Vibration Control in Piping System by Dual Dynamic Absorber*

(3-Dimensional Vibration Analysis and Absorber Design Using Transfer Matrix Method)

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This paper deals with the application of dual dynamic absorbers to nuclear piping systems to accomplish a high damping value and reduce seismic response at resonance frequencies. The transfer matrix method is used for design of the dual dynamic absorbers as well as for determination of the optimum mounting location. The effectiveness of the dynamic absorber is demonstrated by suppressing the first three resonance peaks in the 3-dimensional model piping system.

Key Words: Vibration, Vibration Control, Piping System, Dual Dynamic Absorber, Optimum Design, Modal Analysis, Transfer Matrix

1. Introduction

Piping systems are flexible and affected by many kinds of source vibrations. These vibrations create problems such as fatigue damage, vibration and noise. In the past, support equipment was placed at the main points of the piping system in order to solve these problems. However, this restricts the arrangement of the piping systems. A method for suppressing resonance without support equipment will hopefully be developed in the near future. The design of earthquake-proof piping systems is essential to the safe operation of nuclear power generation plants. Since it is impossible to estimate the damping value of the piping systems, which determines the response against earthquakes, it is difficult to design piping systems rationally.

Thus, the recent trend is to use damping devices in designing nuclear piping systems. Kunieda⁽¹⁾ reported an idea on the application of a dynamic damper to the piping system to reduce the seismic vibration response. Hara and Seto⁽²⁾ also reported the basic concept of the application of dual dynamic absorbers to the seismic design of nuclear piping systems.

However, Seto et al.⁽³⁾ proposed the new method of "vibration control of multi-degree-of-freedom systems", and the effectiveness of this method was confirmed by its application to the beam structure⁽⁴⁾, the Γ -shaped structure⁽⁵⁾, and the plate structure⁽⁶⁾⁽⁷⁾. The following explains the control of the vibration in the structure. First, each mode shape at the resonance frequency is determined by modal analysis. Second, the optimum mounting locations of the dynamic absorber and the equivalent mass of each mounting location are determined on each mode. Finally, the dynamic absorber is designed optimally on each mode.

This paper shows how to control the vibration of piping systems and improve the design safety of such piping systems by using the new method. In order to analyze the vibration mode, estimate the equivalent

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mass on each mode, and examine the effectiveness of controlling vibration, the expanded 3-dimensional transfer matrix method is used. This paper shows theoretically that the first three resonance peaks within 100 Hz are well suppressed by using three dual dynamic absorbers. The effectiveness of the three dual dynamic absorbers is also demonstrated experimentally.

2. The Substance of Vibration Control in Piping Systems by Dynamic Absorbers

In order to establish vibration control in a piping system by using dynamic absorbers, it is essential to combine the modal analysis technique with the established design method of the dynamic absorber for controlling one-degree-of-freedom vibration systems. According to modal analysis, n th-degree-of-freedom systems, such as the piping system with resonance frequencies, are decoupled into the n th one-degree-of-freedom system in the modal domain. This enables better control of the vibration at each one-degree-of-freedom system using the dynamic absorbers designed by the established method.

The modal analysis technique is used to determine a suitable location for installing the dynamic absorber on each vibration mode shape, and to estimate the equivalent mass at the location corresponding to each one-degree-of-freedom system.

Applying the concept for controlling the vibration of the piping system, we propose the following design procedure for the dynamic absorber.

(1) Analyze the vibration mode shapes using modal analysis.

(2) Determine the mounting location of the dynamic absorber at the maximum amplitude point on each vibration mode shape. In principle, the mounting location should be at the vibration node of the other mode in order to obtain the decoupled condition of each mode.

(3) Estimate the equivalent mass at each mounting location of the dynamic absorber.

(4) Design the dynamic absorber using the equivalent mass as a basic design parameter.

3. Vibration Analysis in the Piping System

3.1 The configuration of the piping system

The configuration of the piping system selected in this paper as a vibration-control example is shown in Fig. 1. This system is made of copper tubing, 25 mm in outer diameter and 15 mm in inner diameter; both ends of the tube are fixed. In this paper, we tried to control vibration of the piping system by using only dynamic absorbers and no support equipment.

3.2 The expression of transfer matrix

In order to analyze the vibration characteristics and examine the vibration mode shape of a 3-dimensional piping system, we used the transfer matrix method. This piping system is divided into two fundamental elements which consist of 21 pieces of a piping element and 4 pieces of a coordinate transformation element, as shown in Fig. 2. In applying the transfer matrix method, it is necessary to prepare a 3-dimensional transfer matrix representation of these two fundamental elements.

3.2.1 The transfer matrix of a piping element

In accordance with the definition of coordinates and variables as shown in Fig. 3, the transfer matrix of a piping element between point "a" and point "b" is as follows:

Fig. 2 The division of the piping system

Fig. 1 Construction of a piping system as a vibration control example

Fig. 3 The definition of coordinates and the variables of a piping element

$$\begin{Bmatrix} X \\ N \\ \theta \\ T \\ Y_y \\ \phi_y \\ M_y \\ F_y \\ Y_x \\ \phi_x \\ M_x \\ F_x \\ 1 \end{Bmatrix}_a = \begin{bmatrix} L & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 \\ 0 & 0 & P_y & 0 & 0 \\ 0 & 0 & 0 & P_x & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X \\ N \\ \theta \\ T \\ Y_y \\ \phi_y \\ M_y \\ F_y \\ Y_x \\ \phi_x \\ M_x \\ F_x \\ 1 \end{Bmatrix}_b \quad (1)$$

or

$$\mathbf{Z}_a = \mathbf{B}_a \cdot \mathbf{Z}_b, \quad (1)'$$

where

X : displacement of axial way, N : axial stress,
 θ : angle of twist, T : torque, Y : displacement
of deflection, ϕ : angle of diffraction,
 M : bending moment, F : shearing stress.

Subscripts "y" and "x" indicate the y -direction and x -direction respectively. Each L, M, P_y, P_x is a partial transfer matrix in the flexible beam element which shows axial, twist, and bending directions. The " f_y " and " f_x " are terms of input acting on the beam in the y -direction and x -direction. 0 is a zero matrix and L, M is as follows :

$$L = \begin{bmatrix} \cos \sigma l & \sin \sigma l / (\omega \sqrt{m_b k_b}) \\ -\omega \sqrt{m_b k_b} \sin \sigma l & \cos \sigma l \end{bmatrix} \quad (2)$$

$$M = \begin{bmatrix} \cos \tau l & \sin \tau l / (\omega \sqrt{J_t k_t}) \\ -\omega \sqrt{J_t k_t} \sin \tau l & \cos \tau l \end{bmatrix} \quad (3)$$

where

$$\sigma l = \omega \sqrt{m_b / k_b}, \quad \tau l = \omega \sqrt{J_t / k_t},$$

m_b : weight of a piping element

J_t : inertial moment of a piping element

k_b : stiffness constant in axial direction

k_t : stiffness constant in twist direction

ω : frequency.

P_y, P_x are 4×4 matrices which are defined in the paper "Vibration Control in Beam Structure by Dynamic Absorber⁽⁴⁾".

3.2.2 The transfer matrix of coordinate transformation As shown in Fig. 4, there are three

Fig. 4 The kind of coordinate transformation

kinds of transfer matrices for coordinate transformation, i.e., about the x -axis, y -axis, and the z -axis. Each rotation angle is defined as α, β , and γ , clockwise. In this paper, the definition of coordinate transformation and variable value is shown in Fig. 5, when the coordinate rotates about the x -axis. This transfer matrix results ;

$$\mathbf{Z}_a = \mathbf{T}_a \cdot \mathbf{Z}_b, \quad (4)$$

where

$$\mathbf{T}_a = \begin{bmatrix} C & 0 & 0 & 0 & S & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & 0 & 0 & 0 & S & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 & 0 & 0 & S & 0 & 0 & 0 \\ 0 & 0 & 0 & C & 0 & 0 & 0 & 0 & 0 & -S & 0 & 0 \\ -S & 0 & 0 & 0 & C & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -S & 0 & 0 & 0 & 0 & 0 & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -S & 0 & 0 & 0 & 0 & 0 & 0 & C & 0 & 0 \\ 0 & 0 & 0 & S & 0 & 0 & 0 & 0 & 0 & 0 & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_a$$

where $C = \cos \alpha, S = \sin \alpha$.

The transfer matrices for coordinate transformation about the y -axis and z -axis are similarly prepared.

3.2.3 The transfer matrix of each element and the state vector The concept of transfer matrices in the piping system is shown in Fig. 6. $\mathbf{B}_i$ is the transfer matrix of the i th piping element, $\mathbf{T}_j$ is that of the j th coordinates transformation, and $\mathbf{Z}_k$ is the state vector at the k th point. State vectors $\mathbf{Z}_1$ and $\mathbf{Z}_{26}$ at the ends are derived from the boundary condition as follows :

Fig. 5 The definition of coordinate transformation and the variables about the x -axis

$$Z_1 = \{0 \ N \ 0 \ T \ 0 \ 0 \ M_y \ F_y \ 0 \ 0 \ M_x \ F_x \ 1\}^T \quad (5)$$

$$Z_{26} = \{0 \ N \ 0 \ T \ 0 \ 0 \ M_y \ F_y \ 0 \ 0 \ M_x \ F_x \ 1\}^T \quad (6)$$

3.3 Frequency analysis

A transfer matrix operation based on Fig.6 yields the following state vectors at each point :

$$\left. \begin{aligned} Z_{25} &= H_{25} \cdot Z_{26} \\ Z_{24} &= H_{24} \cdot Z_{26} \\ Z_{23} &= H_{23} \cdot Z_{26} \\ &\vdots \\ Z_2 &= H_2 \cdot Z_{26} \\ Z_1 &= H_1 \cdot Z_{26}, \end{aligned} \right\} \quad (7)$$

$$(8)$$

where

$$H_{25} = B_{21}, \quad H_{24} = B_{20} \cdot H_{25}, \quad H_{23} = T_4 \cdot H_{24}, \dots, \dots,$$

$$H_2 = B_2 \cdot H_3, \quad H_1 = B_1 \cdot H_2.$$

Each state vector is determined by Eq.(7), because unknown variables (N , T , M_y , F_y , M_x , F_x) in Z_{26} are decided by Eq.(5), Eq.(6) and Eq.(8).

Figure 7 shows the compliance obtained at point 11, when point 7 is excited in an up-and-down direction. There are four resonance peaks below 150 Hz. Next, a resonance peak appears around 150 Hz. In this paper, three resonance peaks at $f_{n1}=30.1$ Hz, $f_{n2}=66.3$ Hz, $f_{n3}=90.4$ Hz are selected for control of the object.

Fig. 6 Transfer matrix representation of the piping system

Fig. 7 The compliance calculated at the excitation point

3.4 Vibration mode shape

To control the first three resonance peaks, the corresponding three vibration mode shapes must be examined. Figure 8 shows the natural frequencies and the corresponding mode shapes with front and side views calculated by the transfer matrix method. From this figure, maximum amplitudes are located at :

- node point 12 toward the y axis for the 1st mode ;
- node point 16 toward the z axis for the 2nd mode ;
- node point 6 toward the x axis for the 3rd mode.

3.5 Mounting location for the dynamic absorber and determination of equivalent mass at its point

The location for the dynamic absorbers to be mounted on the piping system must be at the point where the maximum amplitude for each vibration mode shape appears. Since the modal mass is the smallest at that point, it has the advantage of reducing the dynamic absorber mass required. In Fig.8,

Fig. 8 The mode shapes calculated at the first three resonance frequencies

dynamic absorbers are thus mounted at points marked by solid dots.

In order to design the dynamic absorber, we need to know the equivalent mass at the mounting point. The "mass responsive method" developed by Seto et al.⁽⁸⁾ is used to calculate the equivalent mass at the mounting point. The equivalent mass and stiffness of each mode are shown as follows:

$$\begin{aligned} M_1 &= 1.42 \text{ kg}, \quad M_2 = 1.62 \text{ kg}, \quad M_3 = 1.31 \text{ kg}, \\ K_1 &= 5.35 \times 10^4 \text{ N/m}, \quad K_2 = 2.81 \times 10^5 \text{ N/m}, \\ K_3 &= 4.23 \times 10^5 \text{ N/m}. \end{aligned}$$

4. Design of Dynamic Absorber and its Effectiveness of Vibration Control

4.1 Design of D.D.A. (dual dynamic absorbers)

Three D.D.A.s are used to control the vibration of the piping system. The D.D.A. is superior to the S.D.A. (single dynamic absorber) in its stability and ability to control the vibration. A D.D.A. attached for control of the i th vibration mode is shown in Fig. 9, where M_i and K_i are i th-mode equivalent mass and stiffness, and m_{ri} , k_{ri} and c_{ri} are absorber mass, stiffness, and damping coefficient ($r=1,2$), respectively. Giving the mass ratio as the basic parameter, we determine the dimensions of the D.D.A. by the design equations established by Iwanami and Seto⁽⁹⁾. Therefore, mass ratios are selected for each mode as follows:

$$\mu_1 = 0.05, \quad \mu_2 = 0.05, \quad \mu_3 = 0.04$$

The dimensions of the optimally designed D.D.A.s are listed in Table 1.

Fig. 9 Dual dynamic absorbers model in the i th mode

4.2 Response calculation of the piping system with optimally designed D.D.A.

In order to confirm the effectiveness of three D.D.A.s in controlling the vibration in the piping system, it is necessary to prepare the transfer matrix of a dynamic absorber which is represented by a 3-dimensional complex style. The transfer matrix D_r for the D.D.A. located at the point between state vectors Z_a and Z_b is shown as follows:

$$Z_a = D_r \cdot Z_b = D_{1r} \cdot D_{2r} \cdot Z_b \quad (9)$$

where subscripts "1 r " and "2 r " are the 1st and 2nd absorbers in the r -th modes' D.D.A. The construction of D_{1r} and D_{2r} is as described by Eq. (1). But the part matrixes, L , M , P_y , and P_x , are shown as follows:

$$L = \begin{bmatrix} 1 & 0 \\ -m_d \omega^2 & 1 \end{bmatrix} \quad (10)$$

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (11)$$

$$P_x = P_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ D_a & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

" D_a " is the value which is dependent on the direction of the mounted D.D.A. If the operational direction of the dynamic absorber is expressed as "1" and the right-angle direction is expressed as "0", " D_a " is shown as follows:

$$D_a = \begin{cases} (N_R - jN_I)/D & \dots "1" \\ -m_d \omega^2 & \dots "0" \end{cases} \quad (13)$$

where

$$\left. \begin{aligned} N_R &= \omega_d^2 (\omega_a^2 - \omega^2) + (2\zeta_d \omega_d \omega)^2 \\ N_I &= (2\zeta_d \omega_d \omega^3) \\ D &= [(\omega_a^2 - \omega^2) + (2\zeta_d \omega_d \omega)^2] / (-m \omega^2) \end{aligned} \right\} \quad (14)$$

m_d : the total mass of an absorber which includes an absorber's mass, casing, and damping elements
 m : absorber's mass, ζ_d : damping ratio,
 ω_d : absorber's natural frequency

If Eq. (9) is inserted at the mounting point for the D.D.A. of Eq. (7), the compliance of the piping system after setting of the D.D.A. can be calculated.

4.3 The effectiveness of vibration control

Figures 10, 11, and 12 give the results of the numerical calculation for compliance at point 11 on the y axis, at point 19 on the z axis, and at point 5 on

Table 1 Dimensions of the dual dynamic absorbers

Mode	Absorber 1			Absorber 2		
	m (kg)	k (N/m)	c (Ns/m)	m (kg)	k (N/m)	c (Ns/m)
1st	0.071	1920	2.87	0.071	2690	4.15
2nd	0.081	10100	7.04	0.081	14100	10.16
3rd	0.052	12700	5.77	0.052	17200	8.00

the x axis, when point 7 is excited in an up-and-down direction. The solid line is the response of the piping system on which three D.D.A.s were mounted in conformity with the values given in Table 1, and the

Fig. 10 Comparison of the compliance calculated at point 11 on the y -axis

Fig. 11 Comparison of the compliance calculated at point 19 on the z -axis

Fig. 12 Comparison of the compliance calculated at point 5 on the x -axis

broken line represents the piping system without absorbers. Each D.D.A. suppresses the corresponding resonance peak optimally, and the effectiveness of the suppressing of other resonance peaks is also shown. But the resonance frequencies over the 4th peak are moved lower by the attaching of three D.D.A.s.

5. Experimental

5.1 Construction of the D.D.A.

The setup of the D.D.A. constructed in this study is shown in Fig. 13. This D.D.A. is composed of parallel plate-springs, absorber masses that can slide on the spring, and magnetic dampers that use eddy current loss. In the magnetic damper, the poles of a pair of permanent magnets attached to both sides of a U -shaped magnet holder create a uniform magnetic field in the air gap. The absorber mass made of copper traverses the air gap without mechanical contact and generates a magnetic damping force.

The advantages of this dynamic absorber are that optimum damping effected in advance is kept very stable during use, and that spring constant and damping are adjustable for optimum tuning.

5.2 Experimental result

An experiment was conducted to confirm the theoretical results. The 3-dimensional piping system

Fig. 13 Setup of dual dynamic absorbers

Fig. 14 Comparison of the acceleration measured at point 11 on the y -axis

Fig. 15 Comparison of the impulse response measured at point 11 on the y -axis

with three D.D.A.s was excited by an impulse hammer and the acceleration of the piping system was obtained. Figure 14 shows the experimental results for the acceleration measured at point 11 on the y axis, when point 7 is excited in an up-and-down direction. It indicates that the first three resonance peaks were well suppressed by the three D.D.A.s. The experimental result agrees well with the result of theoretical analysis.

Figure 15 compares the impulse response with and without three D.D.A.s. The damping effect of the optimally designed D.D.A.s becomes more evident. Actual measurement in terms of the impulse response also shows the excellent adaptability of the D.D.A. for reducing seismic response in the piping system.

6. Conclusions

The following results were obtained after applying the "method of vibration control of multi-degree of freedom systems by dynamic absorbers" to control of the vibration of the piping system.

(1) When the 3-dimensional transfer matrix method is used, vibration analysis of the piping system with dynamic absorbers is possible.

(2) This shows that the first three resonance peaks are suppressed optimally by three D.D.A.s. It also confirms that the "method of vibration control of multi-degree of freedom systems by dynamic absor-

bers" is suitable for vibration control in piping systems.

(3) The effectiveness of this theory is confirmed theoretically and experimentally.

(4) The proposed method of vibration control in a piping system without supporting equipment will make it possible to design piping systems while considering the response to earthquakes.

References

- (1) Kunieda, M., Positive Use of Damping Devices for Piping Systems, SMiRT 8th, K18/6(1985).
- (2) Hara, F. and Seto, K., Basic Concepts about Application of Nuclear Piping Systems, SMiRT 9th, k2(1987), p. 947.
- (3) Seto, K. and Takita, Y., Vibration Control in Multi-Degree-of-Freedom Systems by Dynamic Absorbers (Report 1: On the Design Method for Dynamic Absorbers), ASME DE-Vol. 4 (1987), p. 169.
- (4) Seto, K., Vibration Control in Multi-Degree-of-Freedom Systems by Dynamic Absorbers (Report 2: On the Design of the Dynamic Absorbers Using the Transfer Matrix), ASME DE-Vol. 4 (1987), p. 177.
- (5) Seto, K. and Yamashita, S., Vibration Control in Multi-Degree-of-Freedom Systems by Dynamic Absorbers (Report 3: On the Vibration Control of Γ -Shaped Structure) (in Japanese), Trans. Jpn. Soc. Mech. Eng., Vol. 50, No. 481(1986), p. 2318.
- (6) Seto, K., Yamashita, S., Ookuma, M. and Nagamatsu, A., Vibration Control in Plate Structures by Dynamic Absorbers, Proc. KSME/JSME Vibr. Conf. '87 (1987), p. 74.
- (7) Seto, K., Ono, T., Yamashita, S. and Suzuki, H., Vibration Control of a Plate Structure by using Dynamic Absorber for Reducing Noise, Proceedings of Inter-Noise 87, Vol. 1(1987), p. 579.
- (8) Seto, K., Ookuma, M., Yamashita, S. and Nagamatsu, A., Method of Estimating Equivalent Mass of Multi-Degree-of-Freedom Systems, JSME Int. J., Vol. 30, No. 268(1987), p. 1638.
- (9) Iwanami, K. and Seto, K., An Optimum Design Method for the Dual Dynamic Damper and Its Effectiveness, Bull. JSME, Vol. 27, No. 231(1984), p. 1965.