The Application of Haar Transform in Signature Extraction and Condition Monitoring of Mechanical Systems*

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In this paper, the Haar transform is applied to vibration signal analysis and system condition monitoring. Based on the Haar transform, the "impulse index" is defined and used as the discriminant for the description of vibration signals. The impulse index offers information about the average power intensity compressed in each equivalent sequency group. It can be obtained conveniently and quickly by means of fast computational algorithm of the Haar transform. The computer simulation data of the output signals of nonlinear models with single degree-of-freedom are used to prove the effectiveness of the impulse index. The results are compared with those obtained from FFT power spectral density and are shown to be effective.

Key Words: Vibration, Fourier Transform, Signature Extraction, Haar Transform, Impulse Index, Equivalent Sequency.

1. Introduction

Vibration signal analysis and signature extraction is the most widely used method for condition monitoring of mechanical systems. In order to meet the requirement of fast on-line monitoring and real-time processing, some amplitude and frequency discriminants, which can be conveniently obtained and can effectively distinguish the signal characteristics, are often preferred.

In the frequency domain, the spectral analysis based on FFT is most commonly used. Although FFT predominates in the signal processing, other transforms such as the Haar transform can also be applied to signal processing. The Haar function set forms a complete set of orthogonal rectangular functions, and

the finite approximation to a function using the Haar function series is more efficient in the convergence features than use of Fourier series. The Haar transform is especially suitable for detecting the signal bursts and has much higher computational efficiency than the Fourier transform. However, the direct use of the squared data of the Haar transform, or the Haar line spectrum, is not appropriate for indicating the signal characteristics. Here, the impulse index is presented based on the Haar transform. It represents the average power intensity contained in the Haar transform components falling into the equivalent sequency group. The equivalent sequency of the Haar function series is defined as one half the average number of zero crossings per unit time interval. The effectiveness of the index is proved by means of the simulation results from two simple nonlinear models.

2. Haar Transform

2.1 The set of Haar functions

Haar functions form an orthogonal system of periodic rectangular waves with a limited set of amplitude values, 0, ± 1 , $\pm\sqrt{2}$, $\pm 2\sqrt{2}$, ± 4 , etc. The partial sum expansion for the finite approximation to

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a function using the Haar series is consistently convergent. The set of the first eight Haar functions is shown in Fig. 1.

Haar functions are expressed as follows⁽¹⁾⁽²⁾

$$\text{har}(0,0,1)=1 \quad \text{for } t \in (0,1)$$

$$\text{har}(r,m,t) = \begin{cases} 2^{r/2} & \frac{2m-2}{2^{r+1}} \leq t < \frac{2m-1}{2^{r+1}} \\ -2^{r/2} & \frac{2m-1}{2^{r+1}} \leq t < \frac{2m}{2^{r+1}} \\ 0 & \text{elsewhere for } t \in [0,1] \end{cases}$$

$$(0 < r < \log_2 N, 1 < m < 2^r). \quad (1)$$

For $t \in [0,1]$, the values are periodical expansions of those for $t \in [0,1]$. According to the definition of the Haar function, it is easy to prove that the Haar function is orthogonal and complete.

2.2 Definition of Haar transform⁽¹⁾

The Haar matrix is composed of a set of Haar functions. For example, the Haar matrix of 8×8 order is

$$[H]_{8 \times 8} = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & \sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \quad (2)$$

For an N dimensional time series $X(N)$, the Haar transform is expressed as

$$H(N) = [H]_{N \times N} \cdot X(N). \quad (3)$$

Compared with the Fourier transform, the Haar

Fig. 1 The set of the first eight Haar functions

transform has the following advantages⁽³⁾

(1) Using a fast algorithm, high computational efficiency is attainable with $2(N-1)$ addition / subtractions and N multiplications, while by means of FFT for an N -point transform, the operation involves $(N/2)\log_2 N$ complex multiplications and $N\log_2 N$ complex addition/subtractions.

(2) The convergence features of the expansion in Haar functions are superior to other orthogonal function series.

(3) The Haar transform is especially suitable for detecting and identifying bursts of energy distributed over time history.

3. Definition of the Impulse Index

Here we define the impulse index as follows :

$$\begin{aligned} I(0) &= H^2(0), \\ I(1) &= H^2(1), \\ I(2) &= [H^2(2) + H^2(3)]/2, \\ I(3) &= [H^2(4) + H^2(5) + H^2(6) + H^2(7)]/4, \end{aligned} \quad (4)$$

⋮

$$I(M) = \left[\sum_{k=2^{M-1}}^{2^M-1} H^2(k) \right] / 2^{M-1},$$

where $M = \log_2 N$ denotes the dimension of the impulse index or the number of sequency groups as stated later. N is the dimension of the time series, or the number of sampling points of the time history.

Sequency, which is analogous to frequency, is defined as one-half the average number of zero crossings per unit time interval. According to the constitution form of the Haar function series, it is seen that the Haar functions can be separated into discrete groups, and in each group the components have the same rectangular shape or the same equivalent sequency. For example, among the first eight Haar functions shown in Fig. 1, component no. 1, $\text{har}(0,1,t)$, forms group 1. Components no. 2 and no. 3, $\text{har}(1,1,t)$ and $\text{har}(1,2,t)$, form group 2. Components no. 4, 5, 6 and 7, $\text{har}(2,1,t)$, $\text{har}(2,2,t)$, $\text{har}(2,3,t)$ and $\text{har}(2,4,t)$, form group 3. The starting number of every group increases in powers of 2 ; that is, the k -th group includes the components numbered from 2^{k-1} to $2^k - 1$ ($k \geq 1$) (see Fig. 2). Therefore, the number of components falling into the k -th group is 2^{k-1} , and the only difference in these components is the location of the rectangular waves. It is easy to know that for an N -point transform, the equivalent sequency number k is analogous to a frequency of $(2^{k-1}/N) \cdot 1/\Delta t$, in which Δt is the sampling interval of time history.

As defined in Eq. (4), impulse index $I(k)$ expresses the average power intensity contained in 2^{k-1} Haar transform components in the k -th group, offering information concerning the power distribution compressed into each sequency group.

Fig. 2 The division of the equivalent sequence group

4. Simulation

Here we carried out a computer simulation of nonlinear systems with single degree-of-freedom subjected to Gaussian white noise in order to prove the effectiveness of the impulse index. The mathematical models are shown in Fig. 3 and Fig. 4. Figure 3 shows a collision model, in which the nonlinearity is caused by the gap between the mass and the spring. In Fig. 4, a hysteresis type is shown. In the simulation, the number of sample points is $N=16\,384$ ($=2^{14}$), and the sampling time interval is $\Delta t=0.1$ sec. It takes about 1.4 sec for the calculation of the impulse index of the output signals, and 15.7 sec for calculation of the FFT power spectral density with a HP-9 000 series computer. That is, for 16 384-point data, the calculation for the impulse index is nearly 11 times faster than that for the FFT spectrum.

4.1 Collision type

The motion equation of the collision type model in a dimensionless formula is expressed as follows:

$$\frac{d^2\xi(\tau)}{d\tau^2} + 2\zeta \frac{d\xi(\tau)}{d\tau} + \text{coll}(\xi, \rho_c, \varepsilon) = \eta(\tau), \quad (5)$$

where

$\xi(\tau) = x(t)/L$, $\tau = \omega_n t$, $\eta(\tau) = u(t)/m\omega_n^2 L$, $L = D/m^2\omega_n^3$, in which $\omega_n = \sqrt{K_0/m}$ is the natural frequency of the system, $u(t)$ is the Gaussian white noise input, and D is the intensity of $u(t)$. Besides, $\zeta = c/2m\omega_n$ is the damping ratio, $\rho_c = (K_0 + K_g)/K_0$ is the stiffness ratio, $\varepsilon = L/x_g$ is selected as the dimensionless nonlinear parameter, and x_g is the gap width.

The dimensionless restoring force is expressed as follows:

$$\text{coll}(\xi, \rho_c, \varepsilon) = \begin{cases} \rho_c \xi + (\rho_c - 1)/\varepsilon & \xi < -1/\varepsilon \\ \xi & -1/\varepsilon \leq \xi \leq 1/\varepsilon \\ \rho_c \xi - (\rho_c - 1)/\varepsilon & 1/\varepsilon < \xi \end{cases} \quad (6)$$

In the simulation, $\rho_c=10$, and $\zeta=0.05$.

Time histories of the system output signals, with the nonlinear parameter $\varepsilon=0.1, 0.2, 0.3, 0.4, 0.5$, and their impulse index distributions are shown in Figs. 5~9. The FFT power spectral density is also shown in these figures for the comparison. The case of $\varepsilon=0.1$

Fig. 3 Nonlinear model of the collision type and its restoring force F_s Fig. 4 Nonlinear model of the hysteresis type and its restoring force F_s

approximately expresses the linear case (Fig. 5). In the time histories, the horizontal axis expresses the sampling number of time series, and in the impulse index curve, it expresses the equivalent sequence number. From the time histories, as the nonlinear parameter increases, we can only see the decrease in the vibration level. In Fig. 5, impulse index $I(9)$ dominates the whole sequence area. We can easily see that sequence number 9 corresponds to a frequency of $2^8/(N \cdot \Delta t)$, that is, $2^8/(2^{14} \cdot 0.1)$, which is approximately 0.16, the dimensionless natural frequency of the system (see FFT spectrum shown in Fig. 5(c)). Figure 6 shows the distribution similar to that of Fig. 5 because of weak nonlinearity. In Fig. 6, value $I(10)$ slightly increased relative to $I(9)$. As the nonlinear parameter increases, the dominating frequency components appear in the higher-frequency area in the FFT spectrum, and the impulse index distribution changes in the same way, that is, two dominating values $I(9)$ and

$I(10)$ appear in Figs. 7 and 8, and $I(10)$ becomes the dominant component in Fig. 9. From these figures, we can easily see that the dominant equivalent frequency gradually moves to a higher frequency area. This change in the distribution of impulse index is clearly caused by the change of nonlinearity.

4.2 Hysteresis type

The motion equation of the hysteresis type model in dimensionless formula is expressed as follows:

$$\frac{d^2\xi(\tau)}{d\tau^2} + 2\xi \frac{d\xi(\tau)}{d\tau} + \text{hyst}(\xi, \epsilon) = \eta(\tau) \quad (7)$$

where

$$\text{hyst}(\xi, \epsilon) = \begin{cases} -1/\epsilon & \xi < a_1/L - 2/\epsilon \\ \xi - (a_1/L - 1/\epsilon) & a_1/L - 2/\epsilon \leq \xi \leq a_1/L \\ \xi - (a_2/L + 1/\epsilon) & a_2/L \leq \xi \leq a_2/L + 2/\epsilon \\ 1/\epsilon & a_2/L + 2/\epsilon < \xi \end{cases} \quad (8)$$

Here, the dimensionless nonlinear parameter is selected to be $\epsilon = L/\lambda$ (see Fig. 4). The other definitions and the parameters used in simulation are the same as the case of the collision type. Time histories, impulse index distributions, and FFT spectra of the system outputs are shown in Figs. 10, 11, 12, and 13 for the nonlinear parameters $\epsilon = 0.2, 0.3, 0.4$, and 0.5 . As ϵ increases, the changes of output signals in the time domain are mainly low-frequency components. In the distribution curve of the impulse index, values $I(0)$, $I(1)$, $I(2)$ and $I(3)$ show significant changes. $I(1)$, especially, increases dominantly; that is, the value $I(1)$ sharply increases for stronger nonlinearity.

5. Conclusions

According to the form of Haar function bases, the authors defined the impulse index which represents the average power intensity contained in the Haar

(a) Time history

(b) Impulse index distribution

(c) FFT power spectral density

Fig. 5 Simulation results of an approximately linear case ($\epsilon = 0.1$)

(a) Time history

(b) Impulse index distribution

(c) FFT power spectral density

Fig. 6 Simulation results of the collision model ($\epsilon = 0.2$)

(a) Time history

(b) Impulse index distribution

(c) FFT power spectral density

Fig. 7 Simulation results of the collision model ($\epsilon=0.3$)

(a) Time history

(b) Impulse index distribution

(c) FFT power spectral density

Fig. 8 Simulation results of the collision model ($\epsilon=0.4$)

(a) Time history

(b) Impulse index distribution

(c) FFT power spectral density

Fig. 9 Simulation results of the collision model ($\epsilon=0.5$)

(a) Time history

(b) Impulse index distribution

(c) FFT power spectral density

(a) Time history

(b) Impulse index distribution

(c) FFT power spectral density

Fig. 10 Simulation results of the hysteresis model($\epsilon=0.2$) Fig. 11 Simulation results of the hysteresis model($\epsilon=0.3$)

(a) Time history

(b) Impulse index distribution

(c) FFT power spectral density

Fig. 12 Simulation results of the hysteresis model($\epsilon=0.4$)

Fig. 13 Simulation results of the hysteresis model($\epsilon=0.5$)

transform components falling into an equivalent sequence group. By means of fast Haar transform, the impulse index can be obtained easily and quickly. In the computer simulation, for 16384-point data, calculation for the impulse index is about 11 times faster than that for the FFT spectrum. The simulation results have shown that the impulse index clearly indicates the change of the system nonlinearity. The impulse index is possibly offering a new method of mechanical signature analysis, acting as a kind of sequence domain discriminant for the description of the signals.

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