Multi-Job Cyclic Scheduling for an Automated Job-Shop-Type Manufacturing System*

(Relationship Between Job Output Interval and Buffer Capacity)

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A multi-job cyclic scheduling problem for an automated job-shop-type manufacturing system was analyzed. In the cyclic scheduling problem, an optimum schedule is a non-idle schedule, where a bottleneck machine produces jobs without idle time in the steady state. A dividing method of the jobs was introduced and the job output interval in the steady state was clarified. A lower bound on the buffer capacity was obtained to construct a non-idle schedule.

Key Words: Production System, FMS, Production Management, Scheduling, Cyclic Production, Buffer, Network

1. Introduction

The multi-job automated manufacturing system is spreading throughout manufacturing industry with the technology development of automated machine tools, automated material handling devices and computer-controlled systems. One of the production methods for multi-job mixed production in this manufacturing system is "cyclic production", where, on the basis of the required amount of respective jobs, a number of different jobs are combined to make a set of jobs, and this set is cyclically produced on the manufacturing system.

The scheduling problem for optimizing the cyclic production is called the "cyclic scheduling problem". An objective of this cyclic scheduling problem is to minimize the job output interval between cycles produced from the manufacturing system in a steady state. An optimum schedule in the cyclic scheduling is a non-idle schedule, where a bottleneck machine produces the jobs without idle time. In the non-idle schedule, the job output interval is equal to the sum of loading times, processing times and unloading times of the jobs produced in one cycle on the bottleneck

machine.

In the case of the manufacturing system with infinite buffers, there is a non-idle schedule and a scheduling method has been developed to generate the non-idle schedule⁽¹⁾. However, in the case of the manufacturing system with finite buffers, there is not always a non-idle schedule for small buffer capacities. The necessary buffer capacities to generate a non-idle schedule have been analyzed for the flow-type manufacturing system⁽²⁾, but the necessary buffer capacities have not been obtained for the job-shop-type manufacturing system. In this paper, the buffer capacities have been clarified for the job-shop-type manufacturing system on the basis of analysis of the relationship between the job output interval and the buffer capacities.

2. Model Description

2.1 Structure of automated manufacturing system

A model of an automated job-shop-type manufacturing system consists of an input station, machines, front local buffers, common buffers, an output station and conveyors to connect the above-mentioned components, as shown in Fig. 1. A front local buffer is provided in front of each machine and at the output station, and accommodates only the jobs which pass that particular equipment. A common buffer accommodates the jobs which are transported to any

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equipment.

2.2 Division of jobs

The process routes through which the jobs pass are expressed by a network. Each position in the system, the input station, machines and the output station, is indicated by a node, and the process route of each job is indicated by an arc. The network, constructed in this way, is called a "process network".

When this process network contains some loops, the non-idle schedule is not always obtained by a normal cyclic production even if the buffers have infinite capacities. A revised cyclic production method has been developed to generate the non-idle schedule on the basis of division of the jobs⁽¹⁾. This paper introduces this production method into the scheduling problem of the automated manufacturing system. In the production method, a portion of a job's process route is completed in a cycle, and the other portions are completed in the successive cycles. These portions of the job are called "divided jobs". This division of a job is carried out by dividing the process route at the point where the arc is an element of a loop in the process network.

The divisions of job A are denoted by the first divided job A_1 , the second divided job A_2 , up to the last divided job A_n according to the process route of job A .

2.3 Input sequence of jobs

On the basis of the required amounts of respective jobs, the set of jobs produced for one cycle is determined. This set of jobs is cyclically inputted into the manufacturing system according to a fixed sequence among the cycles.

The divided jobs generated from a specified job are produced on each different cycle. In order to produce a divided job A_i from the first cycle, the necessary quantity of the divided job A_{i-1} must be produced before the cyclic production⁽¹⁾. This quantity

of the divided job is called the "pre-cyclic-production quantity" of the divided job.

The divided jobs generated from a specified job consist of the processing operations on the same workpiece. When the pre-cyclic-production quantity of the divided job i_j is ξ_{ij} , the divided job i_j on the l -th cycle and the divided job i_{j+1} on the $(l + \xi_{ij})$ th cycle are operations on the same workpiece.

2.4 Production sequence of jobs

(1) The production sequence of jobs on each machine and the output station is the same among the cycles, as is the storing sequence of jobs on each buffer space.

(2) When a job is completed at a position in the manufacturing system, it is immediately transported to the next position in the system if this position is not occupied by the previous job.

3. Work Flow Network Representation

In order to clarify the time when a job is loaded on each position in the manufacturing system, the flow of the processing of the jobs is expressed in the form of a network. The event in which a job is loaded on a position is expressed with a node, and the relationship between nodes is expressed with an arc so that the exact time of the occurrence of the event can be determined. The network expressed in these ways is used to obtain the job output interval in a steady state.

3.1 Setting of notations

The work flow network is defined by the following notations.

- n : number of jobs produced in the manufacturing system in one cycle
- M : number of machines
- I_s : input station
- O_s : output station
- $B_F(i, k)$: i -th buffer space of front local buffer of machine k ($i=1, 2, \dots, b_F(k)$)
- $B_F(i, O_s)$: i -th buffer space of front local buffer of output station ($i=1, 2, \dots, b_F(O_s)$)
- $B_C(i)$: buffer space of common buffer ($i=1, 2, \dots, b_C$)
- $J(l, j, a)$: j -th job in l -th cycle on position a (input station, machines, front local buffer, common buffer, or output station)
- $P(l, j, k)$: processing time of $J(l, j, k)$ on machine k
- t_s : loading time of a job on input station
- t_D : loading time of a job on output station
- t_E : unloading time of a job from output station
- t_r : unloading time of a job from input station and common buffer, or loading time of a job on front local buffer and common buffer
- $t_l(l, j, k)$: loading time of $J(l, j, k)$ on machine k

Fig. 1 Model of an automated manufacturing system

$t_o(l, j, k)$: unloading time of $J(l, j, k)$ from machine k
 t_B : transfer time of a job in front local buffer spaces

$H(a, a')$: transportation time of a job from position a to position a' . $H(a, a')$ does not contain t_T and $t_o(l, j, k)$.

$S(0, a)$: ready time for processing the first job on position a

$N(a)$: number of jobs produced on position a in one cycle

$S(l, j, a)$: time when $J(l, j, a)$ is loaded on position a . When $l \geq 2$, $S(l, 0, a)$ is expressed as $S\{l-1, N(a), a\}$. When $l=1$ and no pre-cyclic-production job is produced on position a , $S(l, 0, a)$ is expressed as $S(0, a)$. Otherwise, $S(l, 0, a)$ represents the time when the last job in the set of pre-cyclic-production jobs is loaded on position a .

$x(0, a)$: node which shows $S(0, a)$

$x(l, j, a)$: node which shows $S(l, j, a)$

3.2 Construction of work flow network

The main difference between the work flow to be analyzed in this paper and the work flow analyzed in the previous paper⁽²⁾ is that in this paper the processing of the divided job is introduced. Since the network representation of the non-divided job is the same as in the previous paper⁽²⁾, this paper represents the network of the divided job.

The machine of the first process of the divided job i_{j+1} is denoted by m_1 , and the production sequence of i_{j+1} on m_1 in each cycle is denoted by s_1 . The machine of the last process of a divided job i_j is denoted by m_2 , and the production sequence of i_j on m_2 in each cycle is denoted by s_2 . The pre-cyclic-production quantity of i_j is ξ_{ij} .

Then both the divided job i_{j+1} in the l -th cycle and the divided job i_j in the $(l - \xi_{ij})$ th cycle consist of operations on the same workpiece. In order to load the job $J(l, s_1, B_F(1, m_1))$ in the buffer space $B_F(1, m_1)$, the space $B_F(1, m_1)$ must be vacant and the job $J(l - \xi_{ij}, s_2, m_2)$ must be arriving at $B_F(1, m_1)$.

When $J(l - \xi_{ij}, s_2, m_2)$ is directly transported from m_2 to $B_F(1, m_1)$, the conditions are expressed by two arcs, as shown in Fig. 2 (a). One is the arc from $x(l - \xi_{ij}, s_2, m_2)$ to $x\{l, s_1, B_F(1, m_1)\}$ and the other is the arc from $x\{l, s_1 - 1, B_F(2, m_1)\}$ to $x\{l, s_1, B_F(1, m_1)\}$. The value of the node $x\{l, s_1, B_F(1, m_1)\}$ is a maximum between the following two values. One is the sum of the value of $x(l - \xi_{ij}, s_2, m_2)$ and the value of the arc connecting $x(l - \xi_{ij}, s_2, m_2)$ and $x\{l, s_1, B_F(1, m_1)\}$, the other is the sum of the value of $x\{l, s_1 - 1, B_F(2, m_1)\}$ and the value of the arc connecting $x\{l, s_1 - 1, B_F(2, m_1)\}$ and $x\{l, s_1, B_F(1, m_1)\}$.

In order to load the job $J(l - \xi_{ij}, s_2 + 1, m_2)$ on m_2 ,

$J(l - \xi_{ij}, s_2, m_2)$ must be unloaded from m_2 . This condition is expressed by the arc from $x\{l, s_1, B_F(1, m_1)\}$ to $x(l - \xi_{ij}, s_2 + 1, m_2)$.

When $J(l - \xi_{ij}, s_2, m_2)$ is transported from m_2 through a common buffer $B_C(c_1)$ to $B_F(1, m_1)$, the network is constructed as in Fig. 2 (b), where the storing sequence of $J(l - \xi_{ij}, s_2, m_2)$ on $B_C(c_1)$ is denoted by s_3 in the $(l - \xi_{ij})$ th cycle.

The network for the pre-cyclic-production job is constructed by the same method as in the previous paper⁽²⁾.

The work flow network for all jobs is constructed by connecting the network of each job on each position in the manufacturing system.

3.3 Features of work flow network

The work flow network constructed has two features which are different from the work flow network of the previous paper⁽²⁾:

(1) A work flow network contains not only an arc from $x\{l, N(a), a\}$ to $x(l+1, 1, a')$, but also an arc from $x(l, j, a) (j < N(a))$ to $x(l+s, j', a')$.

(2) A work flow network contains an arc from $x(l, j, a)$ to $x(l-s, j', a')$.

4. Job Output Interval in a Steady State

On the basis of the work flow network, the job output interval is analyzed. When the work flow network contains a loop and the sum of the values on

Fig. 2 Network representation of the time when the divided job is loaded on the front local buffer

arcs constituting the loop is a positive value, the production sequence is infeasible. Therefore, this kind of the work flow network is excluded in the analysis.

The number of positions in the manufacturing system, which is denoted by d , is given by the following equation.

$$d = 2 + M + \sum_{k=1}^M b_F(k) + b_F(O_s) + b_c. \quad (1)$$

The set of jobs which are divided into the divided jobs is denoted by S_D , and the set of divided jobs which are generated from job i ($i \in S_D$) is denoted by $S_E(i)$.

The work flow network contains an arc connecting the following two nodes. One is the node for the last process of the divided job i_j in the l -th cycle ($l = 1, 2, \dots$), and the other is the node for the first process of the divided job i_{j+1} in the $(l + \xi_{ij})$ th cycle. This is feature (1) in sec. 3.3. The number of arcs from the nodes contained in the l -th cycle or the preceding cycle to nodes contained in the $(l+1)$ th cycle or the following cycle, is denoted by Q , and expressed by the following equation.

$$Q = d + \sum_{i \in S_D} \sum_{j \in S_E(i)} \xi_{ij}. \quad (2)$$

In the work flow network, the sum of the values on arcs contained in a path is called the length of the path. The longest path between two nodes is called the critical path. The set of nodes in a cycle, where each of the nodes is connected with a node in the preceding cycle by an arc, is denoted by S_l . The critical path from a node x contained in the l -th cycle to the node x in the $(l+s)$ th cycle is denoted by $|L(s, x)|$. In the calculation of this length, the nodes in the preceding cycle of the l -th cycle must be taken into account, because the work flow network has feature (2) of sec. 3.3. Using these notations, the value G is defined as follows.

$$G = \max_{\substack{s \in \{1, 2, \dots, Q\} \\ x \in S_l}} \frac{|L(s, x)|}{s}. \quad (3)$$

Then the following property is proved by a similar logic with the previous paper⁽²⁾. The difference between $S(l+Q!, j, a)$ and $S(l, j, a)$ is equal to $GQ!$, when l is a large number.

5. Relationship Between the Job Output Interval and the Buffer Capacity

In the revised cyclic production, the jobs are separated into divided jobs so that the process network has no loop. The process network is reconstructed by considering the divided jobs as different jobs. In this reconstructed network, there are paths from the node representing the input station to the node representing the machine k . When $k = M + 1$, machine k is the output station. The maximum value among the number of nodes contained in these paths is denoted by $N_Q(k)$. The k -th step of the machine in the process

route of the job or the divided job I is denoted by $m(I, k)$, and the number of job processes is denoted by $N_p(I)$, and $m(I, 0)$ expresses the input station and $m(I, N_p(I) + 1)$ expresses the output station.

The necessary buffer capacity to obtain the non-idle schedule is varied by the number of pre-cyclic-production quantities and the production sequence of jobs. When $H(a, a'), t_s, t_D, t_E, t_T$ and t_B are relatively large values compared to $t_l(l, j, k)$, $t_o(l, j, k)$, and $P(l, j, k)$, the processing of jobs on a machine is not always the bottleneck process in the manufacturing system. Therefore, the analysis is carried out under the following assumptions:

(1) The pre-cyclic-production quantity ξ_{ij} of the divided job i_j is $N_Q\{m(i_j, N_p(i_j))\} - N_Q\{m(i_{j+1}, 1)\}$.

(2) The schedule is fixed as a permutation schedule. In this schedule, when job j is produced before job j on a position in the system, job j is produced before job j on the other positions in which both jobs are produced. When the divided job i_j and i_{j+s} ($s = 1, 2, \dots$) generated from job i are produced on a machine, the divided job i_j is produced before i_{j+s} .

(3) The times $H(a, a'), t_s, t_D, t_E, t_T$ and t_B are very small value compared with the processing time $P(l, j, k)$, and these values can be neglected.

The path from any node $x(l, j', a)$ to the node $x(l + s, j', a)$ ($s = 1, 2, \dots, M - 1$) contains the arcs expressing the processing time of jobs on some kinds of machines. When the number of this type of machine is less than $s + 1$, the value G is equal to the sum of loading times, processing times and unloading times of the jobs produced in one cycle on the bottleneck machine, and the non-idle schedule is obtained. A production method to satisfy this condition is as follows. Each job i loaded on the input station passes $k - 1$ machines, $N_Q\{m(i, k)\} - (k + 1)$ common buffers and $N\{m(i, k)\} - 1$ front local buffers, and is loaded on $m(i, k)$. Each divided job i_j produced on $m(i_j, 1)$ passes $k - 2$ machines, $N_Q\{m(i_j, k)\} - N_Q\{m(i_j, 1)\} - (k - 1)$ common buffers and $N\{m(i_j, k)\} - 1$ front local buffers, and is loaded on $m(i_j, k)$. After the divided job i_j is produced on $m(i_j, N_p(i_j))$, the job passes ξ_{ij} common buffers.

The number of necessary common buffers which job i passes between $m(i, k - 1)$ and $m(i, k)$, $N_N\{i, m(i, k - 1), m(i, k)\}$, is given by the following equation.

$$\begin{aligned} N_N\{i, m(i, k - 1), m(i, k)\} \\ = N_Q\{m(i, k)\} - k^* - (k + 1) \\ k = 1, 2, \dots, N_p(i) + 1. \end{aligned} \quad (4)$$

where k^* expresses the total sum of $N_N\{i, m(i, l - 1), m(i, l)\}$ ($l = 1, 2, \dots, k - 1$).

The number of necessary common buffers, which divided job i_j ($j = 2, 3, \dots$) passes between $m(i_j, k - 1)$ and $m(i_j, k)$, $N_N\{i_j, m(i_j, k - 1), m(i_j, k)\}$, is given by the following equation,

$$\begin{aligned}
& N_N\{i_j, m(i_j, k-1), m(i_j, k)\} \\
& = N_Q\{m(i_j, k)\} - N_Q\{m(i_j, 1)\} - k^* - (k-1) \\
& \quad k=2, 3, \dots, N_P(i_j) + 1 \quad (5)
\end{aligned}$$

where k^* expresses the total sum of $N_N\{i_j, m(i_j, l-1), m(i_j, l)\}$ ($l=1, 2, \dots, k-1$). The number for i_1 is given by Eq. (4).

When the capacity of the front local buffer is less than $N\{m(i, k)\} - 1$, $\max\{N\{m(i, k)\} - 1 - b_F\{m(i, k)\}, 0\}$ common buffers are allocated to $m(i, k)$, and $\max\{n - 1 - b_F(O_s), 0\}$ common buffers are allocated to the output station. The allocated common buffers are used as front local buffers.

If the total sum of $N_N\{i, m(i, k-1), m(i, k)\}$, $N_N\{i_j, m(i_j, k-1), m(i_j, k)\}$ and ξ_{ij} is denoted by b^* , then the necessary buffer capacity to obtain the non-idle schedule is as follows.

$$\sum_{k=1}^M \beta(k) + \beta(O_s) + b_c \geq \sum_{k=1}^M \{N(k) - 1\} + (n-1) + b^* \quad (6)$$

$$\beta(k) = \min\{b_F(k), N(k) - 1\} \quad k=1, 2, \dots, M \quad (7)$$

$$\beta(O_s) = \min\{b_F(O_s), n-1\}. \quad (8)$$

6. Numerical Example

A simple numerical example is demonstrated to show the analytical results. The problem is as follows.

There are four machines in the automated manufacturing system. The buffer capacities are $b_F(1) = b_F(2) = b_F(3) = b_F(4) = b_F(O_s) = 2$, and $b_c = 10$. The load-

Table 1 Process routes and processing times (expressed in parentheses; min) of the jobs

Job	1st stage	2nd stage	3rd stage
1	Machine 1 (13)	Machine 2 (8)	Machine 3 (5)
2	Machine 3 (13)	Machine 2 (8)	Machine 1 (13)
3	Machine 3 (6)	Machine 4 (8)	Machine 2 (8)

Fig. 3 Process network constructed for the process routes of Table 1

Fig. 4 Reconstructed process network of Figure 3

ing and unloading times are $t_s = t_D = t_E = 0.5$ min, $t_r = t_B = 0.2$ min, and both $t_l(l, j, k)$ and $t_o(l, j, k)$ are 1 min regardless of the kind of job and machine. The number of jobs produced in one cycle is 3, and the process routes and the processing times of the jobs are given by Table 1. The transportation time $H(a, a')$ is 1 min regardless of the positions a and a' in the manufacturing system. The ready times of the positions are $S(0, 1) = 7$ min, $S(0, 2) = 5$ min, $S(0, 3) = 10$ min, $S(0, 4) = 8$ min, $S\{0, B_F(1, O_s)\} = 11.4$ min, $S\{0, B_F(2, O_s)\} = 11.9$ min, $S(0, O_s) = 12.4$ min, and the ready times of the other positions are 0 min.

The process network is constructed as shown in Fig. 3 using the process routes of Table 1, and contains loops. The jobs are divided to cut the loops of the method⁽¹⁾. Job 1 is divided into divided jobs 1_1 and 1_2 , where the process route of 1_1 is machine 1 and

Table 2 Process route which the jobs pass in the production

Job	Process route
1 ₁	$I_S \rightarrow B_F(1, 1) \rightarrow B_F(2, 1) \rightarrow \text{Machine 1} \rightarrow B_C(1) \rightarrow B_F(1, 2) \rightarrow B_F(2, 2) \rightarrow \text{Machine 2} \rightarrow B_C(2) \rightarrow B_C(3)$
1 ₂	$B_F(1, 3) \rightarrow B_F(2, 3) \rightarrow \text{Machine 3} \rightarrow B_C(4) \rightarrow B_C(5) \rightarrow B_F(1, O_s) \rightarrow B_F(2, O_s) \rightarrow O_s$
2 ₁	$I_S \rightarrow B_F(1, 3) \rightarrow B_F(2, 3) \rightarrow \text{Machine 3} \rightarrow B_C(6) \rightarrow B_F(1, 2) \rightarrow B_F(2, 2) \rightarrow \text{Machine 2} \rightarrow B_C(7) \rightarrow B_C(8)$
2 ₂	$B_F(1, 1) \rightarrow B_F(2, 1) \rightarrow \text{Machine 1} \rightarrow B_C(9) \rightarrow B_C(10) \rightarrow B_F(1, O_s) \rightarrow B_F(2, O_s) \rightarrow O_s$
3	$I_S \rightarrow B_F(1, 3) \rightarrow B_F(2, 3) \rightarrow \text{Machine 3} \rightarrow B_F(1, 4) \rightarrow B_F(2, 4) \rightarrow \text{Machine 4} \rightarrow B_F(1, 2) \rightarrow B_F(2, 2) \rightarrow \text{Machine 2} \rightarrow B_F(1, O_s) \rightarrow B_F(2, O_s) \rightarrow O_s$

Table 3 Production sequence of the jobs on the input station, machines and the output station

Position	Production sequence
I_S	Job 1 ₁ , Job 1 ₁ , Job 2 ₁ , Job 2 ₁ , Job 1 ₁ , Job 2 ₁ , Job 3, Job 1 ₁ , Job 2 ₁ , Job 3,...
Machine 1	Job 1 ₁ , Job 1 ₁ , Job 1 ₁ , Job 2 ₂ , Job 1 ₁ , Job 2 ₂ ,...
Machine 2	Job 1 ₁ , Job 1 ₁ , Job 2 ₁ , Job 2 ₁ , Job 1 ₁ , Job 2 ₁ , Job 3, Job 1 ₁ , Job 2 ₁ , Job 3,...
Machine 3	Job 2 ₁ , Job 2 ₁ , Job 2 ₁ , Job 3, Job 1 ₂ , Job 2 ₁ , Job 3, Job 1 ₂ ,...
Machine 4	Job 3, Job 3,...
O_s	Job 3, Job 1 ₂ , Job 2 ₂ , Job 3, Job 1 ₂ , Job 2 ₂ ,...

Fig. 5 Gantt chart of the schedule obtained for Table 1
 (The number $l_j(l)$ in the square represents
 the l -th produced job of job l_j)

machine 2 and the process route of 1_2 is machine 3. Job 2 is divided into jobs 2_1 and 2_2 , where the process route of 2_1 is machine 3 and machine 2 and the process route of 2_2 is machine 1. When these divided jobs are considered as different jobs, the process network is constructed as in Fig. 4. This reconstructed process network does not contain a loop.

From the network of Fig. 4, $b^*=10, \beta(1)=1, \beta(2)=\beta(3)=\beta(O_s)=2$ and $b(4)=0$. Using the conditions of Eqs. (6)~(8), the number of necessary common buffers is 10. The manufacturing system satisfies this condition.

Both pre-cyclic-production quantities of the divided job 1_1 and 2_1 are 2. The process route of each job is obtained as shown in Table 2, and the job sequence on each position in the manufacturing system is obtained is shown in Table 3. When seven pieces of each job are produced, the Gantt chart is obtained (see Fig. 5). From this chart, it is clear that the job output interval is equal to the sum of the loading times, the processing times and the unloading times on a bottleneck machine, and the non-idle schedule is obtained.

7. Conclusions

In this paper, the cyclic scheduling problem of an automated manufacturing system was analyzed, and the following results were obtained.

- (1) The work flow network was constructed for an automated job-shop-type manufacturing system.
- (2) The job output interval in the steady state was obtained using the work flow network.
- (3) The necessary buffer capacity was clarified to obtain the non-idle schedule.

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