The State of the Art of Rolling Bearing System Technology*

Kazuo KAKUTA**

This paper describes the recent progress of rolling bearing system technology which is a part of the kinematic motion mechanism of machines. This bearing system technology covers the fields of the bearings themselves and also their application technology. These include reliability, accuracy, fatigue life, grease life, integral units, high speed, low torque, low temperature rise, vibration and noise, severe environments, elastohydrodynamic lubrication, dynamics of rolling elements and retainers, FEM, analysis of rolling fatigue, and prediction of bearing life.

Key Words: Machine Element, Bearing, Rolling Bearing, Ball Bearing, Roller Bearing, Tribology, lubrication, Fatigue Life

1. Introduction

A machine is a system that converts energy into mechanical work useful for mankind. Every machine has moving parts, and in this technological age, the number of machines is growing explosively.

Present-day machines are required to meet stringent functional requirements, and are therefore designed to have rolling motion elements in their motion mechanisms. In other words, they are designed to have rolling bearings in their rotating motion mechanisms, rolling linear guides in their linear motion mechanisms, and ball screws in their feeding motion mechanisms. This paper describes recent progress in rolling bearing systems which are components of those kinematic motion mechanisms.

2. Requirements for Rolling Bearings

Machines are developed to meet the needs of mankind. The friction forces exerted on the moving parts of a machine cause power losses that inevitably affect the operating efficiency of a machine and then energy consumption. The wear-resistance and fatigue life of the moving components are crucial to the machine's durability and then resources conservation. Sudden machine trouble due to the seizure of its moving parts can seriously affect man's activities both commercially and socially.

Thus, the ability to support the moving parts of machines is a key technology in the design of efficient and reliable machines.

Table 1 lists the changing requirements for bearings with the progress of society and machinery. The progress of rolling bearings cannot be achieved by hardware technology that just takes the bearings themselves into account. It can only be achieved by the joint development of bearing application technology, i.e. software technology to cover the design and

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** Research and Development, Nippon Seiko K.K. (NSK), 1-5-50 kugenuma-shinmei, Fujisawa, Kanagawa, 251, Japan

performance of a bearing's surrounding parts such as the shaft, housing, damper, lubricant, lubricating system, seals, etc. with which bearings are used. We call this combination of bearing hardware and software "bearing system technology".

For many years since the Industrial Revolution, large- and medium-sized bearings have been used principally in the power transmission systems of large and heavy machines. In recent years, small and miniature bearings have also been increasingly used in small and lightweight communications equipment and instruments.

3. Progress in Rolling Bearing Functions

3.1 Higher reliability

When a bearing's performance under certain operating conditions, including load, speed, lubrication, ambient temperature, and service life are known from experience with conventional bearings, it is possible to quickly design a new bearing system with high reliability. If this is not the case, a number of functional tests and life tests must be performed on test machines that simulate automobiles, railway cars, household electrical appliances, or other small- or medium-sized machines, and conformity to require-

ments must be verified before the design of a new bearing system is implemented. For rolling mills and other large machines that cannot be simulated by test machines, analyses based on finite element methods (FEM) are used in the design of bearing systems.

Quality assurance techniques in the production of rolling bearings have also been making noticeable progress.

The fatigue life of rolling bearings is usually specified on the basis of 90% reliability. But the use of a reliability factor, a_1 , when higher reliability is required, is being studied⁽¹⁾.

3.2 Higher accuracies

The spindles of machine tools, inertial navigation gyroscopes, hard disc drives of computers, rotating heads of video cassette recorders, dental hand pieces, etc. require bearings with high dimensional and running accuracies.

The spindles or shafts of such units or systems are required to have a runout accuracy of not more than 0.5 μm . The balls of ball bearings used to support these spindles or shafts have profile accuracies of ISO Grade 3 or higher. More specifically, the balls are machined to a sphericity of not more than 0.08 μm and a surface roughness not higher than 0.012 μm R_a . The

Table 1 Requirements for bearings

Users' Requirements	Needs for Machines	Requirements for Bearings
o Higher Quality	o Trouble-free	o Higher Reliability
o Resource Conservation	o Longer Endurance	o Higher Load-capacity by Design o Longer Life Using Superior Materials and Heat Treatment
o Automation o Labor-saving	o Maintenance Free	o Use of Sealed or Shielded Bearings o Longer Life of Lubricating Grease
o Energy Conservation	o Smaller Size o Lighter Weight o Higher Efficiency	o Smaller Size o Integrated Bearings o Higher Speed o Less Friction o Lower Temperature Rise
o Pollution-free	o Suitable Environment	o Less Vibration o Less Noise
o Versatility	o Higher Performance	o Larger Size o Higher Accuracy o Higher Rigidity o Higher Temperature Limit o Lower Temperature Limit o Special Environment o Ultraclean o High Vacuum o Corrosive Matter o Radiation

inner and outer races of bearings are also super-precisely machined to accuracies commensurate to those of the balls.

The requirements for the running accuracy of bearings are higher for the runout components that are not repeatable than for those components that are proportional to the running speed of the bearings⁽²⁾.

For the application of superprecision bearings, the dimensional, profile, and assembling accuracies of shafts and housings with which the bearings are used must be higher than the accuracies of the bearings themselves. The cleaning and assembling of super-precision bearings are done in clean rooms to prevent their contamination by dust and other foreign matter that could impair their performance.

3.3 Longer rolling fatigue life

The life of a rolling bearing is restricted by the rolling fatigue of the material of which the bearing is made. The extension of the life of rolling bearings is an endless task for bearing researchers.

During the 1960 s, the process of making steel for bearings was changed from air melting to vacuum degassing, which consequently reduced the oxygen content in steel from the previous levels of 20 to 30 ppm to the current level of less than 10 ppm as shown in Fig. 1⁽³⁾. Oxygen in steel exists in the form of hard oxides such as SiO_2 , which can be the starting point for cracking by the rolling fatigue of the material which shortens the bearings life.

Figure 2 shows the results of a laboratory analysis of the rolling fatigue life of 52100 bearing steel and case-hardened steel in relation to their oxygen content⁽⁴⁾. From this figure, we know that the rolling fatigue life of bearings made of vacuum degassed steel is 3 to 6 times longer than that of air-melted steel.

In fact, the rolling fatigue life of sealed or shielded ball bearings made of this steel and used in electric motors was prolonged by a factor of three, which resulted in changing the design of ball bearings for electric motors from heavy-duty types to light-duty types that were smaller. However, no service data indicating a longer rolling fatigue life were obtained for open bearings used in automobiles and rolling mills.

In the 1970 s, the rolling fatigue life of a bearing was found to have been closely related to the thickness of the elastohydrodynamic lubrication (EHL) oil films formed between the rolling elements and the raceways of the bearing⁽⁵⁾ as shown in Fig. 3⁽⁶⁾. The oil film parameter, Λ , is the ratio of the thickness of the

Fig. 1 Trend of oxygen content in high-carbon chromium-bearing steel

Fig. 2 Oxygen content in bearing steel and rolling fatigue life

oil film formed between the rolling elements and the raceways, h , and the root mean square of the surface roughness of the rolling elements and raceways, σ .

As shown in Fig. 4⁽⁷⁾, the ability of the improved material to extend the rolling fatigue life occurs when $\Lambda > 1.5$. Oil films are formed over the contact surfaces, and flaking due to cracks of subsurface origin, as pointed out by Lundberg & Palmgren⁽⁸⁾, are the cause of bearing failure. The effect of the improved material is unrealizable when $\Lambda > 0.8$, where the rolling elements and the raceways are in solid contact, and flaking due to cracks of surface origin are the cause of bearing failure⁽⁷⁾.

The open bearings used in automobiles and rolling mills did not show any of the above-mentioned improvement in service life because their life was terminated by surface-originated cracks caused by the wear particles, moisture, dust, and other foreign matter when $\Lambda < 0.8$.

3.4 Higher load-carrying capacity

The life of a bearing can be extended by changing

Fig. 3 Oil film parameter Λ and rolling fatigue life

Fig. 4 Effect of oil film parameter Λ and materials and heat treatment on rolling fatigue life of case-hardened steel

its internal design in order to increase its load-carrying capacity. Increasing the size and number of its rolling elements can increase its load-carrying capacity. To increase both their size and number, a design limit is needed because the thickness of the inner and outer rings and retainer must be as thin as possible. Finite element method (FEM) analyses were used to determine the design limit.

The failure mechanism of the retainers in ball bearings is well known⁽⁹⁾, but that of the retainers in roller bearings is still largely unknown. There are two alternatives available to prevent the fatigue failure of retainers: one is to use a metal of greater strength; the other is to use a plastic retainer that can endure high elastic deformation.

3.5 Sealed or shielded bearings, and long-life grease

Deep groove ball bearings are popular because they are easy to use. Sealed or shielded ball bearings, which are standard deep groove ball bearings fitted with a seal or shield and prelubricated with grease by the bearing manufacturer, have recently been increasingly used in household appliances and communications equipment. The life of sealed or shielded ball bearings used in such appliances and equipment is often determined by the life of their prepacked grease, which tends to deteriorate rather than by rolling fatigue life.

For this reason, studies have been undertaken to develop grease with a longer life. The life of grease is halved by its deterioration if the operating temperature of the bearing is increased by 15°C⁽¹⁰⁾. The physical causes of grease deterioration are a sharp increase in the grease leakage rate, the oil bleeding rate, and the amount of steel wear particles in the grease. The chemical causes are a sharp decrease in antioxidant contained in the grease, a sharp increase in the total acid number and a shift in the molecular weight distribution of the base oil to the high molecular weight side⁽¹¹⁾.

3.6 Smaller bearings and bearing units

As machines become smaller in size and lighter in weight, the bearings used in them also become smaller. Not only are the bearings themselves made smaller, but bearings are often combined with their surrounding parts into integrated units. For example, as shown in Fig. 5, sometimes bearings having no inner rings are combined with a shaft with raceway groove directly made in it or in other cases, a raceway is made in the bearing housing to minimize the size of the bearing. Such bearing units have been increasingly used, for instance, as hub units for automobiles and as bearings-with-shafts for the rotating heads of video cassette recorders. With the use of such integral

bearing units, machine manufacturers can save bearing installation and preload-adjusting time, reduce the number of parts engaged with a bearing, and make their machines lighter and smaller.

3.7 Higher speeds

Designing a higher-speed bearing can have two meanings. One means to design a bearing capable of operating at a higher angular velocity, and the other at a higher peripheral speed. A typical example of bearings designed for higher angular velocity is ball bearings (I.D. 3.175 mm, O.D. 6.35 mm, width 2.38 mm) for dental hand pieces that can operate at 630 000 rpm. The main shaft bearings for aircraft jet engines represent an example of bearings developed for higher peripheral speeds.

The progress in developing higher-speed bearings is expressed in terms of the product of the bearing bore diameter, d (mm), and the running speed of the bearing, n (rpm). The speed of the main shaft bearings for jet engines, in terms of dn , has been increased by 37 000 every year as shown in Fig. 6. At present, a dn value of 2.2 million is in service, 3 million has been attained in laboratories, and 3.5 million will be needed in the 1990 s.

Until quite recently, ball bearings⁽¹²⁾⁽¹³⁾ and cylindrical roller bearings⁽¹⁴⁾ have been mainly used as high-speed bearings. At present, attempts are being made to develop tapered roller bearings⁽¹⁵⁾ capable of operating at higher speeds.

Recently, an under-race lubrication system, which ensures the two simultaneous functions of lubrication and cooling by introducing lubricating oil into the bearing against strong air flow and centrifugal force when the bearing is running at high speeds, is sometimes used in place of the conventional jet lubri-

cation^{(13)~(15)}. Grease-lubricated bearings are currently operated at a dn speed level of one million. Other studies currently being made include the study of extratough case-hardened steel for the purpose of preventing fracture of the inner rings by tensile stresses caused by centrifugal force when the bearing is running at high speed⁽¹⁶⁾. Studies of ceramics to develop bearing materials that are lighter and more resistant to high temperature, and studies of dynamics for the purpose of optimizing the design of rolling elements and retainers are in progress⁽¹⁷⁾.

3.8 Lower torque and lower temperature rise

There are two requirements for low torque. One is the requirement that the absolute value of the bearing torque must be low; the other requirement is that the torque variation must be low. The former is demanded by power transmission machines to keep temperature rise and power consumption at low levels, and the latter by information processing systems. However, recent information processing systems such as gyroscopes, hard disc drives, and video cassette recorders demand bearings to meet both of these requirements.

For small-sized and miniature bearings used in such machines, the geometrical accuracy of their balls and raceways is improved and the quality and amount of grease packed in them are kept under strict control. The bearings are fully cleaned and assembled in clean rooms.

Gyroscope rotor bearings have a retainer made of a porous plastic material impregnated with lubricating oil, which is gradually released for lubrication at a very small rate over a long period. Sometimes an oil reservoir made of a porous material is situated near the bearing to provide continuous lubrication for over 10 years⁽¹⁸⁾.

Bearings for the main spindles of machine tools

(a) Integral Shaft Bearings for VCR

(b) Wheel-Hub Bearings for Automobiles

Fig. 5 Integral bearing units

Fig. 6 Trend of mainshaft bearings for aircraft jet engines

are required to have a low temperature rise because high heat generation from the bearings can cause thermal expansion and impair their machining accuracies. To meet this demand and also the demand for higher speeds, studies are being made of oil-air lubrication⁽¹⁹⁾ and oil-jet lubrication as a possible means to replace the conventional oil-mist lubrication.

Some ball bearings for spindles already use ceramic balls designed so that the gyroscopic motion of the balls and the centrifugal force are reduced to increase the bearing speed, and contribute to a lower temperature rise⁽²⁰⁾.

3.9 Lower vibration and less noise

The vibration and noise generation mechanisms of rolling bearings have been studied for many years, and the causes are well known today⁽²¹⁾. In the 1950s, superfinishing techniques were introduced for the finishing of the raceways of most bearings. Machine manufacturers urgently needed low-vibration and low-noise bearings as demonstrated by the fact that methods of measuring the noise of bearings had been established in the form of national standards (JIS).

Modern machines are constructed of thinner materials to reduce size and weight, so they are more liable to resonate with external vibration. A rolling bearing, consisting of inner and outer rings and a number of rolling elements, tends to vibrate structurally at various natural frequencies. To control such vibration, therefore, arrangements are often made when designing a machine to decrease the amplitude of the natural vibration of a bearing that is close to the natural frequencies of the entire system, including the shaft and housing, or to change the natural frequency of the bearing.

3.10 Larger size

The smallest rolling bearing today is 1 mm in bore diameter, 3 mm in outside diameter, and 1 mm in width. The largest so far of a complete type is 8 m in O.D., and that of a segmented type consisting of separate parts to be assembled had an O.D. of 15 m.

3.11 Bearings for extremely severe environments

In certain high-tech applications, bearings are used in high temperature, low temperature, high vacuum, corrosive gas, radioactivity and other environments which are enormously different from those in which conventional types of bearings have been used.

Standard bearings are serviceable at up to 130°C, and those heat-treated for dimensional stabilization can be used at up to approximately 180°C. Bearings that are already in practical service at higher temperatures include high-speed molybdenum steel (AISI-M50) bearings that are used for jet engine main shafts at up to about 350°C, high-speed cobalt steel

(JIS-SKH 4 A) bearings for X-ray tubes at 400°C for CT scanning, and cobalt-chromium alloy "Stellite" bearings serving in liquid sodium for heat exchange at 550°C for nuclear reactors. Silicon nitride (Si_3N_4) is a promising ceramic material for high-temperature bearings.

For low temperature service, some rolling bearings are used in -253°C liquid hydrogen and in -183°C liquid oxygen in the liquid fuel pumps of rockets⁽²²⁾ and at -162°C for liquid natural gas pumps.

These bearings are made of martensitic stainless steel (SUS 440 C), have a retainer made of a composite material of PTFE and molybdenum disulfide (MoS_2) reinforced with glass fibers, and utilize the particles from the worn retainer as a solid lubricant.

In space, semiconductor manufacturing equipment and X-ray tubes, bearings are required to function in a vacuum of 10^{-4} to 10^{-11} Pa or in corrosive silane gas atmospheres, where neither oil nor grease can be used as a lubricant. For such applications, therefore, bearings are coated with a thin solid layer of a soft metal such as gold, silver, or lead, or with a lamellar solid such as molybdenum disulfide (MoS_2), tungsten disulfide (WS_2) or graphite, by electroplating, sputtering or ion plating.

4. Progress of Rolling Bearing Analysis Techniques

4.1 Elastohydrodynamic lubrication

In some applications of rolling bearings, the maximum elastic contact surface pressure of their rolling elements and raceways can reach as high as 3 000 MPa. Even under such a high surface pressure, the lubrication of their rolling contact surfaces is often well maintained. This phenomenon was explained in the 1960s by the theory of elastohydrodynamic lubrication (EHL), which justified the possible presence of an approximately $1\text{ }\mu\text{m}$ -thick lubricant film even under such an extreme condition, and which was later proved by experiments⁽²³⁾⁽²⁴⁾.

The EHL theory furnished rolling bearing lubrication with a theoretical foundation and established a concept for designing bearings with rolling elements and raceways without direct metallic contact with each other, separated by a lubricating film between them.

Considering the three-dimensional distribution of surface roughness, lubricant starvation, and thermal effects caused by the shearing of the oil film, analyses based on the EHL theory were further expanded to include lubrication with grease. Research is also being done actively on the high pressure viscosity of lubricating oils, which plays an important role in the theory of EHL.

4.2 Dynamics of rolling elements and retainers

Through the application of Hertz's elastic contact theory, statics of rolling bearings has been used for the calculation of the load distribution on rolling elements, the displacement and stresses between rolling elements and raceways, the contact angles of balls, and the life, rigidity and torque of bearings.

With the advancement of computers, the analysis evolved into quasi-static analyses⁽²⁵⁾ using traction models of EHL and rolling / sliding contacts between rolling elements and raceways which take into account the inner and outer race controls and centrifugal force on the balls. They were then further developed into quasi-dynamic analyses⁽²⁶⁾ that consider the gyroscopic motion of the balls and the angular momentum of the rolling elements.

Recently, dynamic analyses have been used to analyze the motion of rolling elements and retainers by providing them with six degrees of freedom around their centers of gravity, and to calculate their motion with time under varied loads and running speeds⁽²⁷⁾. Thermal analytical techniques have also been improved and can nowadays calculate the change with time of temperature distribution in bearings and adjacent parts.

The reliability of the calculation of bearing performance using computerized simulation has not been fully confirmed by correlation with experimental data and requires further study.

4.3 Analysis of stresses and deformation using finite element method

The finite element method, an advanced computerized method of numerical computation, permits detailed computation of the deformation and stresses in structures with applied loads.

It was found that the load distribution in bearings calculated using conventional methods assuming that their inner and outer rings, shafts and housings were rigid parts differed considerably from the computation using the FEM which treats them as elastic parts⁽²⁸⁾. This was also confirmed by advanced stress measuring methods. The FEM can also accurately compute the contact stress between crowned rollers and raceways to facilitate the design of their optimum geometry suited for specific bearing operating conditions⁽²⁹⁾. The FEM is particularly useful for the optimum design and accurate life calculation of large bearing systems that cannot be easily simulated experimentally.

4.4 Analysis of rolling fatigue

The progression of fatigue on and under the surface of rolling elements and raceways can be qualitatively determined by the X-ray measurement of residual stress, half-value breadth of diffracted

rays, and retained austenite⁽³⁰⁾. By applying this method to the analysis of rolling fatigue of bearings that were tested in a laboratory or used in actual machines, we can determine where the rolling fatigue occurred, whether at the surface or beneath the surface and to what extent it has progressed. It is known that rolling fatigue at or near the surface is caused by defective lubrication and seals or when $\lambda < 0.8$ and can greatly reduce bearing life as shown in Fig. 3. In practice, therefore, bearing life can be extended by improving the lubrication and seals or shields to ensure that $\lambda > 1.5$. That is, by maintaining conditions such that the end of bearing life results only from flaking caused by subsurface-originated cracking⁽³¹⁾.

4.5 Prediction of bearing life

Machine failure due to bearing failure is of course undesirable. To avoid such unexpected trouble, some machines are equipped with a bearing monitor to constantly check the operating conditions of their vital bearings⁽³²⁾. Such monitors usually sense bearing temperature, vibration, noise, acoustic emission signals, or metallic wear particles in the lubricant.

5. Conclusions

This report summarizes the recently developed functions of rolling bearings and the technology used to analyze them. Rolling bearings, while their appearance has remained almost unchanged, have undergone substantial improvements by the invisible but influential progress of technology, which has resulted in higher dimensional accuracy by superprecision machining, better materials, improved designs to eliminate contact by EHL, thus meeting the increasingly stringent and new requirements of machine manufacturers. Bearings will continue to be improved by promoting technology in order to satisfy the requirements of machines in the future.

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