Reliability Design of the Piping System against Earthquake Excitation*

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This paper deals with the problem of the design of the piping system. Especially when the statistical properties of uncertainties of the system and those of an earthquake (i.e., its period, magnitude and statistical characteristics of the wave) are known, the statistical properties of the piping vibration can be obtained by using the hierarchy method, which is first applied to the reliability problem by one of the authors. Next a period of the first-passage failure of the piping system is calculated in a statistical sense. It is concluded that the statistical property of the seismic wave has more influence on the reliability than does the damping coefficient or other factors in the piping system.

Key Words: Probabilistic Method, Seismic Motion, Reliability Design, First-Passage Problem, Piping System, Uncertain Parameter, Hierarchy Method

1. Introduction

In a piping system, mass, stiffness and damping coefficients of pipes and supports have deviations in the design process or manufacturing process. The piping system is also exposed to the danger of earthquake excitation.

From the viewpoint of the dynamic response of structures against random excitation, there are many investigations. Most of them treat the probability problem against the process of stationary^{(1)~(5)} or nonstationary⁽⁶⁾ excitation. But in all of them, parameters of the structures examined are deterministic. On the other hand, from the viewpoint of the dynamic response of structures with uncertain parameters, there are also many investigations. Iwatsubo et al.^{(7)~(9)}, have reported studies on the dynamic response of a rotating system considering its uncertainty. In addition, for structures with uncertain parameters, the eigenvalue problem⁽¹⁰⁾ or dynamic response against harmonic excitation⁽¹¹⁾ has been studied. But there are very few studies that consider both structural uncertainty and random excitation.

This paper deals with the reliability of the piping system against earthquake excitation. To this end, the dynamic response of structures with uncertain param-

eters against random excitation is first obtained by using the hierarchy method⁽¹²⁾. Then, in the case of regarding earthquake excitation as a nonstationary narrow band random process, the dynamic response against earthquake excitation is presumed from the dynamic response against stationary white noise. Finally, when the statistical properties of uncertainties of the piping system and those of earthquake (i.e., its period, magnitude and statistical characteristics of the wave) are known, a period of the first-passage failure of the piping system is calculated.

2. Equation of Motion

The simplest model of the piping system is shown in Fig.1. The piping system is fixed at both ends, and its supports are modeled as a stiffness and damping system. The equation of motion is expressed as follows;

$$M\ddot{v} + C\dot{v} + Kv = F, \quad (1)$$

where M , C and K are mass, damping, and stiffness matrices, respectively, and v is a vector expressed as

$$v = \{x_1 \ l\theta_1 \ x_2 \ l\theta_2\}^T, \quad (2)$$

where x_i , θ_i ($i=1, 2$) express the vertical displacement and angular displacement of the supported point, respectively, and l is the length of each element. In this paper, the external force F is regarded as a seismic force. As the system is excited in the vertical direction at the supported points, F is expressed as follows:

$$F = MSg(t), \quad (3)$$

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where $g(t)$ is the acceleration of the seismic wave, and S is a vector expressed as

$$S = \{1 \ 0 \ 1 \ 0\}^T. \quad (4)$$

3. Impulsive Response Function

The response $v(t)$ to the random excitation is expressed in integral form by using the impulsive response function matrix $H(t)$ of the system:

$$v(t) = \int_0^t H(t-\tau) F(\tau) d\tau, \quad (5)$$

where the system is assumed to be linear and the characteristic values of the system are all deterministic.

When the mean value of the random excitation is zero, the mean value of the response is also zero, and the auto-covariance function of the responses, $R_v(t_1, t_2)$, is described by the next equation:

$$R_v(t_1, t_2) = \int_0^{t_2} \int_0^{t_1} H(t_1 - \tau_1) R_F(\tau_1, \tau_2) \times H^T(t_2 - \tau_2) d\tau_1 d\tau_2, \quad (6)$$

where R_F is the auto-covariance function matrix of $F(t)$. In this paper, using Eq.(3), the response $v(t)$ shown in Eq.(5) is expressed as follows:

$$v(t) = \int_0^t h(t-\tau) g(\tau) d\tau \quad (7)$$

where $h(t)$ is the impulsive response function vector expressed as follows:

$$h(t) = H(t)MS, \quad (8)$$

Similarly Eq.(6) is rewritten as

$$R_v(t_1, t_2) = \int_0^{t_2} \int_0^{t_1} h(t_1 - \tau_1) h^T(t_2 - \tau_2) \times R_g(\tau_1, \tau_2) d\tau_1 d\tau_2, \quad (9)$$

where R_g is the auto-covariance function of $g(t)$.

The auto-covariance function of the velocity response can be obtained by differentiating Eq.(9) with t_1 and t_2 , and also, the mean square value of response can be obtained by setting $t_1 = t_2 = t$ in Eq.(9). Therefore, when $h(t)$ is obtained, the mean square value of response can be calculated by using Eq.(9).

Further, the method of obtaining $h(t)$ by using the hierarchy method is described more fully. The case in which the impulsive force f is subjected to the system is investigated. The equation of motion is

Fig. 1 Model of the piping system

$$M\ddot{v} + C\dot{v} + Kv = f. \quad (10)$$

Here it is supposed that the uncertainty of the damping coefficient of the support c is important to the dynamic behavior, so this effect is investigated. However, the uncertainty of the other coefficients, for example the stiffness of the support, can be treated similarly. First taking the ensemble averages of Eq.(10) with regard to uncertain coefficient c , the next equation is obtained:

$$M\ddot{\bar{v}} + \bar{C}\dot{\bar{v}} + K\bar{v} = \bar{f}, \quad (11)$$

where the symbol $(-)$ denotes a statistical average.

Further, multiplying Eq.(10) by damping coefficient c , and taking the ensemble averages, the following equation is obtained:

$$M\bar{c}\ddot{\bar{v}} + \bar{C}\bar{c}\dot{\bar{v}} + K\bar{c}\bar{v} = \bar{c}\bar{f}. \quad (12)$$

Since the simultaneous Eqs. (11) and (12) have unknown variables $\bar{v}$, $\bar{c}\bar{v}$ and $\bar{c}^2\bar{v}$, it is impossible to solve the equations. So the following approximate equation, which is accurate within the second order of small deviations, is used:

$$\bar{c}^2\bar{v} \approx (\sigma_c^2 - \bar{c}^2)\bar{v} + 2\bar{c}\bar{c}\bar{v}, \quad (13)$$

where σ_c is the standard deviation of c . As $\bar{c}^2\bar{v}$ in $\bar{C}\bar{c}\bar{v}$ of Eq.(12) is eliminated by using Eq.(13), Eqs.(11) and (12) may be combined as follows:

$$P\ddot{y} + Q\dot{y} + Ry = f', \quad (14)$$

where y and f' are vectors expressed as

$$y = \{\bar{v}^T \bar{c}\bar{v}^T\}^T, \quad f' = \{\bar{f}^T \bar{c}\bar{f}^T\}^T. \quad (15)$$

For the purpose of obtaining the responses governed by Eq.(14), the modal analysis technique is applied. Using the following state vector,

$$z = \{y^T \dot{y}^T\}^T, \quad (16)$$

Eq.(14) is rewritten as

$$\dot{z} = Az + p, \quad (17)$$

where

$$A = \begin{bmatrix} 0 & I \\ -P^{-1}R & -R^{-1}Q \end{bmatrix}, \quad p = \{0 \ f'^T\}^T. \quad (18)$$

The eigenvalues and eigenvectors of matrix A are as follows:

$$\lambda_k = \delta_k \pm i\omega_k, \quad w_k = w'_k \pm iw''_k, \quad (k=1 \sim 4), \quad i = \sqrt{-1}. \quad (19)$$

The next matrices are made from these eigenvalues and eigenvectors:

$$\Psi = [w_k \ \bar{w}_k], \quad \lambda = \text{diag}[\lambda_k], \quad (20)$$

where the matrix Ψ is normalized according to the following equation:

$$\Psi^{-1}A\Psi = \lambda. \quad (21)$$

Using the new coordinate,

$$z = \Psi\eta, \quad (22)$$

the new equation is obtained as follows:

$$\dot{\eta} = \lambda\eta + \Psi^{-1}p. \quad (23)$$

The solutions are

$$\eta = \int_0^t E\Psi^{-1}p d\tau, \quad (24)$$

where

$$\mathbf{E} = \text{diag}[\exp(\lambda_k(t - \tau))]. \quad (25)$$

Using Eq.(22), the response of Eq.(17) is obtained as follows:

$$\mathbf{z} = \int_0^t \Psi \mathbf{E} \Psi^{-1} \mathbf{p} d\tau. \quad (26)$$

Therefore the procedure of obtaining the auto-covariance function in Eq.(9) is as follows:

(1) obtaining the impulsive response function vectors from Eq.(10) by setting the impulsive input. In this paper, since $\mathbf{v}$ is a 4-dimensional vector, the impulsive response function matrix $\mathbf{H}(t)$ is a 4×4 matrix;

(2) obtaining the impulsive response function vector $\mathbf{h}(t)$ by using Eq.(8);

(3) obtaining the auto-covariance function $R_v(t_1, t_2)$ by using Eq.(9).

4. Response Analysis

4.1 Stationary random response

For the sake of simplicity of analysis, a non-stationary wide band noise is applied to the exciting force of the seismic wave.

When the seismic wave is a narrow band, the correspondence of wide band noise to narrow band noise is required. This section is devoted to obtaining the power relation between wide and narrow band noise.

The expression of the two types of stationary random excitation is as follows.

(a) Narrow band random excitation $f_s(t)$.

It is assumed that the excitation is a normal process with zero mean and has the next auto-covariance function:

$$R_s(\tau) = S_n \cdot \exp(-\beta\tau) \cdot \cos(\omega_t \tau), \quad \tau = |t_1 - t_2|, \quad (27)$$

where ω_t is the dominant frequency. The spectral density function of this excitation is

$$\Phi_s(\omega) = \frac{2\beta(\omega_t^2 + \beta^2 + \omega^2)S_n}{(\omega_t^2 + \beta^2 - \omega^2)^2 + 4\beta^2\omega^2}. \quad (28)$$

(b) Wide band random excitation $f_w(t)$.

This excitation is also assumed to be a normal process with zero mean, and its auto-covariance function is

$$R_w(\tau) = \frac{2P}{\tau} \cdot \sin(\omega_n \tau), \quad \tau = |t_1 - t_2|, \quad (29)$$

The spectral density function for this process is given by

$$\Phi_w(\omega) = \begin{cases} P & |\omega| < \omega_n \\ P/2 & |\omega| = \omega_n \\ 0 & |\omega| > \omega_n. \end{cases} \quad (30)$$

The spectral density such as Eqs.(29) and (30) can be replaced with the spectral density of white noise when ω_n is sufficiently greater than the natural frequency of

the system; hence, it is sufficient to obtain the response to the white noise excitation in the present case. That is, if the spectral density I of the white noise is determined to be equal to P , the wide band excitation defined by Eqs.(29) and (30), and the white noise excitation may give the equivalent response.

In order to compare the response to narrow band excitation with that to wide band excitation, we set the mean square values of two excitations to be equal, i.e., $R_s(0) = R_w(0)$; then the following equation is obtained:

$$S_n = 2P\omega_n. \quad (31)$$

This equation gives the power relation between narrow band and wide band random excitation.

4.2 Nonstationary random response

Since the seismic wave is modeled as a non-stationary process, the response to nonstationary random excitation is investigated.

Various models for seismic waves have been proposed in the studies of the ground acceleration in earthquakes, and the seismic design. In this analysis, it may be assumed that the exciting acceleration $f(t)$ is expressed as

$$f(t) = \alpha(t)f_s(t), \quad (32)$$

where $f_s(t)$ is the stationary narrow band process which is defined by Eq.(27), and $\alpha(t)$ is a deterministic function called the modulated function:

$$\alpha(t) = N\{\exp(-d_1 t) - \exp(-d_2 t)\}, \quad (d_1 < d_2), \quad (33)$$

where N is chosen so that the maximum value of $\alpha(t)$ is equal to 1. The intensities of the seismic wave and modified seismic wave are related so that the root mean square of $f_s(t)$ is equal to the maximum acceleration value of the seismic wave, which is expressed as:

$$S_n = \sqrt{G} \quad (34)$$

In this analysis, the response of the piping system is not obtained directly from the input signal of Eq.(32), but it is obtained by changing $f_s(t)$ of Eq.(32) to the white noise $f_w(t)$, whose spectral density is chosen as,

$$I = P = G^2/(2\omega_n). \quad (35)$$

5. Failure Probability

Supposing that the failure of the piping system occurs when the piping system exceeds the limited amplitude for the first time, the probability of the failure can be estimated.

The displacement x_1 is used as the standard point for the evaluation of the first-passage problem in this analysis. The probability that x_1 exceeds r_0 gives the probability of the system failure, in which the limited amplitude is r_0 . Many methods have recently been

proposed to estimate the first-passage probability. Here the approximate solution based on Poisson's excursion rate is used, and the first-passage probability P_f is shown as:

$$P_f = 1 - \exp\left\{-2 \int_0^\infty \nu(t) dt\right\}, \quad (36)$$

$$\nu(t) = \frac{1}{2\pi} \sqrt{\frac{\dot{\bar{x}}_1^2}{x_1^2}} \exp\left(-\frac{r_0^2}{2x_1^2}\right). \quad (37)$$

6. Numerical Examples

Dimensions of the modeled piping system are as follows:

Total Length = 3.00×10^4 [mm],
 Density = 7.85×10^{-10} [kgf sec²/mm⁴],
 Modulus of Elasticity = 2.10×10^4 [kgf/mm²],
 Cross-Section Area = 1.95×10^5 [mm²],
 Second Moment of Area = 2.14×10^{10} [mm⁴].

6.1 Stationary response

The variations of the mean square value of x_1 with respect to the properties of the damping coefficient and stiffness are illustrated in Figs. 2 and 3, when wide band and narrow band stationary excitations with the maximum acceleration of 300 (gal) is applied to the system. Here the uncertainty of the damping coefficient c and stiffness k is, respectively, $\sigma_c/\bar{c}=0.3$ and $\sigma_k/\bar{k}=0.3$, where σ_k is the standard deviation of k . For the narrow band excitations, the parameters are $\beta=0.2$, $\omega_t=14\pi$ and $\omega_n=40\pi$.

It can be seen from Figs. 2 and 3 that the mean square values $\bar{x}_1^2$ containing uncertainty are larger than those of the deterministic cases for both types of stationary random input. Also, the value of $\bar{x}_1^2$ to the narrow band random input, as shown in Fig. 2, is larger than the one to the wide band input because the dominant frequency ω_t is near the natural frequency, as shown in Fig. 4, which illustrates the first mode

Fig. 2 The mean square value of stationary response $\bar{x}_1^2$ (stationary wide band)

shape of the system and the natural frequency.

6.2 Nonstationary response

Let us denote the ratio of the mean square value of the response to the narrow band random input to that of the response for the wide band random input by r for $\sigma_c/\bar{c}=0$ and $\sigma_k/\bar{k}=0$, and by r' for $\sigma_c/\bar{c}=0$ and $\sigma_k/\bar{k}=0.3$, and by r'' for $\sigma_c/\bar{c}=0.3$ and $\sigma_k/\bar{k}=0$, respectively. By using these ratios, the response to the nonstationary narrow band random input is approximated by the response to the nonstationary wide band input with spectrum density which is defined by $P=(S_n/2\omega_n)\gamma$ for a deterministic system and $P=(S_n/2\omega_n)\gamma'$ or $P=(S_n/2\omega_n)\gamma''$ for a stochastic system, respectively, where the nonstationary wide band random input is $f(t)=\alpha(t)f_w(t)$, just as in Eq. (35). Moreover, $f_w(t)$ may be changed by white noise whose spectrum density I is equal to P . Thus, in the analysis, the responses are obtained by changing $f_s(t)$ of Eq. (35) to $(G^2/2\omega_n)\gamma$ for the deterministic system and $(G^2/2\omega_n)\gamma'$ or $(G^2/2\omega_n)\gamma''$ for the stochastic system. The parameters of nonstationary function in Eq. (35) are chosen as $N=9.48$, $d_1=0.9$, and $d_2=0.4$. For this set of parameters, the maximum value of $\alpha(t)$ becomes 1 at $t=3.20$ (sec).

The auto-covariance of the nonstationary response to the excitation $G=300$ (gal) considering

Fig. 3 The mean square value of stationary response $\bar{x}_1^2$ (stationary narrow band)

Fig. 4 The first mode shape

Fig. 5 The mean square value of nonstationary response $\overline{x_1^2}$ (nonstationary narrow band)

the deterministic c and k , and probabilistic c , and probabilistic k is shown in Fig. 5.

6.3 Failure probability

Figure 6 shows the relationship between the maximum acceleration value of seismic waves and the failure probability for the three different limited amplitudes.

7. Conclusions

A method to obtain the random responses of a system having uncertain parameters has been proposed for both stationary and nonstationary excitations. From the results of a numerical example, it was found that the statistical property of a seismic wave had more influence on the reliability than that of the structural parameters for both stationary and nonstationary excitations. The relation between the intensity of seismic waves and the probability of exceeding the limited amplitude of the piping system was obtained by using an approximate solution based on Poisson's excursion rate.

References

- (1) Lin, Y. K., "First-Excursion Failure of Randomly Excited Structures", *AIAA Journal*, Vol. 8, No. 4 (1970), p. 720.
- (2) Lin, Y. K., "First-Excursion Failure of Randomly Excited Structures II", *AIAA Journal*, Vol. 8, No. 10 (1970) p. 1888.
- (3) Yang, J. N. and Shinozuka, M., "On the First-Excursion Probability in Stationary Narrow-Band Random Vibration", *Trans. ASME, Journal of Applied Mechanics*, Vol. 38, No. 4 (1971) p. 1017.
- (4) Yang, J. N. and Shinozuka, M., "On the First-Excursion Probability in Stationary Narrow-

Fig. 6 The system failure probability P_f (nonstationary narrow band)

Band Random Vibration II", *Trans. ASME, Journal of Applied Mechanics*, Vol. 39, No. 3 (1972) p. 733.

- (5) Lennox, W. C. and Fraser, D. A., "On the First-Passage Distribution for the Envelope of a Nonstationary Narrow-Band Stochastic Process", *Trans. ASME, Journal of Applied Mechanics*, Vol. 41, No. 3 (1974) p. 793.
- (6) Roberts, J. B., "Probability of First-Passage Failure for Nonstationary Random Vibration", *Trans. ASME, Journal of Applied Mechanics*, Vol. 42, No. 3 (1975) p. 716.
- (7) Iwatsubo, T., "Analysis of Measurement Errors in Balancing of Rotors", *Proceedings of 20th Japan National Congress for Applied Mechanics* (1970) p. 19.
- (8) Iwatsubo, T., "Error Analysis of Vibration of Rotor/Bearing System", *Conference on Vibrations in Rotating Machinery*, University of Cambridge, Institution of Mech.
- (9) Iwatsubo, T., "Reliability Design of Rotating Machine against Earthquake Excitation", *Bulletin of JSME*, Vol. 22, No. 173 (1979) p. 1632.
- (10) Collins, J. D. and Thomson, W. T., "The Eigenvalue Problem for Structural System with Statistical Properties", *AIAA Journal*, Vol. 7, No. 4 (1969) p. 642.
- (11) Caravani, P. and Thomson, W. T., "Frequency Response of a Dynamic System with Statistical Damping", *AIAA Journal*, Vol. 11, No. 2 (1973) p. 170.
- (12) Bharucha-Reid, A. T., "Probabilistic Methods in Applied Mathematics", Vol. 1, Academic Press, New York and London (1968) p. 48.