the identified degrees of freedom and m is the number of response points. A proposed method, however, needs only $2n$ unknown parameters regardless of the number of response points so that computer time is proportional to about m . Comparing the convergence of the proposed method with that of the partial differential iterative method, 5 degrees of freedom of a simulation model, as Fig.1, is identified. $\alpha\%$ error of the true value is taken as the initial value of ω , and 0.01 times of initial value of ω is taken as that of σ . Point 1 is excited and point 2 is responded. Ninety-one sample data from 50 Hz to 950 Hz are used. The result is shown in Fig. 2. A proposed method shows good convergence when α is both 10% and 20%. A partial differential iterative method, however, does not converge when α is 20%.

4. Proposed New Characteristic Matrix Identification

The mass, stiffness and damping matrices are the physical parameters in a physical coordinate, while curve fitting uses modal parameters in a modal coordinate. If the number of degrees of freedom identified is equal to that of the response point, the modal parameters can be transformed to the physical parameters theoretically, which is possible in simulation data. However, the modal analysis technique makes use of finite degrees of freedom to identify experimental models which have infinite degrees of freedom. When matrices $[D]$ and $[E]$ in Eq.(2) are calculated

Fig. 2 Comparison of convergence

from the modal parameters, the three $[M]$ matrices in Eq.(1) are not equal to one another, and the components of the matrix which must give a null matrix do not yield zero. For example, let us consider a rectangular bar with a free-free boundary condition, as in Fig. 3. The frequency response function at points 1, 2, 3, and 4 are obtained by exciting point 1. The sampled frequency range is from 440 Hz to 1 680 Hz with a sampling interval of 5 Hz. Table 1 shows the result of the $[D]$ and $[E]$ matrices calculated from modal parameters obtained by curve fitting. We can see that the mass, stiffness and damping matrices cannot be obtained from modal parameters directly in an actual continuous model. Hence, this paper proposes a new method of identifying the mass, stiffness and damping matrices from the modal parameters.

From Eq.(5), $\lambda_r = -\sigma_r$ in Eq.(5) if the eigenvector is normalized so that $d_r = 1$. We obtain the inverse of the $[D]$ and $[E]$ matrices using Eq.(3) and Eq.(4)

$$[D]^{-1} = \begin{bmatrix} \phi\phi^T + \phi^*\phi^{*T} & \phi\Lambda\phi^T + \phi^*\Lambda^*\phi^{*T} \\ \phi\Lambda\phi^T + \phi^*\Lambda^*\phi^{*T} & \phi\Lambda^2\phi^T + \phi^*\Lambda^{*2}\phi^{*T} \end{bmatrix}$$

$$[E]^{-1} = \begin{bmatrix} -\phi\Lambda^{-1}\phi^T - \phi^*\Lambda^{*-1}\phi^{*T} & -\phi\phi^T - \phi^*\phi^{*T} \\ -\phi\phi^T - \phi^*\phi^{*T} & -\phi\Lambda\phi^T - \phi^*\Lambda^*\phi^{*T} \end{bmatrix} \quad (15)$$

On the other hand, the inverse of the $[D]$ and $[E]$ matrices can also be obtained directly from Eq.(2)

$$[D]^{-1} = \begin{bmatrix} 0 & M^{-1} \\ M^{-1} & -M^{-1}CM^{-1} \end{bmatrix}$$

$$[E]^{-1} = \begin{bmatrix} K^{-1} & 0 \\ 0 & -M^{-1} \end{bmatrix} \quad (16)$$

We obtain the relation between the $[M]$, $[C]$, and $[K]$ matrices and modal parameters comparing Eq.(15) with Eq.(16)

$$M = (\phi\Lambda\phi^T + \phi^*\Lambda^*\phi^{*T})^{-1}$$

$$K = -(\phi\Lambda^{-1}\phi^T + \phi^*\Lambda^{*-1}\phi^{*T})^{-1}$$

$$C = -M(\phi\Lambda^2\phi^T + \phi^*\Lambda^{*2}\phi^{*T})M$$

$$Re[\phi\phi^T] = [0] \quad (17)$$

In particular, $Re[\phi\phi^T] = 0$ leads to the following relation from Eq.(7)

$$\sum_{k=1}^n [\{U_{kf}(U_{ki}U_{kj} - V_{ki}V_{kj}) + V_{kf}(U_{ki}V_{kj} + V_{ki}U_{kj})\} / (U_{kf}^2 + V_{kf}^2)] = 0$$

$$(i=1 \sim n, j=1 \sim i) \quad (18)$$

Fig. 3 Model of bar (unit is mm)

where f is the excitation point. Let the number of degrees of freedom be equal to that of the response points. Curve fitting identifies $2n(n+1)$ modal parameters, while characteristic matrix identification uses only $3n(n+1)/2$ parameters using the symmetry of $[M]$, $[C]$, and $[K]$ matrices. Hence, $n(n+1)/2$ modal parameters are not independent one another. Equation (18) shows $n(n+1)/2$ constraints which modal parameters must satisfy. In order to identify the mass, stiffness and damping matrices, the modal parameters must minimize the weighted residual sum of squares given by Eq.(9) while satisfying the constraints of Eq.(18). This can be solved by any nonlinear optimization technique with equality constraints such as the gradient projection method^{(10),(11)}. Table 2 shows the $[D]$ and $[E]$ matrices obtained by the proposed method. We can see that the mass, stiffness and damping matrices can be identified from modal parameters without contradiction.

Table 1 Characteristic matrices from curve fitting (bar model)

[D] matrix											
.12E-02	-.89E+00	-.70E+00	.56E+00	.32E-03	.94E-03	.78E-03	-.49E-03				
-.89E+00	-.64E+01	-.48E+01	.34E+01	.90E-03	.56E-02	.45E-02	-.25E-02				
-.70E+00	-.48E+01	-.36E+01	.27E+01	.76E-03	.45E-02	.40E-02	-.21E-02				
.56E+00	.34E+01	.27E+01	-.19E+01	-.46E-03	-.24E-02	-.20E-02	.13E-02				
.32E-03	.90E-03	.76E-03	.46E-03	.21E-07	.19E-06	.14E-06	-.77E-07				
.94E-03	.56E-02	.45E-02	-.24E-02	.19E-06	.12E-05	.91E-06	-.52E-06				
.78E-03	.45E-02	.40E-02	-.20E-02	.14E-06	.91E-06	.68E-06	-.40E-06				
-.49E-03	-.25E-02	-.21E-02	.13E-02	-.77E-07	-.52E-06	-.40E-06	.22E-06				
[E] matrix											
.87E+04	.15E+05	.86E+04	-.95E+04	.30E+00	.35E+01	.25E+01	-.14E+01				
.15E+05	.67E+05	.64E+05	-.50E+05	.16E+01	.12E+02	.86E+01	-.50E+01				
.86E+04	.64E+05	.75E+05	-.55E+05	.11E+01	.81E+01	.60E+01	-.35E+01				
-.95E+04	-.50E+05	-.55E+05	.41E+05	-.93E+00	-.71E+01	-.52E+01	.31E+01				
.30E+00	.16E+01	.11E+01	-.93E+00	-.30E-03	-.78E-03	.66E-03	.42E-03				
.35E+01	.12E+02	.81E+01	-.71E+01	-.78E-03	-.44E-02	-.36E-02	.20E-02				
.25E+01	.86E+01	.60E+01	-.52E+01	-.66E-03	-.36E-02	-.33E-02	.18E-02				
-.14E+01	-.50E+01	-.35E+01	.31E+01	.42E-03	.20E-02	.18E-02	-.12E-02				

Table 2 Characteristic matrices from proposed method (bar model)

[D] matrix											
.43E+00	.18E+01	.99E+00	-.52E+00	.33E-03	.11E-02	.92E-03	-.57E-03				
.18E+01	.22E+01	-.70E+00	.15E+00	.11E-02	.73E-02	.59E-02	-.34E-02				
.99E+00	-.70E+00	-.24E+01	.12E+01	.92E-03	.59E-02	.51E-02	-.28E-02				
-.52E+00	.15E+00	.12E+01	-.64E+00	-.57E-03	-.34E-02	-.28E-02	.18E-02				
.33E-03	.11E-02	.92E-03	-.57E-03	-.14E-14	.62E-15	-.83E-15	.50E-15				
.11E-02	.73E-02	.59E-02	-.34E-02	.62E-15	.14E-14	.74E-15	-.72E-15				
.92E-03	.59E-02	.51E-02	-.28E-02	-.83E-15	.74E-15	.28E-15	.54E-16				
-.57E-03	-.34E-02	-.28E-02	.18E-02	.50E-15	-.72E-15	.54E-16	-.34E-15				
[E] matrix											
.91E+04	.16E+05	.93E+04	-.10E+05	-.43E-07	.26E-07	-.34E-07	.15E-07				
.16E+05	.77E+05	.71E+05	-.56E+05	-.57E-07	.15E-07	-.21E-07	.20E-07				
.93E+04	.71E+05	.83E+05	-.61E+05	-.56E-07	.26E-07	.59E-08	.25E-07				
-.10E+05	-.56E+05	-.61E+05	.46E+05	.55E-07	.54E-08	.10E-07	-.23E-07				
-.43E-07	-.57E-07	-.56E-07	.55E-07	-.33E-03	-.11E-02	-.92E-03	.60E-03				
.26E-07	.15E-07	-.26E-07	.54E-08	-.11E-02	.73E-02	-.59E-02	.34E-02				
-.34E-07	-.21E-07	.59E-08	.10E-07	-.92E-03	-.59E-02	-.51E-02	.28E-02				
.15E-07	.20E-07	.25E-07	-.23E-07	.60E-03	.34E-02	.28E-02	-.18E-02				

5. Application

This proposed method is applied to the experimental data of a bar, cylinder block and crankshaft of an automobile.

One of the results of the bar application in Fig. 3 is shown in Fig. 4 (point 4 is both excited and responded). The solid line represents the experimental data, while the dashed line is one identified result with 4 degrees of freedom and the dot-dashed line is another identified result with 3 degrees of freedom. The unshaded region is that used for identification. The foot of the peak identified with 3 degrees of freedom does not show good agreement with the experimental data since the residual terms are not considered, while the result with 4 degrees of freedom shows good agreement by having a residual peak close to zero Hz.

The cylinder block of an engine with a free-free boundary condition, as shown in Fig. 5 is excited with an impact hammer. The experimental data is averaged 4 times in a digital Fourier transform analyzer.

Fig. 4 Comparison of frequency response function of bar model
(point 4 excited and responded)

Fig. 5 Model of cylinder block

The identified frequency range includes the 1st, 2nd and 3rd natural frequencies of the elastic vibration, which is from 200 Hz to 800 Hz. The result is identified with 4 degrees of freedom and is shown in Fig. 6. The solid line is the experimental data and the broken line is the identified result. The unshaded region is that used for identification.

The crankshaft with a free-free boundary condition as shown in Fig. 7 is also excited by sweep sine excitation. The data used for identification is from 5 Hz to 2 000 Hz with a sampling interval of 5 Hz, which includes that from the 1st to 9th peak, as shown in Fig. 8. The solid line represents the experimental data and the broken line is the identified result. The residual norm calculated from the modal parameters is 3.14×10^{-4} , and that calculated from the physical parameters, considering the proposed constraints (Eq.(18)), is 4.48×10^{-4} (unit is mm/N). We cannot distinguish a difference between them in Bode diagram.

Until recently, it has been very difficult to indicate a good initial value in the characteristic matrix identification. However, this presented method can give a good initial value since curve fitting is used. Hence, a complicated structure can also be identified with good precision if the number of degrees of free-

Fig. 6 Result of cylinder block

Fig. 7 Model of crankshaft

Fig. 8 Result of crankshaft
(point 1 excited and point 4 responded)

dom is equal to that of the response points.

6. Conclusions

In this paper, a new multireference curve fitting method in the frequency domain is presented. The number of unknown parameters depend only on the number of degrees of freedom identified, regardless of that of the natural mode. This is effective when there are a number of response points. Linear parameters (residue, residual terms etc.) can be calculated directly from the identified nonlinear parameters (damped natural frequency, modal damping factor etc.). This method shows good convergence and reduces computer time.

Secondly, this paper proposes the constraints that modal parameters must satisfy in order to be transformed to physical parameters without contradiction. When the number of degrees of freedom identified ($= n$) is equal to that of response points, there must be $n(n+1)/2$ constraints between modal parameters so that they are independent of one another. A nonlinear optimization technique with equality constraints is used to solve these constraints. Some experimental

models are examined to validate proposed method.

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