A Study on Brake Noise*
(Drum Brake Squeal)

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Recently, a theoretical analysis of drum brake squeal was presented by N. Millner. This report has been quoted in many papers. We considered an improved analytical model in order to clarify a practical method of eliminating brake squeal. We obtained an equation which represents $E$, the increase of kinetic energy during 1 cycle. The equation for $E$ often becomes positive under the influence of coupled vibrations between radial and tangential directions in the brake shoe and brake drum. Brake squeal is generated under the condition $E > 0$. The factors having an influence on brake squeal were clarified by considering the above equation for $E$. In accordance with the above considerations it is shown that adopting a material with a low coefficient of friction, varying the boundary conditions of the brake shoe, and changing the position of the lining are effective in eliminating brake noise.

**Key Words:** Automobile, Vibration of Continuous System, Sound, Friction, Numerical Analysis

1. Introduction

Generally, disk brakes and drum brakes are used for vehicles. Eliminating the squeaking or squealing noise produced when the brakes are applied is an important problem for improving comfort. Studies on the causes of brake squeal and the methods of preventing it have been conducted for many years. Studies on drum brakes include, for example, a theoretical study by Fosberry and Holubecki[1], a study using friction vibration models conducted by Wataru and Sugimoto[10], and a study of friction vibration and the natural frequencies of parts carried out by Chikamori[15].

All of these studies considered the friction coefficient $\mu$ incidental to the decrease in rubbing speed as a cause of brake squeal. Recent studies dealt with brake squeal in terms of the coupled vibration of brake component parts. The study by Millner[10], for example, demonstrated for the first time that brake squeal is easily produced if the friction coefficient $\mu$ is high. However, this study used an analytical model consisting only of a single primary shoe, which was different in composition from the actual brake. It is thus not sufficient as a basis for discussing the phenomenon of brake squeal and methods of preventing it. In our study, we conducted an improved analysis and made the following observations regarding the causes of brake squeal and new methods of preventing it.

2. Experimental

2.1 Measurement of vibration mode during brake squeal

A test was conducted using the front 2 L-type drum brakes of a small truck. As shown in Fig. 1, the drum brake uses two iron arc brake shoes that are hydraulically actuated to press a cylindrical brake drum of cast iron, thus generating friction as a braking force. The specifications of the test pieces are as shown in Table 1, with a brake drum inside diameter of 228.6 mm and a piston diameter of 23.81 mm.

When the braking hydraulic pressure $P$ was 3 MPa or more, a brake squeal of 2.92 kHz was generat-
ed; a brake squeal of 3.96 kHz was also generated on occasion. To observe the vibration of the brake shoes and brake drum in the latter case, one measuring point was chosen on the outer side of the contact part of the brake drum and seven measuring points were set on the inner side of a brake shoe. Vibration measurements were then made in two directions, radial and tangential, as shown in Fig. 2. The position of the accelerometer attached to the brake drum was determined from rotary pulse signal input values obtained using a gear fixed to the drum and an electromagnetic pickup. Brake squeal was determined by analyzing the frequency of the sound measured with the precision sound level meter. The test results indicated that the brake drum was in the third vibration mode of 4 diametral nodes at a brake squeal of 2.92 kHz and in the fourth vibration mode of 5 diametral nodes at a brake squeal of 3.96 kHz, as shown in Fig. 3, and that the brake shoe was in the third vibration mode with both ends free arcs, as shown in Fig. 4.

### 2.2 Natural frequencies of brake component parts

The brake shoes and brake drum were excited by a noncontact exciter, and their response was measured using the accelerometer. The points where these parts were excited and the measuring points are shown on the left of Figs. 5 and 6. The test results indicated that, as shown in Fig. 5, the brake shoes differed in vibration mode, though some showed a natural frequency close to the squeal frequency. It was also demonstrated that, as shown in Fig. 6, the brake drum would develop a vibration corresponding to the squeal frequency, as denoted by a dotted line, when

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**Table 1 Brake specifications**

<table>
<thead>
<tr>
<th>Shoe</th>
<th>Drum</th>
<th>Lining</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l, L ) (cm)</td>
<td>10.76</td>
<td>11.93</td>
</tr>
<tr>
<td>( \rho, \alpha ) (kg/m(^3))</td>
<td>7.99 \times 10(^2)</td>
<td>7.23 \times 10(^2)</td>
</tr>
<tr>
<td>( E, \nu ) (deg)</td>
<td>122</td>
<td></td>
</tr>
<tr>
<td>( a, \sigma ) (cm(^2))</td>
<td>2.0</td>
<td>5.18</td>
</tr>
<tr>
<td>( E_r, a_r ) (Nm)</td>
<td>0.98 \times 10(^2)</td>
<td>h (cm)</td>
</tr>
<tr>
<td>( y ) (deg)</td>
<td>138</td>
<td></td>
</tr>
<tr>
<td>( a ) (deg)</td>
<td>180</td>
<td>( \mu )</td>
</tr>
</tbody>
</table>

*Note: See Figure 1 for the symbols*

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**Fig. 1 Analytical model**

**Fig. 2 Method of vibration measurement in brake squeal test**

**Fig. 3 Brake drum vibration modes in brake squeal test**

**Fig. 4 Brake shoe vibration mode in brake squeal test**

the brake shoes were pressed against the brake drum at a hydraulic pressure of \( P = 3 \) MPa. Therefore, it is assumed reasonable to consider brake squeal as a phenomenon occurring due to the coupled vibrations of the brake shoes and brake drum.

3. Theoretical Analysis

3.1 Analytical model

Brake drums are generally structured as described in section 2.1. The analytical model used in our study has been shown in Fig. 1. The brake shoes are arc bars with a neutral axis radius of \( \ell \) and a center angle of \( \gamma \), and are located at angular intervals of \( \alpha \) on the primary and secondary sides. The brake shoes have a lining whose friction coefficient is \( \mu \). The lining is considered to be a radial spring without mass, having a spring coefficient of \( L = E_s t / h \). The brake drum is a circular ring with a neutral axis radius of \( \ell_s \), and is assumed to be rotating in the arrow direction. The symbol \( s \) represents the brake drum; the symbol \( sp \), the primary shoe; the symbol \( ss \), the secondary shoe.

\[
\begin{align*}
\ell & : \text{Brake shoe neutral axis radius} \\
\sigma & : \text{Brake shoe cross-sectional area} \\
\rho & : \text{Brake shoe density} \\
EI & : \text{Brake shoe flexural rigidity} \\
\gamma & : \text{Brake shoe center angle} \\
\gamma_1, \gamma_2 & : \text{Primary shoe lining position} \\
\beta_s, \beta_2 & : \text{Secondary shoe lining position} \\
E_s & : \text{Young's modulus for lining} \\
w & : \text{Lining width} \\
h & : \text{Lining thickness} \\
u & : \text{Deflection in radial direction} \\
v & : \text{Deflection in tangential direction}
\end{align*}
\]

3.2 Fundamental equations

In the coordinate system for analysis, the center of the brake drum was expressed as \( 0 \), representing the original point, the radial direction as \( u \), and the tangential direction as \( v \). In the analytical model, the balance of forces during minute vibration around the point of equilibrium in parts involving minute factors \( d\phi \) and \( d\theta \) was considered. Changes in surface pressure, \( R_{sp} \) and \( R_{ss} \), that occur incidental to the relative displacement of the brake shoes and brake drum in the radial direction can be obtained from Eqs. (1) and (2):

\[
\begin{align*}
R_{sp} &= L(u_{sp} - u_0)d\phi \\
R_{ss} &= L(u_{ss} - u_0)d\theta
\end{align*}
\]

These changes in surface pressure produce changes in frictional force, \( F_{sp} \) and \( F_{ss} \):

\[
\begin{align*}
F_{sp} &= \mu R_{sp} \\
F_{ss} &= \mu R_{ss}
\end{align*}
\]

Next, the kinetic energies, \( dT_{sp} \) and \( dT_{ss} \), of those parts of the brake shoes involving minute factors \( d\phi \) and \( d\theta \), the distortion energies \( dQ_{sp} \) and \( dQ_{ss} \) caused by bending deformation, and the generalized forces, \( dQ_{sp} \) and \( dQ_{ss} \), are obtained, followed by integration in each area. The results are expressed by Eqs. (5) through (10):

\[
\begin{align*}
T_{sp} &= \frac{\rho \sigma}{2} \int_{\phi_0}^{\phi_1} (u_{sp}^2 + v_{sp}^2)d\phi \\
T_{ss} &= \frac{\rho \sigma}{2} \int_{\phi_0}^{\phi_1} (u_{ss}^2 + v_{ss}^2)d\theta \\
U_{sp} &= \int_0^\pi \frac{M_{sp}}{2EI}d\phi \\
U_{ss} &= \int_0^\theta \frac{M_{ss}}{2EI}d\theta \\
Q_{sp} &= \int_0^\pi \left( \frac{\partial R_{sp}}{\partial \phi} \frac{\partial u_{sp}}{\partial \phi} - \frac{\partial F_{sp}}{\partial \phi} \frac{\partial v_{sp}}{\partial \phi} \right) d\phi \\
Q_{ss} &= \int_0^\theta \left( \frac{\partial R_{ss}}{\partial \theta} \frac{\partial u_{ss}}{\partial \theta} - \frac{\partial F_{ss}}{\partial \theta} \frac{\partial v_{ss}}{\partial \theta} \right) d\theta
\end{align*}
\]

where the bending moments \( M_{sp} \) and \( M_{ss} \) are as follows:

\[
\begin{align*}
M_{sp} &= \frac{EI}{\ell_s^2} \left( u_{sp} + \frac{\partial^2 u_{sp}}{\partial \phi^2} \right) \\
M_{ss} &= \frac{EI}{\ell_s^2} \left( u_{ss} + \frac{\partial^2 u_{ss}}{\partial \theta^2} \right)
\end{align*}
\]

The kinetic energies, distortion energies, and generalized forces of the brake drum can be similarly obtained, as expressed in Eqs. (11), (12), and (13):
\[ T_\phi = \frac{L_\phi a_\phi}{2} \int_0^{2\pi} \left( u_\phi^2 + v_\phi^2 \right) d\phi \]  \hspace{1cm} (11)

\[ U_\phi = \int_0^{2\pi} \frac{M_\phi a_\phi}{2} \dot{\phi} d\phi \]  \hspace{1cm} (12)

\[ Q_{ai} = \int_0^{2\pi} \left( \frac{\partial F_{ai}}{\partial \phi} + \frac{\partial F_{ai}}{\partial \theta} \right) d\phi \]  \hspace{1cm} (i=1,2)

\[ + \int_0^{2\pi} \left( \frac{\partial P_{ai}}{\partial \phi} \dot{\phi} + \frac{\partial P_{ai}}{\partial \theta} \dot{\theta} \right) d\phi \]  \hspace{1cm} (13)

where the bending moment \( M_\phi \) is as follows:

\[ M_\phi = \frac{E I_\phi}{L_\phi} \left( u_\phi + \frac{\partial^2 u_\phi}{\partial \phi^2} \right) \]

Now, each vibration solution for the brake shoes and brake drum is assumed as a separable. Because the phenomenon is an elastic vibration, however, the brake shoes and brake drum have a neutral axis. Therefore, the following equations hold true between radial direction \( u \) and tangential direction \( v \):

\[ u_{sp} = -\frac{\partial \phi_{sp}}{\partial \phi} q_{sp}(t) \]  \hspace{1cm} (14)

\[ u_{ss} = -\frac{\partial \phi_{ss}}{\partial \theta} q_{ss}(t) \]  \hspace{1cm} (15)

\[ u_\phi = -\frac{\partial \phi_\phi}{\partial \phi} q_{\phi}(t) \]  \hspace{1cm} (16)

If the brake shoe vibration mode functions that satisfy the above relationships are assumed to be \( \phi_p \) and \( \phi_s \), Eqs. (14), (15), and (16) can be expressed as separable, as in Eqs. (17), (18), and (19):

\[
\begin{bmatrix}
  u_{sp} \\
  v_{sp} \\
  u_{ss} \\
  v_{ss} \\
  u_\phi \\
  v_\phi \\
\end{bmatrix}
= \begin{bmatrix}
  -\frac{\partial \phi_p}{\partial \phi} & \phi_p \\
  -\frac{\partial \phi_s}{\partial \phi} & \phi_s \\
  -\frac{\partial \phi_p}{\partial \theta} & \phi_p \\
  -\frac{\partial \phi_s}{\partial \theta} & \phi_s \\
  -\frac{\partial \phi_\phi}{\partial \phi} & \phi_\phi \\
  -\frac{\partial \phi_\phi}{\partial \theta} & \phi_\phi \\
\end{bmatrix}
\begin{bmatrix}
  q_{sp}(t) \\
  q_{ss}(t) \\
  q_{\phi}(t) \\
\end{bmatrix}
\]

\[ (S=2,3,4,\ldots) \]

where \( \phi_p \) and \( \phi_s \) are functions that satisfy the boundary conditions and can be expressed by the following equations if both ends are free:

Symmetric mode

\[ \phi_p = \sinh \left( c_1 \left( \phi - \frac{\pi}{2} \right) \right) + c_2 \sin \left( c_1 \left( \phi - \frac{\pi}{2} \right) \right) \]  \hspace{1cm} (20)

Antisymmetric mode

\[ \phi_s = \cosh \left( c_1 \left( \phi - \frac{\pi}{2} \right) \right) + c_2 \cos \left( c_1 \left( \phi - \frac{\pi}{2} \right) \right) \]  \hspace{1cm} (21)

where \( c_1 \) is a constant (See the appendix).

The solutions of the assumed separable (17), (18), and (19) are substituted in Eqs. (5) through (13), followed by integration of each area. By substituting the result into Lagrange’s equation of motion, a motion equation (22) can be obtained:

\[ M \ddot{X} + KX = LAX + \mu LBX \]  \hspace{1cm} (22)

where

\[ A = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} & a_{14} \\
  a_{22} & a_{23} & a_{24} & a_{24} \\
  a_{33} & a_{34} & a_{34} & a_{44} \\
  a_{44} & a_{44} & a_{44} & a_{44} \\
\end{bmatrix} \]

\[ B = \begin{bmatrix}
  b_1 & b_2 & b_3 & b_4 \\
  b_2 & b_3 & b_4 & b_4 \\
  b_3 & b_4 & b_4 & b_4 \\
  b_4 & b_4 & b_4 & b_4 \\
\end{bmatrix} \]

\[ M = \begin{bmatrix}
  m_{ss} & 0 & m_{sp} & 0 \\
  0 & m_{ss} & 0 & m_{sp} \\
  m_{sp} & m_{sp} & 0 & 0 \\
  0 & 0 & 0 & 0 \\
\end{bmatrix} \]

\[ K = \begin{bmatrix}
  K_{ss} & 0 & K_{sp} & 0 \\
  0 & K_{ss} & 0 & K_{sp} \\
  K_{sp} & K_{sp} & 0 & 0 \\
  0 & 0 & 0 & 0 \\
\end{bmatrix} \]

\[ X = \begin{bmatrix}
  q_{a1}(t) \\
  q_{a2}(t) \\
  q_{a3}(t) \\
  q_{a4}(t) \\
\end{bmatrix} \]

\[ a_{ij} \text{ and } b_{ij} \text{ are constants dependent on the vibration modes of the brake shoes and brake drum, the brake shoe shape, the lining position, and brake shoe position angle } \alpha. \text{ (See the appendix.)} \]

The following equation can be obtained by substituting the dynamic instability solution \( X \), assumed as the minute vibration solution \( X = e^{\omega t} V \), into motion equation (22):

\[ (K - LA - \mu LB - \omega^2 M)V = 0 \]  \hspace{1cm} (23)

If this equation is to have a solution other than the trivial solution \( V = 0 \), Eq. (24) must be satisfied:

\[ \det(K - LA - \mu LB - \omega^2 M) = 0 \]  \hspace{1cm} (24)

This equation can be rearranged into a characteristic problem, as expressed in Eq. (25):

\[ \lambda^4 K - \lambda^2 M^{-1} A - \mu L M^{-1} B - \omega^2 E = 0 \]  \hspace{1cm} (25)

4. Numerical Calculations and Considerations

4.1 Comparison of test results with calculation results

The brake specifications shown in Table 1 were used in our calculations. The flexural rigidities \( EI \) and \( E_\phi a_\phi \) of the brake shoes and brake drum, and Young’s modulus \( E_\phi \) of the brake lining, shown in Table 1, were determined as follows. The natural frequencies of the experiment shown in Figs. 4 and 5 were considered to be true values, and with attention paid to the natural frequency of 1 974 Hz for the first bending mode of brake shoes and the natural frequency of 2 659 Hz for the brake drum, the flexural rigidities at a point where the calculated values and test values agreed were obtained. Table 2 shows a comparison of the calculations and the experimental values of the natural frequencies of the brake shoes and brake drum.

Young’s modulus \( E_\phi \) was obtained at a point where the experimental result for the natural frequency of the coupled vibration resulting from pressing the brake shoes against the brake drum at the braking hydraulic pressure of \( P = 3 \text{ MPa} \), indicated by a dotted line in Fig. 5, agreed with the calculation result.
4.2 Examination of analytical model

The test results revealed that, when a break squeal of 2.92 kHz was generated, the brake shoes were in the third vibration mode of free bending at both ends, and the brake drum was in the third vibration mode of bending. Thus, the brake specifications shown in Table 1 were substituted into Eq. (25); the natural frequency of coupled vibration of the third bending of brake shoes with each of the first to fourth bendings of the brake drum was calculated, and the results are shown in Table 3. The calculation results show that the first and second bendings of the brake drum produce four natural frequencies in real numbers, while the third and fourth bendings of the same produce two natural frequencies in real numbers and two in conjugate complex numbers, for a total of four. The conjugate complex numbers can be expressed by the solution shown in the note of Table 3, and involve two $\eta$'s, one positive and one negative. If $\eta$ is positive, it indicates that the vibration will diverge exponentially. It is called a dynamic instability solution. The dynamic instability solution for the third bending is 2.91 kHz, and for the fourth bending is 4.46 kHz, each approximately agreeing with the squeal frequency confirmed in the test. The analytical model used in our study may be considered reasonable, and brake squeal may be assumed to be a dynamic instability phenomenon.

4.3 Squeal-generating mechanism

Regarding the causes of brake squeal generation, the stability of the vibration system of the analytical model was observed from the standpoint of kinetic energy increase or decrease. First, Eq. (22) will be represented as follows:

$$MX' = -KX + LA^2X + \mu L^2BX$$

Therefore, the increase or decrease of kinetic energy per cycle of vibration can be calculated from Eq. (27):

$$E = \int X' MX \, dt$$

When Eq. (26) is substituted into Eq. (27), the term of the symmetric matrix will be zero, leaving only the antisymmetric matrix. Hence, Eq. (28):

$$E = \mu L \int X' BX \, dt$$

Eq. (28) can be rewritten as follows:

$$E = W_1 + W_4$$

where

$$W_1 = -\mu L \int \frac{\eta}{2} (u_{SP} - u_a) \, d\phi \, d\psi$$

$$W_4 = \mu L \int \frac{\eta}{2} (u_{SP} - u_a) \, d\phi \, d\psi$$

$$W_2 = -\mu L \int \frac{\eta}{2} (u_{SP} - u_a) \, d\phi \, d\psi$$

$$W_3 = \mu L \int \frac{\eta}{2} (u_{SP} - u_a) \, d\phi \, d\psi$$

If direction $u$ and direction $v$ are independent of each other, $E = 0$. Because the vibrations are coupled in the two directions of $u$ and $v$, $E > 0$; that is, a divergent vibration may occur. If vibration solution $X$ is assumed in terms of the period solution in Eq. (30), and is substituted into Eq. (28), Eq. (31) can be obtained:

$$X = \begin{bmatrix} q_{SS} \\ q_{SP} \\ q_{S1} \\ q_{S2} \end{bmatrix} = \begin{bmatrix} X_{SS} \sin(\omega t + \delta_3) \\ X_{SP} \sin(\omega t + \delta_1) \\ X_{S1} \sin(\omega t + \delta_1) \\ X_{S2} \sin(\omega t + \delta_2) \end{bmatrix}$$

$$E = \pi \mu L ((b_3 - b_2) X_{SP} X_{SS} \sin(\delta_1 - \delta_3)$$

$$+(b_{31} - b_{21}) X_{SP} X_{S1} \sin(\delta_2 - \delta_3)$$

$$+(b_{31} - b_{21}) X_{SP} X_{S2} \sin(\delta_1 - \delta_3)$$

$$+(b_{31} - b_{21}) X_{SP} X_{S2} \sin(\delta_1 - \delta_2)$$

It is known from Eq. (31) that a brake squeal will occur where $b_3 \neq b_2$ and where a phase difference exists among $\delta_1, \delta_2, \delta_3, \delta_4$. An example of calculating the brake drum bending third mode is shown in Figs. 7-10. As $\mu$ increases, the phase of $\delta_2$ delays compared with the phases of $\delta_1$, $\delta_3$, $\delta_4$, so that kinetic energy $E$ increases. As shown in Figs. 7 and 8.

If the lining spring coefficient is viewed in the same way, an increase in Young's modulus gradually increases the phase delay of $\delta_3$, and the kinetic energy increase $E$ reaches the maximum when the phase difference is $90^\circ$ and decreases to 0 when the phase difference is $180^\circ$, as shown in Figs. 9 and 10.

It was found, as described above, that when phase $\delta_2$ delays, there will be a phase difference between the
bending moments to the brake shoes and brake drum, causing a brake squeal.

4.4 Factors affecting brake squeal

It can easily be assumed by referring to Eq. (29) that the factors affecting brake squeal are the friction coefficient $\mu$, Young's modulus $E_t$ of the brake lining, and the lining positions $\gamma_1, \gamma_2, \beta_1, \beta_2$. The effects of the individual factors were calculated by substituting the specifications shown in Table 1 into Eq. (25).

(1) The natural frequencies shown in the first and second lines of Table 3 showing the example of calculation results apply where both the brake shoes and brake drum have a large amplitude, and those in the third and fourth lines apply where only the brake shoes have a large amplitude. The effect of friction coefficient $\mu$ on the natural frequencies in the first and second lines of Table 3 is shown in Fig. 11, where the dotted lines indicate that the natural frequencies are real numbers, and the solid lines denote conjugate complex numbers representing a dynamic instability solution, that is, an area in which brake squeal can occur. It is clear from Fig. 11 that, as the value of $\mu$

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**Fig. 7** Variation of phase difference for $\mu$

**Fig. 8** Variation of energy $E$ for $\mu$

**Fig. 9** Variation of phase difference for $E_t$

**Fig. 10** Variation of energy $E$ for $E_t$

**Fig. 11** Variation of natural frequency of coupled vibration influenced by $\mu$
increases, the natural frequencies in the individual modes come closer to each other until they finally become the same natural frequency indicated by solid lines, thereby causing a brake squeal. It is also indicated that as the value of $\mu$ increases, brake squeals of lower-mode frequencies tend to be generated.

(2) Fig. 12 shows an example of the calculation of $\mu=0$ and $\mu=0.4$ pertaining to the brake drum bending third mode. As shown, the 2.940 Hz indicated by the solid line and the 2.881 Hz indicated by the dotted line are of the same vibration mode when $\mu=0$, but differ in brake drum nodal position. As the value of $\mu$ increases, the natural frequencies indicated by the solid line and dotted line and their brake drum nodal positions come closer to each other until they finally become identical, as is clear from the example of calculation where $\mu=0.4$.

(3) The effect of Young's modulus $E_l$ of the lining is smaller than that of $\mu$. Its effect on brake squeal can be seen only if a logarithmic scale is used as an axis of abscissa, as shown in Fig. 13.

(4) As shown in Fig. 14, changing the positions of the primary and secondary brake shoes vis-à-vis each other will affect brake squeal, even though they are of the same shape. The calculation results shown in Fig. 15 indicate that the positions of brake lining on brake shoes also exert a large effect on brake squeal.

(5) The effect of brake shoe flexural rigidity $EI$ is small as in the same of Young's modulus of brake lining, as shown in Fig. 16. The flexural rigidity $EI$ of the brake drum has a large effect on brake squeal frequency, but only a small effect on the generation of brake squeal, as shown in Fig. 17.

5. Conclusions

This study has brought to light the following as an extension of the study by N. Millner:

(1) There are two types of brake drum vibration, node and antinode, at the center of the lining when friction coefficient $\mu=0$, though the brake drum is in the same diametral nodal mode. As the bending vibrations of the brake shoes and brake drum are coupled in the radial direction and the tangential direction, there occurs a phase difference between the bending moments generated in the brake shoes and brake drum a the value of $\mu$ increases. As a result, both vibration modes become the same, and the nodal position moves in synchronization with the vibration

![Fig. 12 Variation of vibration mode of brake shoe and brake drum influenced by $\mu$ (S=4)](image)

Note: Vibration amplitude is shown with maximum brake drum amplitude as 1

![Fig. 14 Influence of $\alpha$](image)

![Fig. 13 Influence of $E_l$](image)

![Fig. 15 Influence of $\gamma_1$ and $\beta_1$](image)
in the direction opposite to drum rotation, thus generating a standing wave in the brake drum. This is considered to cause a brake squeal.

(2) By inducing Eq. (29), which is used to identify the relationship of increase and decrease of kinetic energy per cycle of vibration, it was clearly demonstrated that the friction coefficient \( \mu \), vibration mode, lining spring coefficient \( L = E_{d}w_{l}/h \), brake shoe center angle \( \gamma \), lining positions \( \gamma, \gamma_{s}, \beta, \beta_{s} \), and the relative angle \( \alpha \) of the two brake shoes affect brake squeal. By observing the results of numeric calculations, the conclusion was drawn that the most effective way of preventing brake squeal would be to select a lining having a low friction coefficient \( \mu \), change the vibration mode by changing the brake shoe boundary conditions, and change lining positions. The study has also shown that an effect can be expected to result from changing the lining spring coefficient, brake shoe relative angle \( \alpha \), and brake shoe flexural rigidity.

Appendix

(1) \( c_{l} \) in the case of a symmetric mode:

\[
\begin{align*}
    c_{1} &= \frac{2}{\gamma} \frac{m_{s} + X_{s}}{2} \frac{3y^{2}}{8m_{s}} \\
    c_{2} &= \frac{2}{\gamma} \frac{m_{s}}{2} + X_{s} + \frac{3y^{2}}{8m_{s}} \\
    c_{s} &= \frac{c_{l}(c_{l} + c_{l} - 2)\cosh\frac{clX}{2}}{c_{l}((c_{l} - c_{l} - 2)\cosh\frac{clX}{2}} \\
    X_{s} &= -\frac{3y^{2}}{4m_{s}} \left( 1 + m_{s} \tan \frac{m_{s}}{2} \right) \tan \frac{m_{s}}{2} \\
    m_{s} &= \frac{2K + 1}{2} \quad (K = 1, 3, 5, \ldots)
\end{align*}
\]

(2) \( c_{l} \) in the case of an antisymmetric mode:

\[
\begin{align*}
    c_{1} &= \frac{2}{\gamma} \frac{m_{s} + X_{s}}{2} \frac{3y^{2}}{8m_{s}} \\
    c_{2} &= \frac{2}{\gamma} \frac{m_{s}}{2} + X_{s} + \frac{3y^{2}}{8m_{s}}
\end{align*}
\]
\begin{align}
(17) \quad a_2 &= - \int_n^s \cos^2 S \phi d\phi + \int_n^s \cos^2 S(\theta + a) d\theta \\
(18) \quad b_{12} &= \int_n^s \phi \frac{d\phi}{d\theta} d\theta \\
(19) \quad b_2 &= \int_n^s \phi \frac{d\phi}{d\phi} d\phi \\
(20) \quad b_{13} &= \int_n^s \sin S(\theta + a) \phi d\theta \\
(21) \quad b_{14} &= - \int_n^s \cos S(\theta + a) \phi d\theta \\
(22) \quad b_{23} &= \int_n^s \sin S \phi d\phi \\
(23) \quad b_{24} &= - \int_n^s \cos S \phi d\phi \\
(24) \quad b_{15} &= - \frac{1}{S} \int_n^s \sin S \phi \cos S \phi d\phi - \frac{1}{S} \int_n^s \sin S(\theta + a) \cos S(\theta + a) d\theta \\
(25) \quad b_4 &= \frac{1}{S} \int_n^s \sin S \phi \cos S \phi d\phi + \frac{1}{S} \int_n^s \sin S(\theta + a) \cos S(\theta + a) d\theta \\
(26) \quad b_{14} &= - \frac{a_4}{S} \\
(27) \quad b_{32} &= - \frac{a_{14}}{S} \\
(28) \quad b_{34} &= - \frac{a_{24}}{S} \\
(29) \quad b_{41} &= \frac{a_{14}}{S} \\
(30) \quad b_{42} &= \frac{a_{24}}{S} \\
(31) \quad b_{43} &= \frac{a_{34}}{S}
\end{align}

References