Dynamic Absorber Using a Hollow Rotor Partially Filled With Liquid*

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This paper describes a technique to suppress vibrations of a structure under excitation by attaching a hollow rotor partially filled with liquid. The rotor rotates about an axis parallel to the direction of vibration. Assuming the fluid motion is axially symmetric, we analyze the motion of the fluid and structure. The influence of the shape of a chamber of the rotor, the amount of liquid and the ratio of rotor speed to exciting frequency on vibration control is investigated. The condition in which the hollow rotor acts as a dynamic absorber is clarified. The effectiveness of the proposed dynamic absorber is demonstrated experimentally. The theoretical results agree with the experimental ones.

Key Words: Vibration Control, Vibration of Rotating Body, Sloshing, Forced Vibration

1. Introduction

Dynamic absorbers are frequently used in controlling the amplitude of vibration of a system. Recently, by applying modal analysis, vibrations of a system with multiple degrees of freedom were controlled successfully by dynamic absorbers^{(1)~(3)}. Rotating machinery, e.g., a water turbine or a pump, causes vibration of a structure on which the machinery is installed. Similarly, rotation of a screw propeller of a ship induces the passengers' quarters to vibrate laterally. In these cases, it is generally found that the structure or the passengers' quarters vibrates at the same frequency as the rotor speed or at multiples of this frequency. If a dynamic absorber has a natural frequency equal to the exciting frequency, the tuning condition is always satisfied. A pendulum absorber for linear vibration described in Ref. (4) is an example of such absorbers. However, pendulum absorbers do not

always operate satisfactorily due to friction and insufficient adjustment. Though a different type of absorber using liquid was studied, effectiveness and performance have not been clarified⁽⁵⁾.

In this paper, we propose a dynamic absorber with a hollow rotor containing liquid. The effects of the size and geometry of a rotor chamber, fill ratio of liquid and rotor speed on vibration control are clarified.

Nomenclature

- a = radius of chamber (Fig. 1)
- b = inner radius of the liquid for a rigid rotation
- B = fill ratio ($= b/a$)
- f_L = liquid force
- f_0 = amplitude of exciting force
- h = half of the chamber height (Fig. 1)
- H = aspect ratio of chamber ($= h/a$)
- k = stiffness (Fig. 1)
- M = mass of the system (Fig. 1)
- m_0 = apparent mass of liquid defined by Eq. (23)
- γ_0 = apparent mass ratio defined by Eq. (23)
($= m_0/M$)
- $\delta = f_0/k$

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- ξ = displacement
 σ = function of Ω/ω defined by Eq. (22)
 ω = exciting frequency
 Ω = angular velocity of rotor
 ω_0 = resonance frequency ($= \omega_0/\sqrt{\sigma}$)
 ω_0 = natural frequency ($= \sqrt{k/M}$)

2. Analysis

The system to be analyzed is shown in Fig. 1. The system consists of a massive body under excitation, a spring k and a rigid hollow rotor which is mounted on the vibrating body. The shaft axis is parallel to the direction of an exciting force. The rotor has an axisymmetric cylindrical rigid chamber with radius a and height of $2h$. The chamber is partially filled with liquid. The equation of motion for the vibrating body is given as:

$$M\ddot{\xi} + k\xi = f_0 \sin \omega t + f_L, \quad (1)$$

where $f_0 \sin \omega t$ is an exciting force and f_L is a liquid force acting on the rotor. In Eq. (1), M includes the masses of the body, hollow rotor and casing. Rotor unbalance and external damping are neglected.

The following assumptions are used in the analysis of the liquid force:

- (a) The hollow rotor is driven at constant angular speed
- (b) The contained liquid is incompressible and inviscid. Surface tension effects are negligible.
- (c) Gravity force is negligible.

(a) Analytical model

(b) Hollow rotor

Fig. 1

- (d) Liquid motion is axially symmetric.

The z -axis is the line of bearing centers. The r -, θ - and z -axes are fixed to the rotor. For an inviscid liquid, the equations of motion referred to a cylindrical coordinate system fixed to the rotor are:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} - 2\Omega v - r\Omega^2 = -\frac{1}{\rho} \frac{\partial p}{\partial r}, \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + w \frac{\partial v}{\partial z} + \frac{uv}{r} + 2\Omega u = -\frac{1}{\rho} \frac{\partial p}{r \partial \theta} \quad (3)$$

and

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \theta} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z}. \quad (4)$$

The continuity equation takes the following form:

$$\frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0. \quad (5)$$

The boundary conditions are:

$$u=0 \quad \text{at} \quad r=a, \quad (6)$$

$$u - \frac{\partial \eta}{\partial t} - w \frac{\partial \eta}{\partial z} = 0, \quad p=0 \quad \text{at} \quad r=\eta(z, t) \quad (7)$$

and

$$w = \dot{\xi} \quad \text{at} \quad z = \pm h - \xi(t), \quad (8)$$

where η is the inner radius of the liquid.

A perturbation is made on liquid velocities, pressure and η . For example, the pressure is written as $p = p_0 + p_1$, where the subscript "0" denotes the steady-state condition, and the subscript "1" denotes the perturbed condition. After substitution into Eq. (2) through (8), terms of like order are grouped for solution. The zero-order solutions are given by

$$u_0 = v_0 = w_0 = 0, \quad \eta_0 = b, \quad p_0 = \rho \Omega^2 (r^2 - b^2)/2, \quad (9)$$

where b is the inner radius of the liquid. These solutions represent a rigid rotation of the enclosed liquid. They are independent of time.

For periodic motion, consider

$$\left. \begin{aligned} u_1 &= U(r, z) \cos \omega t \\ v_1 &= V(r, z) \sin \omega t \\ w_1 &= W(r, z) \cos \omega t \\ p_1 &= \rho P(r, z) \sin \omega t \\ \eta_1 &= \epsilon(r, z) \sin \omega t \end{aligned} \right\} \quad (10)$$

Substituting Eqs. (9) and (10) into the momentum and continuity equations for perturbed velocities and pressure, and eliminating U , V , and W , we obtain a differential equation for P :

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r} \right) - \{ (2\Omega/\omega)^2 - 1 \} \frac{\partial^2 P}{\partial z^2} = 0. \quad (11)$$

The boundary conditions are:

$$\left. \begin{aligned} U &= 0 \quad \text{at} \quad r=a, \\ W &= A\omega \quad \text{at} \quad z = \pm h \quad \text{and} \end{aligned} \right\} \quad (12)$$

$$U - \epsilon\omega = 0, \quad P + \epsilon b \Omega^2 = 0 \quad \text{at} \quad r=b,$$

where A is the amplitude of the rotor or the vibrating body in the z direction.

In order that the rotor containing liquid acts as a dynamic absorber, the pressure distribution and the free surface of the liquid must be antisymmetric with respect to the midplane of the rotor (i.e., $z=0$). Then applying the boundary conditions (12), we find the solution of Eq. (11),

$$P = \begin{cases} A\omega^2 z + (8hA\omega^2/\pi^2) \sum_{n=1,3,5}^{\infty} \{J_0(\alpha_n r/h) - J_1(\alpha_n a/h) Y_0(\alpha_n r/h)/Y_1(\alpha_n a/h)\} \\ \times \frac{(-1)^{(n-1)/2}}{n^2 C_n} \sin \frac{n\pi z}{2h} & : (\Omega/\omega > 1/2) \\ A\omega^2 z + (8hA\omega^2/\pi^2) \sum_{n=1,3,5}^{\infty} \{I_0(\beta_n r/h) + I_1(\beta_n a/h) K_0(\beta_n r/h)/K_1(\beta_n a/h)\} \\ \times \frac{(-1)^{(n-1)/2}}{n^2 D_n} \sin \frac{n\pi z}{2h} & : (\Omega/\omega < 1/2) \end{cases}, \quad (13)$$

where

$$\alpha_n = (n\pi/2) \sqrt{(2\Omega/\omega)^2 - 1}, \quad (14)$$

$$C_n = \{\Omega^2/(4\Omega^2 - \omega^2) \cdot (\alpha_n b/h) Y_1(\alpha_n b/h) + Y_0(\alpha_n b/h) \times J_1(\alpha_n a/h)/Y_1(\alpha_n a/h) - \{\Omega^2/(4\Omega^2 - \omega^2) \cdot (\alpha_n b/h) J_1(\alpha_n b/h) + J_0(\alpha_n b/h)\}, \quad (15)$$

$$\beta_n = (n\pi/2) \sqrt{1 - (2\Omega/\omega)^2}, \quad (16)$$

$$D_n = -\{\Omega^2/(\omega^2 - 4\Omega^2) \cdot (\beta_n b/h) K_1(\beta_n b/h) + K_0(\beta_n b/h)\} I_1(\beta_n a/h)/K_1(\beta_n a/h) - \{\Omega^2/(\omega^2 - 4\Omega^2) \cdot (\beta_n b/h) I_1(\beta_n b/h) + I_0(\beta_n b/h)\}, \quad (17)$$

and J_i and Y_i are the Bessel functions of the first and the second kind, and I_i and K_i are the modified Bessel functions of the first and the second kind, respectively.

Making use of Eq. (13), the perturbed displacement of the free surface is expressed as:

$$\eta_1 = \frac{P(b, z)}{b\Omega^2} \sin \omega t. \quad (18)$$

The liquid force acting on the rotor in the z -direction is obtained by integrating the pressure.

$$f_L = \int_b^a (p_1|_{z=h} - p_1|_{z=-h}) 2\pi r dr. \quad (19)$$

Assuming the solution of Eq. (1) to be

$$\xi = A \sin \omega t, \quad (20)$$

and substituting Eqs. (19) and (20) into Eq. (1), we obtain the amplitude of the massive body:

$$A = \frac{\delta}{1 - \sigma(\omega/\omega_0)^2}, \quad (21)$$

where

$$\sigma = 1 + \gamma_0 \{ (1 - B^2) + (4/\pi)^2 BH \sum_{n=1,3,5}^{\infty} S_n \}, \quad (22)$$

$$\left. \begin{aligned} \delta &= f_0/k, \quad \gamma_0 = m_0/M, \quad m_0 = \rho\pi a^2(2h) \\ \omega_0 &= \sqrt{k/M}, \quad B = b/a, \quad H = h/a \end{aligned} \right\}, \quad (23)$$

$$S_n = \begin{cases} \{J_1(\alpha_n a/h) Y_1(\alpha_n b/h) - J_1(\alpha_n b/h) Y_1(\alpha_n a/h)\} / \{n^2 \alpha_n C_n Y_1(\alpha_n a/h)\} & : (\Omega/\omega > 1/2) \\ \{I_1(\beta_n a/h) K_1(\beta_n b/h) - I_1(\beta_n b/h) K_1(\beta_n a/h)\} / \{n^2 \beta_n D_n K_1(\beta_n a/h)\} & : (\Omega/\omega < 1/2) \end{cases}, \quad (24)$$

and m_0 denotes the mass when liquid is fully filled in the rotor chamber. The actual liquid mass is $m_0(1 - B^2)$, which is smaller than m_0 . ω_0 is the natural frequency of the system.

Fig. 2 Response curves for $0 \leq B \leq 1$ ($\Omega/\omega = 1$, $H = 2$, $\gamma_0 = 0.2$)

3. Results and Discussion

Figure 2 shows the response curves of the system when the exciting frequency is equal to the rotor speed, i.e., $\Omega/\omega=1$. The other conditions are described in the figure. When the rotor is empty ($B=1$), a resonance occurs at $\omega_0/\omega=1$. As the amount of liquid increases, that is, as B is reduced, resonance frequency increases. When $B \approx 0.29$, the resonance frequency becomes infinite. Resonance does not occur in the range where $0.26 \leq B \leq 0.29$. At $B \approx 0.25$, resonance occurs again at a frequency lower than ω_0 . As the value of B is further reduced to zero, resonance frequency approaches the frequency $\omega_0/\sqrt{1+\gamma_0}$, which is the natural frequency of a system where a mass $M+m_0$ is supported by spring k . Since the ratio Ω/ω is fixed, the resonance frequency is uniquely determined. In Fig. 3, the value of σ is shown as a function of B . For $0.26 \leq B \leq 0.29$, σ is negative. At $B \approx 0.26$, σ becomes infinite.

From these figures and Eq. (21) we note that a resonance takes place at $\omega=\omega_0/\sqrt{\sigma}$ for $\sigma>0$. For $\sigma \leq 0$, amplitude does not increase with increase of exciting frequency ω , i.e., resonance vanishes. Therefore, the condition for neutralizing vibrations, or tuning condition, is $\sigma \leq 0$.

$(\sigma-1)$ is regarded as the liquid force divided by $A\omega^2 \sin \omega t$. In the range where $0.3 \leq B \leq 1$, the liquid force acts in the same direction as the spring restoring force since $(\sigma-1)<0$. Therefore, resonance frequency ω_0 is higher than ω_0 for this range of B . On the other hand, in the range where $B \leq 0.25$, the liquid force acts in the opposite direction to the spring restoring force since $(\sigma-1)>0$. Therefore, the resonance frequency is lower than ω_0 .

Fig. 3 σ versus B

Figure 4 shows the resonance frequency versus B for various values of γ_0 . The range where the resonance frequency changes rapidly corresponds to the range where resonance vanishes. As γ_0 increases, the tuning conditions for B become wider. From a practical viewpoint, however, the mass ratio γ_0 should be small. We set $\gamma=0.2$ in the following analysis.

In Figs. 5 and 6, the tuning conditions are shown. When the tuning conditions are satisfied, liquid motion is one of the natural modes. In the figures only the first, second and third modes are shown. The mode order represents the number of z -nodes of liquid motion. Except for the hatched regions the tuning allowance for B is too narrow. The tuning allowance of higher modes is much severer. Therefore, they are not shown in the figures.

So far, we have mainly investigated the tuning condition for B . Next, we examine the condition for

Fig. 4 Variation of resonance frequency versus B

Fig. 5 Tuning conditions for H and B

suppressing the vibration of a system in resonance, i.e., $\omega = \omega_0$, by changing only rotor speed, keeping the amount of enclosed liquid constant. The response curve to rotor speed is shown in Fig. 7. At $\Omega/\omega_0 \approx 0.73$ and 2.02, vibrations are suppressed. However, the tuning allowance at $\Omega/\omega_0 \approx 0.73$ is very narrow. At $\Omega/\omega_0 \approx 1.18$ and 1.58, there are other tuning conditions for Ω which are not shown since these tuning allowances are too narrow. In order to suppress vibrations completely, i.e., $|A|/\delta = 0$, rotor speed Ω must be tuned very carefully and accurately. Practically speaking, vibrations within some limit are allowable. Figure 8 shows tuning allowance for Ω/ω_0 (the hatched regions) when the allowable amplitude $|A|$ is set at the static deflection $\delta = Mg/k$. The tuning allowance for Ω/ω_0 increases with H .

4. Experiment

Figure 9 shows an experimental apparatus. A

Fig. 6 Tuning conditions for Ω/ω and B

Fig. 7 Amplitude variation of a system in resonance versus rotor speed

hollow rotor and a casing are mounted on two leaf springs, 1 300 mm long by 50 mm wide by 6 mm thick. The radius and height of the rotor chamber are 50 mm and 82 mm, respectively. Therefore, the aspect ratio H is 0.82. The rotor is attached to a shaft whose diameter and span are 10 mm and 270 mm, respectively, and is driven by a DC motor. The shaft is supported by two ball bearings. The stiffness of the bearings is much higher than that of the leaf springs, so we assume that the casing vibrates together with the hollow rotor. The total mass of the rotor, casing and DC motor is 9.7 kg. The apparent mass ratio is 0.051. The transient vibrations were measured after blowing in the vertical direction. The damping ratio was calculated to be 0.001.

Since the casing is supported by two leaf springs, there are two modes of vibrations: i.e., the two leaf springs vibrate in phase, and they vibrate 180° out of phase. The two natural frequencies are 8.9 Hz and 12 Hz, respectively, when the rotor is empty. As rotor

Fig. 8 Tuning condition for Ω/ω with the allowed amplitude, $|A| = \delta$

Fig. 9 Test apparatus

Fig. 10 Response of the casing in the vertical direction

speed is increased, vertical vibrations take place. Their frequency is the same as the rotor speed. A resonance occurs at 534 rpm (8.9 rps). There is no exciting force in the vertical direction. However, due to the elasticity coupling of the apparatus, rotor-unbalanced force induces vortical vibration.

To demonstrate the effectiveness of the proposed dynamic absorber, we suppress the vertical vibrations by changing the amount of the enclosed water. Thus, in the experiment, the ratio $\Omega/\omega=1$. After balancing the rotor, the amplitude of horizontal vibrations at the top of the casing is less than $2\text{ }\mu\text{m}$ at any rotor speed under 660 rpm. At 720 rpm (12 rps), large vibrations are observed. This resonance is considered to be the second mode of the apparatus. Therefore, measurement is performed at speeds ranging from 280 to 660 rpm.

Figure 10 shows representative experimental results of the casing response in the vertical direction. In the figure, ● represents vertical translational vibrations and ○ represents conical vibrations where the two leaf springs move 180° out of phase. At first, resonance frequency increases and its maximum amplitude decreases as B is reduced. Then, no resonance occurs. Decreasing the value of B further produces a resonance at a speed lower than the natural frequency of 8.9 Hz. Due to structural damping and viscous shear of water, which are neglected in the analysis, the amplitude at resonance is finite. Since the exciting force was not estimated, the amplitude could not be calculated. However, the experimental results and calculated results shown in Fig. 2 demonstrate similar tendencies. Amplitude begins to increase over 650 rpm due to the second resonance at 720 rpm.

Figure 11 shows resonance rotor speed versus B . The rotor speed at which amplitude reaches a maximum is regarded as an experimental resonance frequency ω_c since $\Omega/\omega=1$. From Eq. (21), theoretical

Fig. 11 Theoretical and experimental resonance frequencies versus B

resonance frequency ω_c is $\omega_0/\sqrt{\sigma}$. The experimental results agree with the theoretical ones. Although we did not observe the surface motion of water, the mode of water motion at the tuning condition is considered to be the first mode from experimental agreement with analytical resonance frequency.

5. Conclusions

We have proposed a dynamic absorber which consists of a hollow rotor partially filled with liquid. Tuning conditions were analyzed. The results are summarized as follows:

(1) By choosing the amount of liquid and keeping the ratio Ω/ω of rotor speed to exciting frequency constant, vibration can be suppressed at any exciting frequency.

(2) The tuning allowance for fill ratio B increases with apparent mass ratio γ_0 .

(3) It is possible to control vibration of a system in resonance by changing the rotor speed of an absorber containing a fixed amount of liquid.

The effectiveness of the proposed dynamic absorber is demonstrated experimentally.

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