Stiffness of a Ball Screw with Consideration of Deformation of the Screw, Nut and Screw Thread* (Preloaded Double Nut)

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The stiffness in the axial direction of preloaded ball screws constructed from double nuts is calculated considering the elastic deformation of the screw shaft, the nut body, and the screw threads, in addition to the deformation due to Hertzian contact. The basic equations derived here could be applied in many cases of preloading schemes and the various states of an externally applied load. Moreover, the calculation can be carried out both for single- and double-circuit nuts. The calculated stiffness is low compared with that obtained when only the Hertzian contact effect is considered. The difference between the two calculated values becomes large as the spacing disc rigidity increases. At high rigidity of spacing disc, the externally applied load which releases the preload also becomes lower than that obtained with Hertzian contact deformation model. The calculated values of both the stiffness and the releasing load approximately agree with the experimental results.

Key Words : Machine Element, Ball Screw, Stiffness, Preload, Double Nut, Hertzian Contact, Elastic Deformation, Screw Shaft, Nut, Thread

1. Introduction

The axial stiffness of ball screws has been calculated considering Hertzian contact deformation alone^{(1)~(2)} and by considering both elastic deformation of the screw shaft and the nut body together with the Hertzian contact deformation^{(3)~(5)}. However, the effect of the elastic deformation of the screw threads on the ball screw stiffness cannot be ignored. The calculation of the ball screw stiffness is consequently performed in a previous paper⁽⁶⁾ taking into account the above three kinds of deformation for ball screws with a single nut, based on the load distribution calculation method in screw threads presented by Kubo⁽⁷⁾. The results show that the deformation of screw threads has only slight influence on the load distribution, but it does have a certain effect on the stiffness.

Ball screws are usually used as a preloaded double nut to eliminate backlash and to improve the

stiffness. Hence, in this paper, the axial stiffness of ball screws used with a preloaded double nut construction is investigated by extending the single nut theory. In deducing the basic equations, it is considered that they can be applied to the cases with typical preloadings and various combinations of external forces. For comparison, another two calculations are performed: one considers the elastic deformations of the screw shaft and the nut body, and the Hertzian contact deformation; the other considers only the Hertzian contact deformation. These three calculated results are compared with the experimental results.

2. Nomenclature

A_b : Stress area of screw shaft
 A_n : Stress area of nut
 d : Ball diameter
 E_b : Young's modulus of screw shaft material
 E_n : Young's modulus of nut material
 F_0 : Preload
 F_b : Internal force of screw shaft
 F_n : Internal force of nut body
 K_D : Rigidity of spacing disc

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- K_b : Constant of deformation of external screw thread
 K_n : Constant of deformation of nut thread
 P : External force ($P_1 \sim P_4$, P_a , P_b)
 Q : Normal load acting on a ball at contact point ($=wd$)
 w : Normal load/unit width on thread surface
 α : Lead angle
 β : Contact angle between balls and thread groove
 δ_b : Deformation of external screw thread
 δ_H : Axial displacement due to Hertzian contact
 δ_n : Deformation of nut thread
 δ_T : $\delta_b + \delta_n$
 ϵ : Factor depending on the ratio of ball diameter to groove radius and material
 λ_b : Deformation of screw shaft
 λ_D : Deformation of spacing disc
 λ_n : Deformation of nut body

Deformations mentioned above are elastic ones in the axial direction.

3. Basic Equations

3.1 Equations of individual deformation

3.1.1 Deformation of screw threads The cross section of a thread of a ball screw consists of a circular arc. However, for simplicity, it is considered as a trapezoidal thread in the analysis; that is, the circular arc is replaced by the tangent to the arc at the point of contact between the ball and the thread groove.

Concentrated loads may act on the thread groove at every ball space in the real ball screw. Considering, however, that there are a great number of balls, the load is assumed in the analysis to be uniformly distributed over the thread groove.

Then, the equations derived by Yamamoto⁽⁸⁾ are applied for the elastic deformation in the axial direction of the thread. They are given by

$$\delta_b = \frac{K_b}{E_b} w \sin \beta \quad (1)$$

$$\delta_n = \frac{K_n}{E_n} w \sin \beta \quad (2)$$

$$\delta_T = \delta_b + \delta_n \quad (2)$$

3.1.2 Deformation of screw shaft and nut body

Let the each internal force of screw shaft and nut at distance x from the contact beginning position be F_b and F_n , respectively, as shown in Fig. 1. Then, elongation of the screw shaft λ_b and shrinkage of the nut λ_n in a distance x are

$$\lambda_b = \int_0^x \frac{F_b}{A_b E_b} dx \quad (3)$$

$$\lambda_n = \int_0^x \frac{F_n}{A_n E_n} dx$$

3.1.3 Hertzian contact deformation The axial displacement due to Hertzian contact between a ball and thread groove is

$$\delta_H = \frac{\epsilon}{\sin \beta \cos \alpha} d^{1/3} w^{2/3} \quad (4)$$

3.2 The relationship between three kinds of deformation and deduction of basic equations

Figure 2 shows preloading types and operating states of external forces. The preloading is classified into two types, tensile preloading and compressive preloading. However, a different situation of working forces may occur in each of screw shaft and nut as shown in Fig. 2(a) and Fig. 2(b). For example, for a given tensile preloading, with (a), the tensile force occurs in the screw shaft and compressive force occurs in the nut, but with (b), tensile force occurs in both screw shaft and nut.

The external forces to the nut are considered to

Fig. 1 Forces acting on the ball screw

Fig. 2 Schemes of preloading and external forces on ball screw

operate at the positions expressed as P_1 to P_4 . On the screw shaft side, the external forces are received at P_a and/or P_b . The external forces, however, do not operate at these positions simultaneously, but usually operate at only one of these positions for both of screw shaft and nut, depending upon the design and operating conditions. To derive a generalized equation which is effective for all combinations of external forces in Fig. 2, it is assumed that the external forces operate at every position in Fig. 2. In calculation, only the actually working external force is taken into account and the others are ignored. The single-circuit nut is considered first and the result is applied to the double-circuit nut.

For ease of understanding, Fig. 2 is simplified in Fig. 3. Let the left side of the contact beginning position be the origin of nut 1 and the right side of it be the origin of nut 2. Then, the equations derived here become common both for nut 1 and for nut 2, and as a result, the calculation becomes simple.

The difference between the thread part deformation—the sum of thread deformation and Hertzian contact deformation ($\delta_T + \delta_H$)—at the origin 0 and at the distance x from the origin should be equal to the deformation of the screw shaft and nut body in that length, $(\lambda_b + \lambda_n)_{x=x}$.

$$\pm(\delta_H + \delta_T)_{x=0} \mp (\delta_H + \delta_T)_{x=x} + (\lambda_b + \lambda_n)_{x=x} = 0 \quad (5)$$

The upper and lower signs of above equation apply to

Fig. 3 Models of preloaded double nut

tensile preloading and compressive preloading, respectively. Signs appearing hereafter express the same meaning. The relationship between the normal load/unit width on thread surface w and the internal force gradient dF_b/dx is

$$w = \pm \frac{\tan \alpha}{\sin \beta} \cdot \frac{dF_b}{dx} \quad (6)$$

Substituting Eqs. (1)~(3) and (4) into Eq. (5), taking care of Eq. (6) and differentiating it with respect to x , we obtain

$$\left\{ A \pm B \left(\frac{dF_b}{dx} \right)^{-1/3} \right\} \frac{d^2 F_b}{dx^2} - \left(\frac{F_b}{A_b E_b} + \frac{F_n}{A_n E_n} \right) = 0 \quad (7)$$

$$\left. \begin{aligned} A &= \left(\frac{K_b}{E_b} + \frac{K_n}{E_n} \right) \tan \alpha \\ B &= \frac{2}{3} \epsilon d^{1/3} \left(\frac{\sin^2 \alpha}{\sin^5 \beta \cos^5 \alpha} \right)^{1/3} \end{aligned} \right\} \quad (8)$$

Equation (7) can be solved with F_b or F_n under the equilibrium equations and boundary conditions described in the next section. Although Eq. (7) is a nonlinear differential equation, it can be solved numerically by a combination of the Runge-Kutta integration and the shooting method.

3.3 Force equilibrium and boundary conditions

Assume the force acting on the spacing disc changes from preloading load F_0 to $F_0 + F$ when an external force operates, as shown in Fig. 3. Under these circumstances, equilibrium equations of force and boundary conditions at both contact beginning position $x=0$ and contact ending position $x=L_1, L_2$ will be established in the following.

3.3.1 For Fig. 3(a) The relationship between external forces operating on screw shaft and preloaded double nut is

$$P_a + P_b = P_1 + P_2 + P_3 + P_4 \quad (9)$$

The respective relations between internal and external forces for nut 1 or nut 2 are

$$F_n = F_b + P_1 - P_a \quad (10)$$

$$F_n = F_b + P_b - P_4 \quad (11)$$

The boundary conditions are:

For nut 1,

$$\left. \begin{aligned} x &= 0 \\ (F_b)_{x=0} &= P_a \\ (F_n)_{x=0} &= P_1 \\ x &= L_1 \\ (F_b)_{x=L_1} &= (F_0 + F) + P_a - P_1 - P_2 \\ (F_n)_{x=L_1} &= F_0 + F - P_2 \end{aligned} \right\} \quad (12)$$

For nut 2,

$$\left. \begin{aligned} x &= 0 \\ (F_b)_{x=0} &= -P_b \\ (F_n)_{x=0} &= -P_4 \\ x &= L_2 \\ (F_b)_{x=L_2} &= (F_0 + F) - P_b + P_3 + P_4 \\ (F_n)_{x=L_2} &= F_0 + F + P_3 \end{aligned} \right\} \quad (13)$$

3.3.2 For Fig. 3(b) The relationship between external forces operating on the screw shaft and preloaded double nut is

$$P_a + P_b = P_1 + P_2 \quad (14)$$

The respective relations between internal and external forces for nut 1 or nut 2 are

$$F_n = F_b + P_1 - P_a - (F_0 + F) \quad (15)$$

$$F_n = F_b - P_2 + P_b - (F_0 + F) \quad (16)$$

The boundary conditions are:

For nut 1,

$$\left. \begin{aligned} x=0 \\ (F_b)_{x=0} &= P_a \\ (F_n)_{x=0} &= P_1 - (F_0 + F) \\ x=L_1 \\ (F_b)_{x=L_1} &= P_a - P_1 + (F_0 + F) \\ (F_n)_{x=L_1} &= 0 \end{aligned} \right\} \quad (17)$$

For nut 2,

$$\left. \begin{aligned} x=0 \\ (F_b)_{x=0} &= -P_b \\ (F_n)_{x=0} &= -P_2 - (F_0 + F) \\ x=L_2 \\ (F_b)_{x=L_2} &= P_2 - P_b + (F_0 + F) \\ (F_n)_{x=L_2} &= 0 \end{aligned} \right\} \quad (18)$$

3.4 Method of calculation

In solving Eq. (7), a force variation F acting on the spacing disc, which arises owing to external force, is assumed in the boundary conditions. Hence, internal forces F_b or F_n are not obtained directly. Consequently, the actual calculation is performed by following procedure.

At first, the equations are solved for each of nut 1 and nut 2 under the condition that only the preload is applied. If both nuts have the same dimensions, it is sufficient to solve one side only. Secondly, for the conditions of external force operating, the equations are solved under an assumed force variation F . The correctness of the assumed value of F is evaluated by checking the next deflection relationship which must be satisfied before and after external force operation. For Fig. 3(a),

$$\{(\delta_{H1} + \delta_{T1})_P - (\delta_{H1} + \delta_{T1})_0\}_{x=L_1}$$

$$\begin{aligned} &+ \{(\delta_{H2} + \delta_{T2})_P - (\delta_{H2} + \delta_{T2})_0\}_{x=L_2} \\ &\pm \{(\lambda_D)_P - (\lambda_D)_0\} \pm \{(\lambda_b)_P - (\lambda_b)_0\} = 0 \end{aligned} \quad (19)$$

For Fig. 3(b),

$$\begin{aligned} &\{(\delta_{H1} + \delta_{T1})_P - (\delta_{H1} + \delta_{T1})_0\}_{x=0} \\ &+ \{(\delta_{H2} + \delta_{T2})_P - (\delta_{H2} + \delta_{T2})_0\}_{x=0} \\ &\pm \{(\lambda_D)_P - (\lambda_D)_0\} \pm \{(\lambda_b)_P - (\lambda_b)_0\} = 0 \end{aligned} \quad (20)$$

The sum of the first and second terms is the variation of the thread part deformation, the third term is the variation of spacing disc deformation and the fourth term is the variation of screw shaft elongation or shrinkage in the part intercepted by the spacing disc. Subscripts 1 and 2 of δ_H and δ_T correspond to nut 1 and nut 2, respectively. Subscripts 0 and P express the condition of no external force and external force operating, respectively.

The correct axial load distribution in the screw thread can be obtained when the assumed force variation F satisfies Eq. (19) or Eq. (20). Then, the individual deformation and axial displacement of the nut against the screw shaft are calculated through Eqs. (1)~(4) and Eq. (6). The force gradient dF_b/dx of Eq. (6) is obtained during the Runge-Kutta calculation process, leading to the calculation of the normal load distribution on thread. The stiffness of the ball screws is derived from the axial displacement at the external force operating position on the nut.

The calculation mentioned above is effective until the preload is lost, which means the occurrence of a no-contact condition between the balls and the thread groove. As a result, the releasing load of preload can also be obtained.

3.5 Double circuit nut

Figure 4 illustrates the preloaded double nut with double circuit. Although this expresses the case of Fig. 2(a), the procedure described hereafter can also be applied to Fig. 2(b).

In the intermediate part of each circuit in a nut, the force operates on the screw shaft and the nut body, but there is no force on the screw thread. Accordingly, this part is analogous to a spacing disc. If the load variation F of spacing disc, and axial loads P'_1 and P'_2 acting on both intermediate parts of nut 1

Fig. 4 Model of double-circuit nut

and nut 2, respectively, under the given external force are known, then the axial load distribution in screw thread and axial displacement of nut can be calculated in the same way as described in the previous sections. However, the values of F , P'_1 and P'_2 cannot be obtained directly. Consequently, at first, F is assumed and secondly, P'_1 is assumed. Then, the axial load distribution is calculated and is checked to determine whether or not the deformation condition in nut 1 is satisfied. If it is not satisfied, then P'_1 is changed and the above procedure is repeated until the deformation condition is satisfied. Thirdly, the value of P'_2 is assumed and we search for the value P'_2 which satisfies the deformation condition in nut 2. After that, the deformation condition of the double nut is checked under the values of assumed F and obtained P'_1 and P'_2 . If it is not satisfied, then the value of F is changed and the above calculation is repeated until the value F which satisfies the deformation condition of the double nut is obtained. Thus, the correct values of F , P'_1 and P'_2 are determined and then the axial load distribution in the screw thread and axial displacement are obtained.

4. Experiment

The same apparatus described in a previous paper⁽²⁾ is used here. The experiment is performed under tensile preloading as shown in Fig. 2(b). The applied external forces are P_1 and P_a , and others are taken as zero. The specifications of the test ball screw are listed in Table 1. Both of a single-circuit nut and a double-circuit nut are used, and their number of thread turns is 2.5. Two kinds of rigidity of spacing disc are adopted; one is 21 kN/mm and the other is 1360 kN/mm. The rigidity of 21 kN/mm is equivalent

to constant preloading case, where preload is maintained at almost the same value even if some pitch error in the screw exists. The rigidity of 1360 kN/mm corresponds to a state which is intermediate between constant preloading case and fixed position preloading case, which means constant spacing disc thickness. The latter value is selected as large as possible within the limit of detecting the preload and the load variation with strain gauge.

Figures 5 and 6 represent the experimental results of nut displacement in the axial direction at the external force operating position against the screw shaft. In these figures, three calculated results are shown together; first, the values calculated by considering the three kinds of deformation δ_H , λ and δ_T ; second, the values calculated considering two kinds of deformation δ_H and λ ; finally, the values calculated considering only δ_H , where δ_H is the Hertzian contact deformation, λ is the deformation of the screw shaft and the nut body and δ_T is the thread deformation. The values obtained from the first calculation are closest to the experimental values. The difference between the first calculation and the last calculation

(a) Single-circuit nut

(b) Double-circuit nut

Fig. 5 Displacement of loading point ($K_D = 21$ kN/mm)

Table 1 Specifications of test ball screw

Lead	mm	6.0
Ball diameter	mm	3.175
Ball center diameter	mm	36.5
Major diam. of screw	mm	36.0
Root diam. of screw	mm	33.2
Minor diam. of nut	mm	37.0
Major diam. of nut	mm	39.8
Outer diam. of nut	mm	65.0
Lead error	mm/300mm	0.003
Contact angle β	degree	45.0
ϵ	mm ^{4/3} N ^{-2/3}	5.24×10^{-4}
Number of thread turns		2.5

(a) Single-circuit nut

(b) Double-circuit nut

Fig. 6 Displacement of loading point ($K_D = 1360 \text{ kN/mm}$)

becomes large as the spacing disc rigidity increases.

The releasing load, the load where no-contact portion between balls and thread groove in a preloaded double nut appears, is also shown in Figs. 5 and 6. The releasing loads by the three calculations exhibit about the same value for constant preloading case ($K_D = 21 \text{ kN/mm}$). However, for high spacing disc rigidity case ($K_D = 1360 \text{ kN/mm}$), the releasing loads calculated by the first and second calculations are lower than the value by the third calculation. A rapid increase of displacement can be seen experimentally at beyond the releasing loads calculated by first and second calculations.

Figure 7 represents the stiffness against the dimensionless load P/F_0 . Experimental values are lower than the calculated values, but the difference is small. For constant preloading case ($K_D = 21 \text{ kN/mm}$), the releasing of preload occurs when the external force just reaches the preload value, regardless of the circuit number. The axial displacement near this region tends toward unstable behavior during the

(a) $K_D = 21 \text{ kN/mm}$ (b) $K_D = 1360 \text{ kN/mm}$

Fig. 7 Stiffness of ball screw

experiment. Under high spacing disc rigidity case ($K_D = 1360 \text{ kN/mm}$), it becomes stable.

Although the applied preload value F_0 for the high spacing disc rigidity case is half that of the constant preloading case, the axial stiffness of the preloaded double nut results in a higher value at higher spacing disc rigidity case. It is possible in the high spacing disc rigidity case to maintain this higher stiffness until near the releasing load value.

5. Conclusions

Axial stiffness of the ball screws with the preloaded double nut is calculated by considering Hertzian contact deformation, elastic deformation of the screw shaft and of the nut body, and elastic deformation of the screw thread. The calculated values are compared with the experimental values. The results obtained are summarized as follows:

(1) The generalized basic equation for a stiffness calculation is presented; it is applicable for typical preloading and for various combinations of external

forces. It is also applicable for both single- and double-circuit nuts ;

(2) The calculated stiffness is lower than that calculated considering Hertzian contact deformation alone. The difference increases with increasing spacing disc rigidity. The releasing load for constant preloading case is almost the same for both calculations. However, when the spacing disc rigidity becomes higher, the values calculated here becomes low and they are close to the experimental values ;

(3) A slight lower stiffness values are obtained by considering the thread deformation compared with that of no considering case. The effect of this is small for the double-circuit nut ;

(4) The experimental values are slightly lower than the calculated values, but the calculated values are sufficiently useful for most practical applications.

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