A Study on Strength Design Methods for Plastic Gears *

(Estimation of Operation Life of Plastic Gears in Elevated-Temperature Environment)

Naohisa TSUKAMOTO**, Hiroki MARUYAMA*,
Tokihiko TAKI *** and Noriteru NISHIDA ****

In the previous report, the method of calculating the fatigue strength of plastic gears using strain energy was proposed ; however, the conditions only extended to gear life in the ordinary-temperature environment. Recently, the use of plastic gears has expanded, so that they are now used frequently in elevated-temperature environments. For example, the plastic gears used to drive the drying drum in paper-making machines perform at the ambient temperature of about 90 °C. Owing to these facts, a method of estimating the operation life of plastic gears is proposed in this report. Here, the change of the properties of teeth due to working temperature was taken into account using the calculation method developed in the previous report in order to be able to cope with the plastic gears operated in the environment of both elevated-temperature and ordinary-temperature.

Key Words : Machine Element, Gear, Plastic Gear, Nylon Gear, Fatigue Strength, Strain Energy

1. Introduction

Concerning the strength design method for plastic gears, various calculation methods have been proposed so far. In most of them, however, the allowable load, etc., are evaluated from the bending stress of the root of the tooth in accordance with the theory of cantilever represented by the equation of Lewis. In view of a phenomenon peculiar to the plastic gear that its operation life is affected by the duration while a load is applied to its tooth, these methods seem to have many features which are not suitable for the actual circumstances. Therefore, the previous report⁽¹⁾ of this study proposed a calculation method for fatigue strength using strain energy, but this calculation method handles only the operation life of a plastic gear under the ordinary-temperature environment.

Recently, the use for plastic gears has expanded, and plastic gears have been frequently used even under elevated-temperature environments, for example, as driving gears for drying drums in paper-making machines. Therefore it has become necessary even in the strength design method for plastic gears to take into consideration the influences of variation in the physical properties such as rigidity and strength of the plastic material under the elevated-temperature environment.

Under these circumstances, this report proposes an estimation method for the operation life of a plastic gear suitable for both elevated- and ordinary-temperature environments. We conducted operation tests for the plastic gear under an elevated-temperature environment (80 to 120 °C), and on the basis of the results, we took into consideration the variation in the physical properties due according to the working temperature of the tooth in the calculation method in the previous report⁽¹⁾.

* Received 16 th July, 1990. Paper No.89-0553 B

** Department of Mechanical Engineering, Chiba Institute of Technology, 2-17-1 Tsudanuma, Narashino, 275, Japan

*** Ishikawajima-Harima Heavy Industrial Co., Ltd., 3-5 Mukodaicho, Tanashi, 188, Japan

**** Department of Mechanical Engineering, Nagasaki university, 1-14 Bunkyocho, Nagasaki, 852, Japan

2. Experimental Device and Conditions

2.1 Gear tester

To investigate the operation life and tooth temperature of the plastic gear under the elevated-temperature environment in the experiment (operation test), we manufactured a power circulating gear tester with a center distance of 135 mm as shown in Fig. 1 for use. The power was supplied through a V-belt and V-belt pulley ⑫ from a variable-speed motor, and the load was given by twisting the load joint ⑥. The value was detected by torque pickup ⑦ and was measured with a torque meter. Slip rings ⑧ and ⑨ were used for the measurement of the tooth temperature. This test device is provided with a heating wire heater ⑪ within the gear box of a test gear ④, and is constructed so that the temperature is detected with a thermistor ⑩ and the temperature in

the gear box can be freely set by a temperature regulator. Although we measured the temperature in the gear box during heating for a different location of thermistor, the temperature variation due to the location could hardly be detected.

2.2 Test and mating gears

To study the calculation method of fatigue strength as mentioned later together with the results under the ordinary-temperature environment shown in the previous reports⁽²⁾⁻⁽⁴⁾, we conducted experiments with three types of gears with specifications shown in Table 1, using MC nylon as the plastic gear (driving gear) material and carbon steel (JIS S 45 C) as the mating gear material similarly as in the previous report. Each of those gears is of a disk type with the same thickness as the face width, and a backlash of 0.08 module is provided for both plastic and steel gears. These gears are all hobbled to a precision of

Table 1 Main characteristics of gears used in experiments

	Type I gear		Type I gear		Type II gear	
	Driving gear	Driven gear	Driving gear	Driven gear	Driving gear	Driven gear
Module	3		5		5	
Standard pressure angle	20°		20°		20°	
Number of teeth	30	60	17	37	34	64
Addendum modification coefficient	0	0	0	0	0	0
Diameter of standard pitch circle	90	180	85	185	170	320
Clearance coefficient	0.25		0.25		0.25	
Face width	10	10	10	10	20	20
Center distance	135		135		245	
Teeth finishing	Hobbing	Hobbing	Hobbing	Hobbing	Hobbing	Hobbing

N.B., A unit of module, diameter of standard pitch circle, face width and center distance is mm.

Fig. 1 Gear tester

Class 4 of JIS B 1702.

2.3 Experimental conditions

The experiments were conducted for each type of gears under two kinds of torque (input side), T_r , as 9.8 and 14.7 N·m, two kinds of rotational speeds (input side), n , as 500 and 1000 rpm, and three kinds of temperature in the gear box, T_m , as 80, 100 and 120 °C, as shown in Table 2. For symbols in the Table, A and B represent classification of T_r , L and F represent classification of n , and 1, 2 and 3 represent classification of T_m , respectively. In an experiment with 14.7 N·m in T_r and 120 °C in T_m , the plastic gear was likely to be damaged immediately after the operation, and therefore this experiment was excluded. To make the temperature in the gear box equal to that of the gear, we sent current through the heating wire heater, after installing both driving and driven gears, and operated the device at 1000 rpm for about 30 minutes with no load before the experiments. All experiments were conducted without lubrication.

3. Experimental Results

3.1 operation life of plastic gears

Table 3 shows the results of experiments for operation life of plastic gears conducted under the

Table 2 Experimental conditions and symbols

Tr	n tem	500			1000		
		80	100	120	80	100	120
Type I gear							
9.8	I-A-L-1	I-A-L-2	I-A-L-3	I-A-F-1	I-A-F-2	I-A-F-3	
14.7	I-B-L-1	I-B-L-2	————	I-B-F-1	I-B-F-2	————	
Type II gear							
9.8	II-A-L-1	II-A-L-2	II-A-L-3	II-A-F-1	II-A-F-2	II-A-F-3	
14.7	II-B-L-1	II-B-L-2	————	II-B-F-1	II-B-F-2	————	
Type III gear							
9.8	III-A-L-1	III-A-L-2	III-A-L-3	III-A-F-1	III-A-F-2	III-A-F-3	
14.7	III-B-L-1	III-B-L-2	————	III-B-F-1	III-B-F-2	————	

N.B., n : Rotational speed (rpm) , tem : Temperature in gear box (°C), Tr: Torque (N·m).

Table 3 Operation life of plastic gears

Type I gear		Type II gear		Type III gear	
Experi. mark	Life of gear $\times 10^6$	Experi. mark	Life of gear $\times 10^6$	Experi. mark	Life of gear $\times 10^6$
I-A-L-1	1.162	II-A-L-1	10 (Non)	III-A-L-1	0.952
I-A-L-2	0.804	II-A-L-2	7.681	III-A-L-2	0.799
I-A-L-3	0.163	II-A-L-3	0.545	III-A-L-3	0.301
I-A-F-1	2.142	II-A-F-1	10 (Non)	III-A-F-1	1.463
I-A-F-2	0.603	II-A-F-2	10 (Non)	III-A-F-2	1.302
I-A-F-3	0.109	II-A-F-3	0.511	III-A-F-3	0.767
I-B-L-1	0.443	II-B-L-1	1.091	III-B-L-1	0.101
I-B-L-2	0.054	II-B-L-2	0.382	III-B-L-2	0.010
I-B-F-1	0.282	II-B-F-1	1.284	III-B-F-1	0.219
I-B-F-2	0.220	II-B-F-2	0.866	III-B-F-2	0.162

N.B., (Non) indicates nonbreakage.

conditions specified in Table 2. The operation life of a gear is defined as the total number of rotations (cumulative number of rotations) of the gear until cracks to such a degree that they could be noticed with the naked eye occurred on the tooth surface of the plastic gear. As the operation conditions in Table 2, we set the conditions to a torque and rotational speeds that allow the plastic gear to have an operation life exceeding the total number of rotations $N_r=10^7$ without damage under the ordinary-temperature environment. However, all gears except II-A-L-1, II-A-F-1 and II-A-F-2 were damaged before 10^7 was reached as shown in Table 3 under such an environment that the temperature in the gear box was elevated to more than 80 °C. This damage was caused by a breakage in the vicinity of the pitch point, as shown in the previous report⁽⁵⁾.

3.2 Temperature rise in plastic gear tooth

In this experiment, the temperature of the plastic gear tooth during operation was measured by a thermocouple through a slip ring. The thermocouple was embedded in a hole of 1.2 mm in diameter drilled from the side of the tooth to the middle of the face width as shown in Fig. 2. Figure 3 shows the results. The lower limit temperature in each experiment indicates the temperature in the gear box, and the upper limit temperature the steady state temperature of the tooth. Since the tooth temperature is the same as that in the gear box before the experiment, the difference between the upper limit temperature and the lower limit temperature is the temperature rise of the plastic

Fig. 2 Position for measuring tooth temperature of plastic gear

Fig. 3 Temperature rise in plastic gear tooth

gear tooth. Although there is some difference according to the experimental conditions, the tooth temperature became balanced within about 10 minutes after the start of operation for any of the gears. As shown in Fig. 3, the temperature rise of the plastic gear tooth under elevated-temperature environment appears higher with lower temperature in the gear box irrespective of the type of the gear and operation condition. In the experiment at $T_m = 120^\circ\text{C}$, there is little difference between the tooth temperature and that in the gear box. Such a tendency seems to be explained by the fact that the higher is the temperature in the gear box, the smaller is the temperature difference between the gear box and the heat source such as friction heat.

4. Calculation of Fatigue Strength Using Strain Energy

The previous report proposed a new calculation method using strain energy as a fatigue strength equation for plastic gears. This calculation method, however, is based on the strength and rigidity value of plastic at 23°C , and does not take into consideration the influences of variation in physical properties of plastic due to the tooth temperature rise involved in the operation and the difference in the environment temperature.

Figure 4 shows the temperature of the plastic gear tooth used for calculation in the previous reports^{(2)~(4)}. This has been obtained by rearranging the experimental results conducted with 7.35 to 23.52 N · m in torque and in a range of 500 to 1 500 rpm in the rotational speeds under ordinary-temperature environment, using PV value (P : Hertzian stress equivalent to a steel gear^{(2),(4)}, V : average sliding velocity on the tooth surface). The rigidity⁽⁶⁾ (compressive elastic modulus) and strength⁽⁶⁾ (compressive strength) of plastic (MC nylon) are greatly dependent on the temperature, as shown in Figs. 5 and 6. In view of this fact, it is necessary to build up a strength design method for plastic gears taking into consideration the temperature rise of the tooth as shown in Fig.

Fig. 4 Tooth temperature of plastic gears in last experiments (room temp.: $22\sim 24^\circ\text{C}$)

4 under the ordinary-temperature environment as well as the results under the elevated-temperature environment in Fig. 3. Thus, on the basis of the fatigue strength equation using strain energy in the previous report, this report will study a fatigue strength equation taking into consideration the variation in the rigidity and strength of plastic caused by the difference in the plastic gear tooth temperature.

The strain energy, U , can be represented as shown in Eq.(1) with reference to stress, σ , and strain ϵ :

$$U = \int_0^\epsilon \sigma d\epsilon = \frac{1}{2} \sigma \epsilon \quad (1)$$

Although the relation between stress and strain cannot be simply formulated in such viscoelastic materials as plastic, it will be expressed by Eq.(2) with C as a proportional constant in the same manner as in the previous report for simplification's sake:

$$\epsilon = C \sigma t^{1/4} \quad (2)$$

Accordingly, strain energy, U , is expressed by Eq.(3) from Eqs.(1) and (2):

$$U = \frac{1}{2} C \sigma^2 t^{1/4} \quad (3)$$

Adopting in this report the concept that the breakage in the vicinity of the pitch point, where most of the plastic gear failure occurs, is resulted⁽⁵⁾ from the tooth surface fatigue, we substitute σ in Eq.(3) by the maximum Hertzian stress, P_H , during meshing at the pitch point. Hence, we derive Eq.(4) from Eq.(3):

Fig. 5 Temperature characteristics of compressive elastic modulus of MC nylon

Fig. 6 Temperature characteristics of compressive strength of MC nylon

$$U = \frac{1}{2} C P_H^2 t^{1/4} \quad (4)$$

where t is a contact time (time of passing the contact width in Hertzian contact).

For plastic gears, the higher the environment temperature, that is, the higher the tooth temperature, the shorter the operation life of the plastic gear, as shown in Fig. 3. This is because the higher the tooth temperature, more the fatigue strength of the plastic reduces. Therefore, it is necessary to consider the influences of variation in the fatigue strength of plastic according to the working temperature of the tooth in order to evaluate the fatigue strength of the plastic gear using the strain energy. The relation between strain energy, U , used to evaluate the fatigue strength of the plastic gear and the fatigue strength cannot be simply formulated. Since, however, there is also a report⁽⁷⁾ that the compressive fatigue limit of plastic is 30 to 40 % of the compressive strength, we have decided to correct the value of U using the rate of variation (with respect to the value at 23 °C) in compressive strength due to the temperature variation although it is a very rough method. Denoting the temperature coefficient as K_T , we provided $1/K_T = (\text{compressive strength at any temperature}) / (\text{compressive strength at 23 °C})$ using Fig. 6, and defined the corrected strain energy, U' , taking into consideration the variation in fatigue strength of plastic due to the temperature variation as $U' = K_T \cdot U$ to evaluate the fatigue strength of the plastic gear. The relation between the temperature and K_T is as shown in Fig. 7.

Although the previous report handled Hertzian stress P_H , and contact time, t , as values not related to temperature, we have decided to express P_H and t as $P_{H(T)}$, and $t_{(T)}$ respectively as a function of temperature because the plastic rigidity (compressive elastic

Fig. 7 Temperature coefficient

modulus) greatly varies with temperature, as shown in Fig. 5.

From the foregoing, U' , the corrected strain energy taking the temperature into consideration, is expressed by Eq. (5) :

$$U' = \frac{1}{2} C K_T P_{H(T)}^2 t_{(T)}^{1/4} \quad (5)$$

In this equation, $P_{H(T)}$, $t_{(T)}$ and $E_{(T)}$ are given by the following equations :

$$P_{H(T)} = \sqrt{\frac{2}{\pi} p_n E_{(T)} \frac{Z_1 + Z_2}{m Z_1 Z_2 \sin \alpha}}$$

$$t_{(T)} = \frac{120 p_n}{\pi m Z_1 n P_{H(T)}}$$

$$\frac{1}{E_{(T)}} = \frac{(1 - \nu_1^2)}{E_{1(T)}} + \frac{(1 - \nu_2^2)}{E_{2(T)}},$$

where p_n is the tooth surface load per unit face width, $E_{(T)}$ is the reduced modulus taking temperature into consideration, Z_1 and Z_2 are the number of teeth of driving and driven gears, respectively, m is the module, α is the pressure angle, n is the rotational speeds (rpm), and $E_{1(T)}$, ν_1 and $E_{2(T)}$, ν_2 are the compressive elastic modulus and Poisson's ratio of the driving and driven gears, respectively, taking into consideration the temperature. Since Poisson's ratio has little variation⁽⁶⁾ with temperature, we have decided to treat it as a value not related to temperature.

since the constant, C , in Eq. (5) is an uncertain value as in the previous report, we consider an expression dividing both sides of Eq. (5) by C , and using this U'/C , we rearrange the experimental results conducted under the elevated - temperature environment in this report and the results obtained under the ordinary-temperature environment in the previous report. In this study, $E_{1(T)}$ was determined by the value of $E_{p(T)}$ obtained from Fig. 5 with the temperature of plastic gear tooth (Figs. 3 and 4), and $E_{2(T)}$ was regarded to be constant as $E_{2(T)} = 206$ GPa because the rigidity of S 45 C hardly changes at about 150 °C. ν_1 and ν_2 were regarded as 0.35 and 0.3, respectively, for calculation.

Figure 8 shows the results of the value of U'/C calculated using the above method versus the opera-

Fig. 8 Relation between operation life of plastic gears and U'/C

tion life (total number of rotations to damage), N_{T0} , of the plastic gear. All points on $N_{T0}=10^7$ have been experimented on without damage. The same correlation appears between N_{T0} and U'/C under both elevated- and ordinary-temperature environments although some dispersions are seen in the results, and the relation is represented as shown in Eq.(6).

$$U'/C = -80 \log N_{T0} + 730 \quad (6)$$

If, therefore, the temperature of the plastic gear tooth is known, it seems fully possible to roughly estimate the operation life irrespective of the environment temperature through this method. In view of this degree of resulting accuracy, there seem to be no great problems in handling Eq.(2) and K_T .

If we estimate the operation life of a MC nylon gear from the above results, it seems to be able to operate for more than 10^7 in the total number of rotations without damage until U'/C reaches about $200 \text{ MPa}^2 \cdot \text{sec}^{1/4}$ under an environment temperature of less than 120°C .

5. Conclusions

on the basis of the fatigue strength equation for plastic gears using strain energy proposed in the previous report, this report has proposed a fatigue strength equation taking into consideration the tooth temperature in the calculation equation in order to apply it to the operation of a plastic gear under the elevated-temperature environment. we formulated the operation life from data obtained from the operation test of the plastic gear at 80 , 100 and 120°C in environment temperature (temperature in the gear box) as well as the data under the ordinary-temperature environment used in the previous report. As a

result, we verified that the operation life can be estimated using this strength equation irrespective of the environment temperature if the temperature of a plastic gear tooth is known.

To further improve the accuracy of this design method, however, the relation between stress and strain in Eq.(2), temperature coefficient, value of contact, C , and the like should be more strictly investigated.

References

- (1) Tsukamoto, N., Taki, T. and Nishida, N., A Study on Strength Design Methods of Plastic Gears (1st Report), Trans. Jpn. Soc. Mech. Engrs., (in Japanese), Vol. 53, No. 486, C(1987), p. 432.
- (2) Tsukamoto, N., Investigation about the Strength of Plastic Gears (2nd Report), Bulletin Jpn. Soc. Mech. Engrs., Vol. 24, No. 191, (1981), p. 872.
- (3) Tsukamoto, N., Investigation about Load Capacity of Nylon Gears, when Tooth Surface Finishing of Mating Steel Gears is Different (1st Report), Bulletin Jpn. Soc. Mech. Engrs., Vol. 24, No. 198, (1981), p. 2184.
- (4) Tsukamoto, N., Yano, T. and Sakai, H., Noise and Transmission Efficiency under Deformation of Tooth Form of Nylon Gear, Bulletin Jpn. Soc. Mech. Engrs., Vol. 25, No. 207, (1982), p. 1465.
- (5) Tsukamoto, N., Investigation about the Strength of Plastic Gear (1st Report), Bulletin Jpn. Soc. Mech. Engrs., Vol. 22, No. 173, (1979), p. 1685.
- (6) Technical Paper, (in Japanese), (1984), p. 4, Nippon Polypenco Co., Ltd.
- (7) Terashima, K., Tsukamoto, N. and Nishida, N., Development of Plastic Gears for Power Transmission, Bulletin Jpn. Soc. Mech. Engrs., Vol. 29, No. 250, (1986), p.1326.