State Estimation of Road Surface and Vehicle System Using a Kalman Filter*

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In this paper, an approach to estimating the state variables of a vehicle system and the absolute displacement of a road surface is presented. The Kalman filter theory is applied to an augmented system which consists of the vehicle system and the shaping filter. The shaping filter is taken so that the power spectral density of output is equal to that of the displacement of the road surface, while white noise is taken as input. The Padé approximation is used in the augmented system to take account of the time delay between the input values of the front and rear wheels. The results obtained using the Padé approximation are compared with the results obtained by assuming that the inputs of the four wheels are independent of each other. A method for identifying the shaping filter in the augmented system, which may be effective in constructing the augmented system adaptively in actual driving case, is also presented. Some simulation models are examined to validate the presented methods.

Key Words: Automobile, Filter, Random Vibration, Identification, Automatic, Control

1. Introduction

Recently, the active control of motion and vibration of a vehicle has been studied extensively. The state estimation of vehicle motion is very important to realize better control performance. All of the state variables of a vehicle system should be measured or estimated in order to apply the optimal state feedback control. In general, we need to design the state estimator for estimating all state variables from limited measurable variables, since all of the state variables may not be measured by measurement equipment. It is very difficult to measure the absolute roughness of a road surface from a vehicle. The vehicle system does not satisfy the condition of observability even though the relative displacement is measured. Therefore, the deterministic state estimator such as the Luenberger observer is not available.

An approach to estimating state variables of a

four-wheeled vehicle system and a road surface by the Kalman filter theory is presented. However, the Kalman filter requires white noise input, while the displacement excitation of the road surface may be considered as colored noise. If the power spectral density of the road surface on which a vehicle is running is known, we can overcome this difficulty by introducing the shaping filter. The shaping filter is selected so that the power spectral density of output is the same as that of the true road surface, while the white noise is taken as input. The Kalman filter theory is applied to an augmented system which consists of the vehicle system and the shaping filter. Time delay terms of road input cannot be formulated exactly into the ordinary state space equation. The Padé approximation is used in the augmented system to take account of the time delay relation between the road input of the front wheel and of the rear wheel. Two kinds of Kalman filters are constructed. One takes into account the time delay relation of road input, and the other takes into account the fact that the four road inputs are independent from one another. The estimated properties of two kinds of Kalman filters are compared by numerical simulation. A vehicle system is modeled with seven degrees of freedom.

The characteristics of the road surface on which a vehicle is running will vary continuously. We cannot apply the Kalman filter theory with fixed characteris-

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tics of the shaping filter in this case. A method for identifying the shaping filter from the measurable time-historied data is also presented. This method may be effective in constructing an adaptively augmented system in an actual driving case.

2. Modeling a Vehicle System

Consider a four-wheeled vehicle system subjected to four displacement excitations of the road surface, one at each wheel. A mathematical model of this vehicle system is given by Fig. 1. A vehicle body is assumed to be a rigid body, and the horizontal component of road input is small enough to be neglected. m , k , and c denote the equivalent mass, stiffness, and damping parameters. I_x , I_y , and I_z denote the mass moment of inertia with respect to the x , y , and z axes at the center of mass of a vehicle body. x_g , y_g , and z_g denote the displacement of the center of mass of a vehicle body. θ_x , θ_y , and θ_z denote the angular displacement of rolling, pitching, and yawing. z_1 to z_4 denote the vertical displacement of the equivalent tires. δ_1 to δ_4 denote the relative displacement of a suspension system. r_1 to r_4 denote the vertical displacement excitation of a road surface. It may be necessary to take into account the horizontal displacement of a suspension system since the center of mass of a vehicle body is separated from the suspension plane. If the horizontal component of stiffness and damping of a suspension are far larger than the vertical component of those, however, x_g and y_g become equal to $h_g\theta_y$ and $-h_g\theta_x$, and θ_z becomes small enough to be neglected. Therefore, a model given by Fig. 1 can be considered as a system with seven degrees of freedom. The equation of motion of a vehicle model is expressed as

$$\begin{aligned} m\ddot{z}_g = & -2(k_f + k_r)z_g + 2(k_f l_f - k_r l_r)\theta_y + k_f(z_1 + z_2) \\ & + k_r(z_3 + z_4) - 2(c_f + c_r)\dot{z}_g \\ & + 2(c_f l_f - c_r l_r)\dot{\theta}_y + c_f(\dot{z}_1 + \dot{z}_2) + c_r(\dot{z}_3 + \dot{z}_4) \end{aligned}$$

Fig. 1 Model of vehicle system

$$\begin{aligned} (I_x + mh_g^2)\ddot{\theta}_x = & -2(s_f^2 k_f + s_r^2 k_r)\theta_x + s_f k_f(z_1 - z_2) \\ & + s_r k_r(z_3 - z_4) - 2(s_f^2 c_f + s_r^2 c_r)\dot{\theta}_x \\ & + s_f c_f(\dot{z}_1 - \dot{z}_2) + s_r c_r(\dot{z}_3 - \dot{z}_4) + mh_g a_y \\ (I_y + mh_g^2)\ddot{\theta}_y = & 2(l_f k_f - l_r k_r)z_g - 2(l_f^2 k_f + l_r^2 k_r)\theta_y \\ & - l_f k_f(z_1 + z_2) + l_r k_r(z_3 + z_4) \\ & + 2(l_f c_f - l_r c_r)\dot{z}_g - 2(l_f^2 c_f + l_r^2 c_r)\dot{\theta}_y \\ & - l_f c_f(\dot{z}_1 + \dot{z}_2) + l_r c_r(\dot{z}_3 + \dot{z}_4) - mh_g a_x \\ m_f \ddot{z}_1 = & -k(z_1 - r_1) + k_f(z_g + s_f \theta_x - l_f \theta_y - z_1) \\ & + c_f(\dot{z}_g + s_f \dot{\theta}_x - l_f \dot{\theta}_y - \dot{z}_1) \\ m_f \ddot{z}_2 = & -k(z_2 - r_2) + k_f(z_g - s_f \theta_x - l_f \theta_y - z_2) \\ & + c_f(\dot{z}_g - s_f \dot{\theta}_x - l_f \dot{\theta}_y - \dot{z}_2) \\ m_f \ddot{z}_3 = & -k(z_3 - r_3) + k_r(z_g + s_r \theta_x + l_r \theta_y - z_3) \\ & + c_r(\dot{z}_g + s_r \dot{\theta}_x + l_r \dot{\theta}_y - \dot{z}_3) \\ m_f \ddot{z}_4 = & -k(z_4 - r_4) + k_r(z_g - s_r \theta_x + l_r \theta_y - z_4) \\ & + c_r(\dot{z}_g - s_r \dot{\theta}_x + l_r \dot{\theta}_y - \dot{z}_4) \end{aligned} \quad (1)$$

where a_x and a_y represent the components of acceleration at the center of the suspension plane in the direction of x and y axes. The pitching and rolling motion caused by accelerating, decelerating and a turning operation can be expressed approximately by a_x and a_y values. Define the state space variable $\{x\}$ and displacement excitation of road surface $\{r\}$ as

$$\begin{aligned} \{x\}^T = & \begin{bmatrix} z_g & \theta_x & \theta_y & \dot{z}_g & \dot{\theta}_x & \dot{\theta}_y & z_1 \\ z_2 & z_3 & z_4 & \dot{z}_1 & \dot{z}_2 & \dot{z}_3 & \dot{z}_4 \end{bmatrix} \\ \{r\} = & [r_1 \ r_2 \ r_3 \ r_4]^T. \end{aligned} \quad (2)$$

The state space equation is expressed as

$$\begin{aligned} \dot{\{x\}} = & [A]\{x\} + [B]\{r\} \\ [B] = & [B_f \ B_r], \quad \{r\} = \{r_f^T \ r_r^T\}^T, \end{aligned} \quad (3)$$

where the $[A]$ and $[B]$ matrices represent the 14×14 and 14×4 matrices obtained easily from Eq. (1). It should be noticed that the road input $\{r\}$ cannot be measured from a vehicle system. Assume measurable variables as

$$\{y\}^T = [\ddot{z}_g \ \ddot{\theta}_x \ \ddot{\theta}_y \ \ddot{z}_1 \ \ddot{z}_2 \ \ddot{z}_3 \ \ddot{z}_4 \ \delta_1 \ \delta_2 \ \delta_3 \ \delta_4]. \quad (4)$$

The output equation can be written as

$$\begin{aligned} \{y\} = & [C]\{x\} + [D]\{r\} \\ [D] = & [D_f \ D_r], \end{aligned} \quad (5)$$

where the $[C]$ and $[D]$ matrices represent the 11×14 and 11×4 matrices.

3. Shaping Filter for Road Surface

It is very difficult to estimate the state variables of a vehicle system by the Luenberger observer since the displacement input of the road surface cannot be measured easily from a vehicle. Road surface roughness appears to be a random process, provided that the effects of such occasional large irregularities as potholes are removed from the analysis. The introduction of a statistical estimator such as a Kalman filter may be effective in this statistical case. The Kalman filter requires, however, input data as white noise, while a displacement input of the road surface may be

considered as colored noise. If a power spectral density of the road surface as a statistical property is known, we can apply the Kalman filter theory by introducing a shaping filter. The shaping filter should be constructed so that its output variable becomes a colored noise which has the same power spectral density as the actual displacement of a road surface on which the vehicle is running. The procedure for constructing a shaping filter is as follows.

The power spectral density of the road surface is assumed to be given approximately by the following analytical form

$$S_\nu(\nu) = \frac{\sum_{j=0}^m b_{\nu j} \nu^{2j}}{\sum_{i=0}^n a_{\nu i} \nu^{2i}}, \quad (6)$$

where ν is spatial frequency (wavelength per unit length) of the road surface, and $a_{\nu j}$ and $b_{\nu j}$ are parameters of the shaping filter, while $b_{\nu m} = 1$. When a vehicle is running along the road with velocity v , the excitation frequency of the road input ω [rad/sec] becomes $\omega = 2\pi\nu v$. The mean squared value of road surface roughness, that is total area of the power spectral density function, does not change with the velocity of a vehicle. Thus we have the relation of the form

$$S(\omega)d\omega = S_\nu(\nu)d\nu, \quad (7)$$

where $S(\omega)$ represents the power spectral density of road input with respect to displacement excitation frequency. We can express the power spectral density of road input as

$$S(\omega) = \frac{\sum_{j=0}^m b_j \omega^{2j}}{\sum_{i=0}^n a_i \omega^{2i}}, \quad (8)$$

where

$$a_i = a_{\nu i} (2\pi v)^{-2i}$$

$$b_j = b_{\nu j} (2\pi v)^{-2j-1}.$$

Determine the transfer function $G(j\omega)$ of the shaping filter given by Fig. 2 so that the power spectral density of the output of the shaping filter is the same as that of the road input imposed at each wheel. The relation of power spectral density between input and output of the shaping filter is represented as

$$S_r(\omega) = |G(j\omega)|^2 S_w(\omega), \quad (9)$$

where $S_w(\omega)$ and $S_r(\omega)$ are the power spectral density functions of input (white noise) and output (colored noise). If we normalize the power spectral density of white noise so that $S_w(\omega) = 1$, the transfer function of the shaping filter can be determined as

$$G(j\omega) = \frac{b_n^{0.5} \prod_{j=1}^m (j\omega - z_j)}{a_n^{0.5} \prod_{i=1}^n (j\omega - p_i)}, \quad (10)$$

where p_i is a negative root of the following equation :

$$\sum_{i=0}^n (-1)^i a_i s^{2i} = 0, \quad (11)$$

and z_j is a negative root of the following equation :

$$\sum_{j=0}^m (-1)^j b_j s^{2j} = 0. \quad (12)$$

Minimal realization of Eq. (10) leads to the state space equation in the form

$$\begin{aligned} \dot{\{x_s\}} &= \{A_s\}\{x_s\} + \{B_s\}\{w\} \\ \{r\} &= \{C_s\}\{x_s\}. \end{aligned} \quad (13)$$

Here the system noise $\{w\}$ is white noise with the property of the form

$$E[\{w(t)\}\{w(t+\tau)\}^T] = [W]\delta(\tau). \quad (14)$$

Increasing the order of Eq. (8) will give a more accurate approximation for statistical properties of road input. However, increasing the order of Eq. (8) requires the increasing of the number of the state variables, which may deteriorate the control performance.

4. Construction of Kalman Filter

4.1 Kalman filter for independent road input

Imaginary road surfaces were shaped by a shaping filter. Imaginary road surfaces imposed at four wheels, one at each wheel, are assumed to be independent from one another. We can construct an augmented system by combining a shaping filter given by Eq. (13) with the vehicle system given by Eq. (3) and Eq. (5). The system dynamic equation of the augmented system is represented as

$$\begin{aligned} \dot{\{x_{a1}\}} &= [A_{a1}]\{x_{a1}\} + [B_{a1}]\{w_s\} \\ \{y\} &= [C_{a1}]\{x_{a1}\} + \{v\}, \end{aligned} \quad (15)$$

where

$$\begin{aligned} \{x_{a1}\} &= [x^T \quad x_s^T]^T \\ [A_{a1}] &= \begin{bmatrix} A & BC_s \\ 0 & A_s \end{bmatrix} \\ [B_{a1}] &= [0 \quad B_s^T]^T \\ [C_{a1}] &= [C \quad DC_s]. \end{aligned}$$

System noise $\{w_s\}$ and measurement noise $\{v\}$ are zero-mean Gaussian white noise vectors and their covariances are assumed as

$$\begin{aligned} E[\{w_s(t)\}\{w_s(t+\tau)\}^T] &= [W_s]\delta(\tau) \\ E[\{v(t)\}\{v(t+\tau)\}^T] &= [V]\delta(\tau) \\ E[\{w_s(t)\}\{v(t+\tau)\}^T] &= [0]. \end{aligned} \quad (16)$$

The Kalman filter equation for the best estimate $\{\hat{x}_{a1}\}$ of $\{x_{a1}\}$ in the mean squared error sense is given as follows⁽¹⁾ :

$$\begin{aligned} \dot{\{\hat{x}_{a1}\}} &= [A_{a1}]\{\hat{x}_{a1}\} + [K]([y] - [C_{a1}]\{\hat{x}_{a1}\}) \\ \{\hat{x}_{a1}(t_0)\} &= E[\{x_{a1}(t_0)\}]. \end{aligned} \quad (17)$$

The filter gain $[K]$ is of the form

Fig. 2 Model of shaping filter

$$[K] = [\tilde{X}_{a1}][C_{a1}]^T [V]^{-1}, \quad (18)$$

and the covariance matrix of estimated error $[\tilde{X}_{a1}]$ is given by the solution of the Riccati equation given by

$$\begin{aligned} [\dot{\tilde{X}}_{a1}] &= [A_{a1}][\tilde{X}_{a1}] + [\tilde{X}_{a1}][A_{a1}]^T + [B_{a1}][W][B_{a1}]^T \\ &\quad - [\tilde{X}_{a1}][C_{a1}]^T [V]^{-1} [C_{a1}][\tilde{X}_{a1}] \\ [\tilde{X}_{a1}(t_0)] &= \text{cov}\{x_{a1}(t_0) - \hat{x}_{a1}(t_0)\}, \\ \{x_{a1}(t_0) - \hat{x}_{a1}(t_0)\}^T & \end{aligned} \quad (19)$$

4.2 Kalman filter for road input with time delay

Road input imposed at the front wheel may also be imposed at the rear wheel after a certain period of time if a vehicle is running a straight line. If time delay T [sec] is considered, the Laplace transform gives the relation of time delay as

$$R_r(s) = e^{-sT} R_f(s), \quad (20)$$

where $R_f(s)$ and $R_r(s)$ represent the Laplace transformed values of road input imposed at the front and rear wheels. It is very difficult, however, to fit Eq.(20) into the state space equation exactly. The Padé approximation is applied to realize the state space equation of Eqs.(3), (5) and (15). The second- and third-order Padé approximation⁽²⁾ are applied. The second-order Padé approximation is represented as

$$e^{-sT} = \frac{1 - \frac{1}{2}sT + \frac{1}{12}(sT)^2}{1 + \frac{1}{2}sT + \frac{1}{12}(sT)^2}, \quad (21)$$

and the third order Padé approximation is represented as

$$e^{-sT} = \frac{1 - \frac{1}{2}sT + \frac{1}{10}(sT)^2 - \frac{1}{120}(sT)^3}{1 + \frac{1}{2}sT + \frac{1}{10}(sT)^2 + \frac{1}{120}(sT)^3}. \quad (22)$$

Increasing the order of the Padé approximation will give a more exact expression of time delay, but the number of auxiliary state variables is also increased. Time delay T becomes $(l_f + l_r)/v$ when a vehicle is running with velocity v . A minimal realization of the Padé approximation gives the state space equation as

$$\begin{aligned} \{\dot{x}_p\} &= [A_p]\{x_p\} + [B_p]\{r_f\} \\ \{r_r\} &= [C_p]\{x_p\} + [D_p]\{r_f\}. \end{aligned} \quad (23)$$

We need both the shaping filter for road input imposed at the front wheels and the Padé approximated filter for road input imposed at the rear wheels. By combining the Padé approximated filter given by Eq.(23) and the shaping filter given by Eq.(13) with a vehicle system given by Eq.(3) and Eq.(5), we have an augmented system such that

$$\begin{aligned} \{\dot{x}_{a2}\} &= [A_{a2}]\{x_{a2}\} + [B_{a2}]\{w_{s2}\} \\ \{y\} &= [C_{a2}]\{x_{a2}\} + \{v\}, \end{aligned} \quad (24)$$

where

$$\begin{aligned} \{x_{a2}\} &= [x^T \quad x_p^T \quad x_s^T]^T \\ [A_{a2}] &= \begin{bmatrix} A & B_r C_p & B_r C_s + B_r D_p C_s \\ 0 & A_p & B_p C_s \\ 0 & 0 & A_s \end{bmatrix} \\ [B_{a2}] &= [0 \quad 0 \quad B_s^T]^T \end{aligned}$$

$$[C_{a2}] = [C \quad D_r C_p \quad D_r C_s + D_r D_p C_s].$$

The Kalman filter equation for estimate $\{\hat{x}_{a2}\}$ of $\{x_{a2}\}$ can be given by the similar equation as that described in the section 4.1.

5. Adaptive Identification of Shaping Filter

The statistic properties of the road surface on which a vehicle is running will vary from time to time. We cannot apply the Kalman filter theory with the fixed property of the shaping filter in this case. We need to change the parameters of the shaping filter adaptively according to the change of properties of the road surface. The parameters of the shaping filter will be determined from time-historied measured variables of a vehicle system.

Firstly, the power spectral density of the road surface will be estimated from output values of a vehicle system. The procedure is presented as follows.

A transfer function between the output value $\{y(t)\}$ and road input $\{r(t)\}$ is

$$\begin{aligned} \frac{Y(j\omega)}{R(j\omega)} &= [H_f(j\omega) \quad H_r(j\omega)] \\ &= [C][j\omega I - [A]]^{-1}[B] + [D], \end{aligned} \quad (25)$$

where $[A]$, $[B]$, $[C]$ and $[D]$ matrices are defined by Eq.(3) and Eq.(5), and subscripts f and r denote the front and rear wheels. It is assumed that there is a time delay relation between the road input imposed at the front wheel and that imposed at the rear wheel. The relation between the power spectral density of output variable $\{S_y(\omega)\}$ and that of road input $\{S_r(\omega)\}$ becomes

$$\{S_y(\omega)\} = [P(\omega)]\{S_r(\omega)\}, \quad (26)$$

where

$$P_{ij}(\omega) = |H_{fij}(j\omega) + H_{rij}(j\omega)e^{-j\omega T}|^2,$$

where subscripts i ($i=1 \sim p$, p is number of output variable) and j ($j=1 \sim 2$) denote the i -th row and the j -th column of the matrices. The least square estimation gives the estimate of power spectral density of the road surface as

$$\{S_r(\omega)\} = ([P(\omega)]^T [P(\omega)])^{-1} [P(\omega)]^T \{S_y(\omega)\}. \quad (27)$$

Secondly, the parameters a_i ($i=0 \sim n$), b_i ($i=0 \sim m-1$) should be identified so that the residual sum of the squares of the form

$$F' = \sum_{i=1}^N [S_r(\omega_i) - S(\omega_i)]^2 \quad (28)$$

is minimum. Here, N is the number of data used. Let γ_i ($i=0 \sim n+m+1$) represent the unknown parameters a_i , b_i . Applying the Marquardt method⁽³⁾ leads to the increment of parameters to the solution in the form

$$[\alpha + \lambda I]\{\delta\gamma\} = \{\beta\}, \quad (29)$$

where

$$\alpha_{jk} = 2 \sum_{i=1}^N \left[\frac{\partial S(\omega_i)}{\partial \gamma_j} \frac{\partial S(\omega_i)}{\partial \gamma_k} \right]$$

$$\beta_j = 2 \sum_{i=1}^N \left[\frac{\partial S(\omega_i)}{\partial \gamma_j} \{ S_R(\omega_i) - S(\omega_i) \} \right].$$

Here, λ is a positive constant. This minimization approach becomes the steepest descent method with sufficient large λ , and becomes the Gauss-Newton method with sufficient small λ . If increment of parameters $\{\delta\gamma\}$ is determined, modified parameters $\{\gamma'\}$ is obtained as

$$\{\gamma'\} = \{\gamma\} + \{\delta\gamma\}. \quad (30)$$

6. Numerical Simulation

From the report presented by I.S.O.^{(4),(5)}, the power spectral density of the road surface is approximately decreasing in proportion to ν^2 in the range $\nu < 1/2\pi$ cycle/m, where ν denotes the spatial frequency. The power spectral density of the road surface is assumed as

$$S_\nu(\nu) = \frac{\nu_3^2}{(\nu^2 + \nu_1^2)(\nu^2 + \nu_2^2)} \quad (31)$$

The specifications of a vehicle system used for simulation are as follows.

$m = 1389$ kg, $m_f = 40$ kg, $m_r = 35$ kg, $k = 980$ kN/m, $k_f = 19208$ N/m, $k_r = 19796$ N/m, $c_f = 5390$ NS/m, $c_r = 4469$ NS/m, $h_a = 0.185$ m, $I_x = 649.91$ Nms², $I_y = 1872.95$ Nms², $I_z = 2304.15$ Nms², $I_f = 1.061$ m, $I_r = 1.519$ m, $s_f = 0.731$ m, $s_r = 0.733$ m.

Numerical simulation for state estimation is carried out as follows. Firstly, the data of the road surface are obtained from the ordinary differential equation given by Eq.(31) by using the Runge-Kutta method of the second order. The sampling interval, Δt , is taken as 5×10^{-3} sec, and the transient response terms are removed. The parameters of the shaping filter used are $2\pi\nu_1 = 0.36$ m⁻¹, $2\pi\nu_2 = 36$ m⁻¹ and $\nu_3 = 9.33 \times 10^{-4}$. The time delay of displacement excitation of the road surface imposed at the wheel is considered. The velocity of a vehicle is assumed to be 60 km/hr. Secondly, two kinds of Kalman filter are constructed: one with four independent road inputs, and the other

Fig. 3 Model of augmented system (Time delay of road input is taken into account)

taking account of the time delay of road input. Thirdly, the state estimation of a road surface and a vehicle system is carried out by the Runge-Kutta method of the second order. The time-historied output variables given by Eq.(4) are used for input values of the Kalman filter. The sampling interval used is 5×10^{-3} sec. The state variables estimated by the Kalman filter are compared with the true values.

The results of numerical simulation are shown in Figs. 4 to 9. Figure 4 represents the results of state estimation by the Kalman filter theory in which four road inputs are assumed to be independent from one another. Figure 5 represents the results of state estimation in which time delay terms are approximated by the third-order Padé approximation. The Kalman filter does not show a good estimate of z_θ . The power spectral density of the angular displacement and velocity of rolling, θ_x , $\dot{\theta}_x$, are shown by Fig. 6 and Fig. 7. The solid line represents the true value and the dashed line represents the estimated value. The dot-dashed line represents the power spectral density of estimated error. From the results of θ_x , the estimation with the Padé approximation shows better estimates in the lower-frequency range than that with independent road input. From the result of $\dot{\theta}_x$, however, estimation with the Padé approximation shows a worse estimate in the higher-frequency range than that with independent road input. The reason for this is considered to be the approximated error of the Padé

Fig. 4 Time history of state variables estimated by Kalman filter theory with independent road input

Fig. 5 Time history of state variables estimated by Kalman filter theory with third-order Padé approximation

(a) With independent road input

(b) With third-order Padé approximation

Fig. 6 Power spectral density of angular displacement of rolling θ_x

(a) With independent road input

(b) With third-order Padé approximation

Fig. 7 Power spectral density of angular velocity of rolling θ_x

Fig. 8 Power spectral density of angular displacement of pitching θ_y (with third-order Padé approximation)

approximation becomes larger as frequency increases. The power spectral density of the angular displacement of pitching θ_y , is shown by Fig. 8. The power spectral density of the absolute displacement of the road surface imposed at the rear wheel is shown by Fig. 9, where the Padé approximation of the second-

(a) With second-order Padé approximation

(b) With third-order Padé approximation

Fig. 9 Power spectral density of absolute displacement of road input imposed on rear wheel

order is used in (a) and that of the third order is used in (b). The estimation with the Padé approximation of third-order shows a good estimate up to a higher frequency compared with the result of second-order Padé approximation. The estimate with the Padé approximation shows a good estimate up to about 10 Hz. From the results above, we can recognize that not only the state of a vehicle system but also the state of the road surface, can be estimated with sufficient accuracy by the Kalman filter theory.

The shaping filter, whose transfer function is $0.1/(s^2 + 4s + 328)$, is used for generating the data of the road surface. White noise with an intensity of 1 is used as the input data of the shaping filter. The time-historied data of vertical acceleration of the four wheels are obtained by the Runge-Kutta method of the second order in order to be used as the input data for the identification of the shaping filter. The transfer function of the shaping filter identified by the proposed method becomes $0.074/(s^2 + 4.44s + 278)$. The values of

Fig. 10 Identification of shaping filter

parameters of the shaping filter show some difference between true and identified values. However, from Fig. 10, we can recognize that the identified shaping filter represents the characteristics of the road surface with sufficient accuracy.

7. Conclusions

A method for the state estimation of a road surface and a vehicle system is presented.

(1) Not only the state of a vehicle system but also the state of the road surface can be estimated with sufficient accuracy up to about 10 Hz by the presented method.

(2) A Kalman filter with independent road input shows almost uniform accuracy of estimation over the entire frequency range.

(3) The estimation taking into account the time delay shows better accuracy than that with independent road input in lower frequencies, but is worse for higher frequencies.

(4) An adaptive identification of the shaping filter from the measurable output variables is presented.

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