Fluid-Structure Interaction Analysis Program for Axisymmetric Structures*

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This paper presents an elementary version of a fluid-structure interaction analysis program which has been developed for vibration analysis and seismic response analysis for pool type LMFBR's. This program is based on a finite-element method combined with a boundary element method for axisymmetric structures, in which structures are mainly treated by the finite-element method and fluid regions by the boundary element method. The formulation of the present method is described, and the numerical examples are compared with analytical solutions and experimental data. Good agreements between them were obtained.

Key Words: Vibration, Numerical Analysis, Boundary Element Method, Finite-Element Method, Fluid-Structure Interaction, Axisymmetric Structure, Fast Breeder Reactor, Seismic Response, Seismic Design

1. Introduction

Fluid effects on seismic responses of a reactor vessel and internal components are of great importance in the seismic design of pool type liquid metal fast breeder reactors (LMFBR's) since the reactor vessel, which is a large shell structure of relatively thin walls, contains a large volume of sodium coolant as well as internal components. Therefore, the fluid effects must be taken into account in the seismic design to accurately evaluate seismic responses of the reactor components and to ensure structural integrity during earthquakes.

Previously, many experimental and analytical studies were performed to investigate the fluid-structure interaction in the pool type LMFBR's and to examine the validity of analytical models. The analytical models can be classified into three types, a one-dimensional analytical model, an axisymmetric (two-

dimensional) analytical model and a three-dimensional analytical model. In the one-dimensional model, structures are treated as beam elements, and the fluid as virtual mass (which is usually expressed by simple formulae) is added to structure nodes. This model is considered to be most economical but is not suitable for detailed analysis, as the oval deformation of thin shells is not taken into account. While the three-dimensional model, in which structures are usually modeled as plate elements and the fluid as solid elements by the finite-element method, is considered to give more accurate results, but is not practical because it requires a large number of discretized elements, especially in fluid domains. As the axisymmetric model can be analysed two-dimensionally by assuming the circumferential periodicity, it is considered to be more practical than the three-dimensional model. Although, in the pool-type LMFBR, the axisymmetry is destroyed by IHXs and pumps, which are regarded as beam structures, they can be taken into account by combining the one-dimensional model with the axisymmetric model.

The purpose of this study is to develop the fluid-structure interaction analysis program, which can

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efficiently analyse the fluid coupled effect of axisymmetric structures.

The fluid regions are generally analysed by the finite-element method⁽¹⁾⁽²⁾ or the boundary element method⁽³⁾⁽⁴⁾. In this analysis program, the boundary element method is chosen to analyse the fluid regions, except for thin fluid gaps. The reason is that the boundary element method which can analyse the higher circumferential modes of the axisymmetric fluid-structure coupled systems has not been much treated in previous studies⁽⁵⁾.

This analysis program has the following features.

(1) The structural parts are mainly modeled as ring shell elements by the finite-element method. (2) The fluid regions are treated by the boundary element method, in which linear shape functions are used under the assumption that fluid is inviscid and incompressible. (3) Not only axisymmetric modes but also nonaxisymmetric modes, can be analysed for fluid-structure-coupled problems. (4) Free surface oscillations (sloshing) can also be analysed. (5) This program can be applied to both eigenvalue analyses and seismic response analyses. (6) Another analytical technique, which can efficiently treat the thin fluid gaps, is combined with the afore-mentioned method.

In this paper, the formulation of the present method is described except for the technique for thin fluid gaps, which will be presented in a subsequent paper. Then, the validity of the program is examined by comparing numerical results with analytical solutions and experimental data.

Nomenclature

- [C] : Damping matrix
 f : Eigenfrequency
 g : Gravitational acceleration
[K] : Stiffness matrix
 l : Length of element along the meridian
[M] : Mass matrix
 m : Meridional mode number
 N : Number of joints of the fluid elements
 n : Circumferential wave number
 $\mathbf{n}$: Unit vector along the outer normal to the surface
 T : Kinetic energy
 U : Potential energy
 $\{u\}$: Displacement vector
 a_g : Acceleration of ground motion

Greek Symbols

- Γ : Boundary of fluid region
 δ_{ij} : Kronecker's symbol
 $=1; i=j, =0; i \neq j$
 $[\lambda]$: Coordinate transformation matrix

- ξ : Nondimensionalized coordinates
 ρ : Density
 ϕ : Velocity potential
 ω : Eigen-circular frequency

Subscripts

- f : Refers to a fluid
 i, j : Refer to a joint number of a fluid element
 k : Refers to a number of a fluid element
 p : Refers to spatial point p
 q : Refers to spatial point q
 s : Refers to shell

2. Fluid Mass Matrix

2.1 Axisymmetric boundary element method

The boundary element method is applied to derive the fluid mass matrix for the axisymmetric fluid region. The great advantage of this method is that it is easy to prepare the input data, as it does not require internal grid data for the fluid region.

In this analysis, the fluid is assumed to be inviscid, incompressible and irrotational. Velocity potential ϕ exists, which is subjected to the Laplace equation,

$$\Delta\phi=0, \quad (1)$$

where Δ is Laplacian. Then, the boundary integral equation can be derived from Green's formula for the velocity potential ϕ as follows;

$$\int_{\Gamma} \phi_q \frac{\cos \gamma_{pq}}{(r_{pq})^2} d\Gamma + \int_{\Gamma} \frac{\partial \phi_q}{\partial n} \frac{1}{r_{pq}} d\Gamma = 0, \quad (2)$$

where r_{pq} is the distance between points p and q . p is the point where ϕ is evaluated and q is the boundary point. γ_{pq} is the angle subtended by n and the position vector from p to q , as shown in Fig. 1. Assuming Γ as the boundary of an axisymmetric fluid region, ϕ_p and ϕ_q are expressed as

Fig. 1 Axisymmetric fluid region

$$\phi_p = \phi_i^{(n)} \cos n\psi_p \quad (3)$$

$$\phi_q = \{(1-\xi)\phi_j^{(n)} + \xi\phi_{j+1}^{(n)}\} \cos n\psi_q, \quad (4)$$

referring to Figs. 1 and 2, where ψ_p and ψ_q are position angles, ξ is a nondimensionalized coordinate along the meridian, and n is the circumferential wave number. The matrix equation describing the relation between ϕ and $\partial\phi/\partial n$ can be obtained from Eq. (2) as

$$[H]\{\phi^{(n)}\} = [G]\{\partial\phi^{(n)}/\partial n\}, \quad (5)$$

where the ij elements of the matrices $[H]$ and $[G]$ are given by

$$H_{ij} = -2 \int_0^1 \int_0^\pi \frac{\cos\gamma \cos n\theta}{r^2} \{(1-\xi)R_k l_k + \xi R_{k-1} l_{k-1}\} d\theta d\xi + 2\mu_i \delta_{ij} \quad (6)$$

$$G_{ij} = 2 \int_0^1 \int_0^\pi \frac{\cos n\theta}{r} \{(1-\xi)R_k l_k + \xi R_{k-1} l_{k-1}\} d\theta d\xi, \quad (7)$$

where R_k , r and $\cos\gamma$ are defined by

$$R_k = (1-\xi)R_j + \xi R_{j+1} \quad (8)$$

$$r = \{R_i^2 + R_k^2 - 2R_i R_k \cos\theta + (z_k - z_i)^2\}^{1/2} \quad (9)$$

$$\cos\gamma = \{(R_k - R_i \cos\theta) \cos\tau_k - (z_k - z_i) \sin\tau_k\} / r. \quad (10)$$

The kinetic energy of fluid element k can be expressed as

$$T_k = \frac{\rho_f}{2} \int_0^1 \int_0^{2\pi} \phi_k \frac{\partial\phi_k}{\partial n} R_k l_k d\theta d\xi, \quad (11)$$

where ϕ_k can be given by

$$\phi_k = \{(1-\xi, \xi)\{\phi_j^{(n)}, \phi_{j+1}^{(n)}\}\}^T \cos n\theta. \quad (12)$$

Substituting Eq. (12) into Eq. (11), T_k can be written as

$$T_k = \frac{1}{2} \{\phi_j^{(n)}, \phi_{j+1}^{(n)}\} [a] \{\partial\phi_j^{(n)}/\partial n, \partial\phi_{j+1}^{(n)}/\partial n\}^T, \quad (13)$$

where $[a]$ is given by

$$[a] = \rho_f \int_0^1 \int_0^{2\pi} \{1-\xi, \xi\}^T \{1-\xi, \xi\} R_k l_k \cos^2 n\theta d\theta d\xi. \quad (14)$$

Fig. 2 Boundary elements

Summing T_k up throughout the whole region, the total kinetic energy can be obtained as

$$T = \frac{1}{2} \{\phi^{(n)}\}^T [A] \{\partial\phi^{(n)}/\partial n\}, \quad (15)$$

where $[A]$ is the total matrix constructed from $[a]$ by the direct stiffness method. From Eqs. (5) and (15), T can be expressed in the following expression for the nonaxisymmetric vibration modes ($n \geq 1$).

$$T = \frac{1}{2} \{\partial\phi^{(n)}/\partial n\}^T \{[H]^{-1}[G]^T[A]\{\partial\phi^{(n)}/\partial n\}; n \geq 1 \quad (16)$$

For the case of the axisymmetric mode ($n=0$), introducing matrix $[S]$ derived from the fluid incompressibility, the kinetic energy is modified as

$$T = \frac{1}{2} \{\partial\phi^{(n)}/\partial n\}^T [S]^T \{[\hat{H}]^{-1}[\hat{G}]\}^T [\hat{A}][S] \{\partial\phi^{(n)}/\partial n\}; n=0, \quad (17)$$

where $[\hat{H}]$ is an $(N-1) \times (N-1)$ matrix reduced from $[H]$. $[\hat{G}]$ and $[\hat{A}]$ are $(N-1) \times N$ matrices reduced from $[G]$ and $[A]$, respectively. $\{\partial\phi^{(n)}/\partial n\}$ is a column vector of $(N-1)$ elements reduced from $\{\partial\phi^{(n)}/\partial n\}$. $[S]$ relates $\{\partial\phi^{(n)}/\partial n\}$ to $\{\partial\phi^{(n)}/\partial n\}$ as follows.

$$\{\partial\phi^{(n)}/\partial n\} = [S] \{\partial\hat{\phi}^{(n)}/\partial n\} \quad (18)$$

Therefore, the fluid mass matrix is obtained as

$$[\bar{M}_f] = ([H]^{-1}[G])^T [A] \quad ; \text{ for } n \geq 1 \\ = [S]^T ([\hat{H}]^{-1}[\hat{G}])^T [\hat{A}][S] \quad ; \text{ for } n=0. \quad (19)$$

Although $[\bar{M}_f]$ is apparently a nonsymmetric matrix, the fluid mass matrix can be symmetrized as

$$[M_f] = \frac{1}{2} ([\bar{M}_f] + [\bar{M}_f]^T). \quad (20)$$

The fluid mass matrix obtained above is used in the analysis for the fluid-structure interaction including free surface oscillation (sloshing).

Another form of a fluid mass matrix can be derived to analyse the fluid-structure interaction, provided that free surface sloshing might be neglected. First, Eq. (5) can also be expressed as

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{Bmatrix} \phi_1^{(n)} \\ \phi_2^{(n)} \end{Bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{Bmatrix} \partial\phi_1^{(n)}/\partial n \\ \partial\phi_2^{(n)}/\partial n \end{Bmatrix}, \quad (21)$$

where subscripts 1 and 2 refer to the free surface and the contact surface between the fluid and the structure, respectively. Kinetic energy T can also be written as

$$T = \frac{1}{2} \begin{Bmatrix} \phi_1^{(n)} \\ \phi_2^{(n)} \end{Bmatrix}^T \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix} \begin{Bmatrix} \partial\phi_1^{(n)}/\partial n \\ \partial\phi_2^{(n)}/\partial n \end{Bmatrix}. \quad (22)$$

Assuming the fluid pressure on the free surface to be zero ($\phi^{(n)}=0$), the fluid mass matrix $[\bar{M}_f]$ can be derived from Eqs. (21) and (22) as follows:

$$[\bar{M}_f] = \{[G_{22}] - [G_{21}]\{[G_{11}] - [H_{12}][H_{22}]^{-1}[G_{21}]\}^{-1} \\ \times ([G_{12}] - [H_{12}][H_{22}]^{-1}[G_{22}])\}^T ([H_{22}]^{-1})^T [A_{22}]. \quad (23)$$

In Eq. (23), $[A_{12}]$ has been neglected. The fluid mass matrix $[\bar{M}_f]$, derived in Eq. (23), can also be modified to the symmetric matrix $[M_f]$ using Eq. (20).

2.2 Calculation for fluid mass matrix

As the integration for G_{ij} given by Eq. (7) includes a singular point in the case where $i=j$, $i=j+1$ and $i=j-1$, it cannot be implemented efficiently by a numerical integration technique, such as Gauss-Legendre's method. However the singular point for H_{ij} given by Eq. (6) is removed by the term $2\mu_i\delta_{ij}$. Therefore, the integration for H_{ij} is implemented by the numerical integration method of Gauss-Legendre. The integration domain for G_{ij} in the case where $i=j$, $i=j+1$ and $i=j-1$, is divided into two parts: i.e., the domain involving the singular point and the remainder. The analytical integration is carried out for the former as described below, and the numerical integration is accomplished for the latter in the same way as for H_{ij} . Introducing parameter ϵ , which divides the integration domain into the two parts, G_{ij} can be expressed as

$$\begin{aligned} G_{ij} &= 2I_G + 2 \int_0^1 \int_{\epsilon}^{\pi} \frac{\cos n\theta}{r} \{(1-\xi)R_k l_k \\ &\quad + \xi R_{k-1} l_{k-1}\} d\theta d\xi \quad ; \text{ if } i=j \\ &= 2I_{G1} + 2 \int_0^1 \int_0^{\pi} \frac{\cos n\theta}{r} (1-\xi) R_k l_k d\theta d\xi \\ &\quad + 2 \int_0^1 \int_{\epsilon}^{\pi} \frac{\cos n\theta}{r} \xi R_{k-1} l_{k-1} d\theta d\xi \\ &\quad ; \text{ if } i=j-1 \\ &= 2I_{G2} + 2 \int_0^1 \int_{\epsilon}^{\pi} \frac{\cos n\theta}{r} (1-\xi) R_k l_k d\theta d\xi \\ &\quad + 2 \int_0^1 \int_0^{\pi} \frac{\cos n\theta}{r} \xi R_{k-1} l_{k-1} d\theta d\xi \\ &\quad \text{if } i=j+1 \end{aligned} \quad (24)$$

I_G , I_{G1} and I_{G2} are analytically integrated in the rectangular domain, as shown in Fig. 3, with the following approximation.

$$\cos n\theta = 1 - (1/2)(nr/R_j)^2 \cos^2 \eta \quad (25)$$

Fig. 3 Analytical integration domain

$$\xi = (r/l_k) \sin \eta ; \text{ if } \eta \geq 0, = 1 + (r/l_k) \sin \eta ; \text{ if } \eta < 0. \quad (26)$$

For the example, I_G is given by

$$\begin{aligned} I_G &= I_G(k) + I_G(k-1) \quad (27) \\ I_G(k) &= R_j \epsilon \left(1 - \frac{n^2 \epsilon^2}{6} \right) \ln \left\{ \tan \left(\frac{\alpha_k}{2} + \frac{\pi}{4} \right) \right\} - \frac{(R_j \epsilon)^2}{2 l_k} \\ &\quad \left(1 - \frac{n^2 \epsilon^2}{4} \right) \left(\frac{1}{\cos \alpha_k} - 1 \right) \\ &\quad - \left(\frac{l_k}{2} + \frac{n^2 l_k^3}{48 R_j^2} \right) \ln \left\{ \tan \frac{\alpha_k}{2} \right\} - \frac{n^2 l_k^3}{48 R_j^2} \frac{\cos \alpha_k}{\sin^2 \alpha_k}, \end{aligned}$$

where

$$\alpha_k = \tan^{-1}(l_k/R_j \epsilon). \quad (28)$$

I_{G1} and I_{G2} are similarly evaluated. The number of numerical integration points is varied depending on the distance r , as shown in Table 1.

3. Fluid Stiffness Matrix

The fluid stiffness matrix is required for the analysis of free surface sloshing. The potential energy for fluid element k is given by

$$U_k = \frac{\rho g}{2} \int_0^1 \int_0^{2\pi} w_k^2 R_k l_k d\theta d\xi \quad (29)$$

at the free surface, which is assumed to be horizontal. w_k is the vertical displacement of element k .

Substituting Eq. (12) into Eq. (29), U_k for circumferential wave number n can be written as

$$U_k = \frac{1}{2} \{w_j^{(n)}, w_{j+1}^{(n)}\} [b] \{w_j^{(n)}, w_{j+1}^{(n)}\} \quad (30)$$

where $[b] = g[a]$. Summing up U_k throughout the whole region, the total potential energy can be written as

$$U = \frac{1}{2} \{w^{(n)}\}^T [B] \{w^{(n)}\}, \quad (31)$$

where $[B]$ is the total matrix constructed from $[b]$. From Eq. (31), the fluid stiffness matrix is given by

$$[K_f] = [B]; \text{ for } n \geq 1, = [S]^T [B] [S]; \text{ for } n = 0. \quad (32)$$

4. Seismic Response Analysis

The structure is treated as the aggregation of conical frusta in this analysis program. The derivations of the mass matrix $[M_s]$ and the stiffness matrix $[K_s]$ are not described here because they are well-

Table 1 Number of numerical integration points

Direction	Number of Numerical Integration Points			
	Meridional : $d = \min(r)/l_k$ Circumferential : $d = \min(r)/\max(R_k)$			
	$0 < d < 2$	$2 < d < 4$	$4 < d < 8$	$8 < d < \infty$
Meridional	4	3	2	1
Circumferential	16	12	8	4

known. The equation of motion for the fluid-structure coupled systems can be written by

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{e\}a_g, \quad (33)$$

where $[M]$, $[K]$, $\{u\}$ and $\{e\}$ are given by

$$[M] = [M_s] + [\lambda]^T [M_f] [\lambda] \quad (34)$$

$$[K] = [K_s] + [\lambda]^T [K_f] [\lambda] \quad (35)$$

$$\{u\} = \{\dots, u_{rj}, u_{\theta j}, u_{zj}, \beta_j, \dots\}^T \quad (36)$$

$$\{e\} = \{\dots, 1, -1, 0, 0, \dots\}^T \quad ; \text{ for } n=1$$

$$= \{\dots, 0, 0, 1, 0, \dots\}^T \quad ; \text{ for } n=0. \quad (37)$$

$[\lambda]$ is the coordinate transformation matrix which relates $\{u\}$ to $\{\partial\phi^{(n)}/\partial n\}$.

The eigenvalue analysis can be performed by using Eq. (33), eliminating the damping term and the seismic input term. Applying the model analysis to Eq. (33), the following equation for meridional mode number m can be obtained:

$$\ddot{q}_m + 2\xi_m\omega_m\dot{q}_m + \omega_m^2q_m = -\beta_m a_g, \quad (38)$$

where β_m is the participation factor, ξ_m is the damping factor and q_m is the generalized coordinate. Seismic responses can be obtained from Eq. (38), and q_m is related to $\{u\}$ by the following equation:

$$\{u\} = [U]\{q\}, \quad (39)$$

where $[U]$ is modal matrix.

5. Numerical Examples and Discussions

5.1 Sloshing in cylindrical tank

Sloshing in a rigid cylindrical tank, of which the exact solution has been obtained⁽⁶⁾, is analysed to demonstrate the accuracy of this analysis program. The cylindrical tank has radius-to-length ratio, $R/L = 1$, as shown in Fig. 4. All the elements have an identical meridional length.

First, the effect of parameter ε , introduced in Eq. (24), on calculation accuracy is investigated to determine the adequate value of ε by comparing the numerical results with the exact solution. Figure 5 shows the relationship between ε and $(f/f_{ex})^2$, which is the eigenvalue ratio of the numerical result to the exact value for the case $(m,n)=(1,1)$. The curve for the eigenvalue ratio rapidly tends to a limit in the range where $\varepsilon < \pi/90$ as ε increases, and rises gradually in

Fig. 4 Rigid cylindrical tank filled with fluid

the range where $\varepsilon > \pi/9$. The reason is considered to be that the accuracy of the numerical integration is not sufficient in the range where $\varepsilon < \pi/90$, and the accuracy of the analytical integration for I_G , I_{G1} and I_{G2} is not sufficient in the range where $\varepsilon > \pi/9$. In the range where $\pi/90 < \varepsilon < \pi/9$, the eigenvalue ratio is found to be nearly constant. Therefore, $\varepsilon = \pi/18$ is adopted for the first circumferential mode in the examples shown below.

Figure 6 shows the convergence of the numerical results versus the number of boundary elements for the first circumferential mode. It can be confirmed that lower eigenfrequencies are more accurately calculated, and that the numerical results agree well with the exact solution, as the number of boundary elements increases. In Fig. 6, the numerical result from another method (Ref. (5)) is also shown, taking the

Fig. 5 Effect of integration parameter ε

Fig. 6 Convergence of numerical results

first meridional mode. The result of the present method is found to be improved in convergence.

With regard to the higher circumferential modes, it might be supposed that the accuracy of the numerical integration could not be maintained without increasing the number of numerical integration points. However, the accuracy of the numerical results is shown to be maintained by lowering the upper limit of the circumferential integration instead of increasing the number of numerical integration points, based on the fact that the interaction between two distant points is small. Figure 7 shows the effect of the upper limit of the circumferential integration on the accuracy of the eigenfrequency for the case of the twentieth circumferential mode with the first meridional mode. In Fig. 7, parameter ϵ , which divides the circumferential integration into two areas, is assumed as $\epsilon = \pi/18$; $n=0, =\pi/18n$; $n \geq 1$, (40)

in order to maintain the accuracy of Eq. (25). From Fig. 7, the numerical errors can be regarded to be very small in the range from $3\pi/20$ to $16\pi/20$. It is also considered that, if the upper limit of the integration is less than $3\pi/20$, the integration range is not wide enough to evaluate accurately $[H]$ and $[G]$, and that, if the limit is more than $16\pi/20$, the number of numerical integration points is not sufficient.

Figure 8 shows a comparison of the dimensionless eigen-circular frequencies $\omega\sqrt{R/g}$ between the numerical results and the exact solution, taking the first twenty-one circumferential modes with the first three meridional modes. The circumferential integration domain is limited as

$$0 \leq \theta \leq \pi; n < 3, 0 \leq \theta \leq 3\pi/n; n \geq 3. \quad (41)$$

A good agreement between the numerical results and the exact solution is shown up to the high circumferential modes in Fig. 8.

5.2 Fluid-structure-coupled vibration of a cylindrical shell

A fluid-structure-coupled vibration of a cylindrical shell is analysed to compare the numerical results

with the analytical solution, which was obtained by the energy method⁽⁷⁾⁽⁸⁾. Figure 9 shows the cylindrical shell containing the fluid, of which the boundary conditions are that the cylindrical shell is freely supported at both ends and that the perturbed fluid pressure is zero at both ends. The frequency equation can be given as

$$\begin{aligned} |A_{ij}| &= 0; i, j = 1, 2, 3 \\ A_{11} &= b^2 + n^2(1-\nu)/2 - a, A_{12} = A_{21} = -bn(1+\nu)/2, \\ A_{13} &= A_{31} = \nu b \\ A_{22} &= b^2(1-\nu)/2 + n^2 + c(n^2 + 2b^2(1-\nu)) - a \\ A_{23} &= A_{32} = -n - c(n^3 + b^2n(2-\nu)), \\ A_{33} &= 1 + c(b^2 + n^2)^2 - (1+d)a \\ a &= \rho_s R^2 \omega^2 (1-\nu)^2 / E, b = \pi R / L, c = (t/R)^2 / 12 \\ d &= (\rho_f / \rho_s) (R/t) \{I_n(b)\} / \{bI_n'(b)\}, \end{aligned} \quad (42)$$

where E is Young's modulus, ν is Poisson's ratio and I_n is modified Bessel function of the first kind of the order n . Considering the symmetry, only the upper half, which has five identical ring shell elements and twenty-five fluid boundary elements with identical meridional length, is treated. The material properties of the coupled system are shown in Table 2. The fluid

Fig. 8 Comparison between numerical results and exact solution for sloshing

Fig. 7 Effect of upper limit of circumferential integration

Fig. 9 A cylindrical shell containing a fluid

mass matrix described with Eq. (23) is applied here. Figure 10 shows a comparison of the eigenfrequencies between the numerical results and the analytical solutions, including the cylindrical shell without fluid. The numerical results are shown to agree well with the analytical solution, up to high circumferential modes.

5.3 Simplified pool type LMFBR model

A simplified pool type LMFBR test model, whose experimental data has been obtained⁽⁹⁾, is treated as a more practical structure. The test model, which is shown in Fig. 11, consists of two coaxial cylindrical vessels filled with water. The outer and inner cylindrical vessels are a 1/20 scaled model of the reactor vessel and the core suspension cylinder, respectively. Both vessels are concentrically suspended from the test frame. The test model is made of polyvinyl chloride resin, except for an aluminum bottom plate and a steel weight simulating the core. The material properties used in the calculation are shown in Table 3. As the circular plate suspending the inner cylinder at the top is not completely rigid, the circular plate is modeled as one element and its stiffness is adjusted to make the lowest eigenfrequency of the inner cylinder

without water equal to the experimental resonant frequency of 11.0 Hz. The fluid mass matrix described Eq. (23) is applied here.

Figure 12 shows the discretized model and the

Table 3 Material Properties of the Simplified Pool-Type LMFBR Model

Water	ρ_f	1.0×10^3	kg/m ³
Polyvinyl Chloride Resin	E	3.53×10^9	Pa
	ν	0.3	
Aluminium	ρ_s	1.38×10^3	kg/m ³
	E	6.86×10^{10}	Pa
	ν	0.33	
	ρ_s	2.7×10^3	kg/m ³

Table 2 Material properties of the coupled system

Fluid	ρ_f	8.33×10^2	kg/m ³
Shell	E	2.06×10^{11}	Pa
	ν	0.3	
	ρ_s	7.84×10^3	kg/m ³

Fig. 10 Comparison between numerical results and analytical solution

Fig. 11 Simplified pool type LMFBR model

Fig. 12 Numerical model and mode shapes

Table 4 Comparison between numerical results and experimental data

Mode	Eigenfrequency Hz		
	Experiment	Present	NASTRAN
Out-of-phase	8.6	8.70	9.00
In-phase	15.4	15.23	15.33

calculated mode shapes, which consist of an out-of-phase mode and an in-phase mode. Table 4 shows a comparison between the numerical results and experimental data of the eigenfrequencies. The results predicted by NASTRAN's virtual mass method⁽⁹⁾ are also shown in Table 4. A good agreement between the NASTRAN's virtual mass method, the present analysis program and the experimental data are obtained.

6. Conclusions

The fluid-structure interaction analysis program, which is based on the finite-element method combined with the boundary element method, was developed to analyse the vibration characteristics and seismic responses of axisymmetric structures.

The accuracy and the convergence of the present program were confirmed to be good through parametric studies on sloshing in the rigid cylindrical tank. Also, the present program was shown to be applicable to the fluid-structure interaction problems, even for high-circumferential deformation modes, comparing the numerical results with the analytical solution and the experimental data.

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