An Evaluation Method for Sampling Intervals in Measurements of Sound Pressure Distribution*

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This paper deals with an evaluation method for spatial sampling intervals in the measurement or the calculation of sound pressure distribution. In this method, a sampling coefficient is newly defined as the ratio of a sampling interval to the half-wavelength of the sound in question. On the assumption that the sound pressure distribution along an arbitrary straight spatial line in a sound field can be represented by a one-dimensional spatial coordinate in the line direction, it can be theoretically shown that the spatial sampling interval should be set so as to make the sampling coefficient less than 1. In order to examine the influence of the sampling coefficient, three models of sound sources have been used in experimentation. The sound pressure distribution around each model has been measured and calculated for different sampling coefficients. All the results obtained show that the evaluation method using the sampling coefficient is effective and applicable to actual three-dimensional sound fields.

Key Words: Sound, Measurement, Calculation, Evaluation Method, Sampling Interval, Sampling Coefficient, Sound Pressure Distribution

1. Introduction

With the intention of investigating the noise radiation characteristics of equipment and machinery, the spatial distributions of sound level and sound pressure level around them are frequently measured. In the study of theoretical analysis and numerical analysis technique on the prediction of a sound field around equipment and machinery, on the other hand, whether the analysis technique is appropriate or not is verified by carefully comparing the results of calculation and measurement of the sound pressure distribution around the equipment and machinery in question^{(1)~(5)}. Discussion here will concentrate on the sampling interval for computing and measuring the sound pressure distribution in such a case as this.

The sound pressure distribution around a noise source such as a machine is generally quite complicated but depends on the frequency. How the interval

between measuring points, as well as that between computing points (both referred to hereinafter as the sampling interval), is set affects the accuracy of the results obtained for such a sound pressure distribution as this. It is accepted, however, that the task of acquiring spacewise data is, in general, far more difficult than that of processing time-series data only⁽⁶⁾. The authors investigated 93 selections of relevant Japanese and foreign literature in order to determine how the sampling interval was practically handled in connection with the computation and measurement of a sound field. As a result, it could be seen that there were a large number of papers in which the sampling interval was unknown and there was no explicit description about it. Even if the interval was explicitly described, the appropriateness of the set value was scarcely mentioned and there were also a few examples which seemed to be inaccurate because of the excessively large sampling interval for each frequency in question. The results of this investigation suggest that a sampling interval is quite ambiguously set in practice in spite of the importance of the sampling interval for evaluating the result.

In the present study, the authors take into consid-

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eration this situation and propose an evaluation method, of which the criterion is the "sampling coefficient", defined as the ratio of a sampling interval to the half-wavelength of the sound in question. This method makes it possible to suitably evaluate the appropriateness of the sampling interval to be set when a sound field is measured and computed. The introduction of the coefficient will first be described. Next, the sound pressure distribution around typical sound source models will be computed and measured at various values of their sampling coefficient, and the authors will investigate the influence of the sampling coefficient on the accuracy of the results obtained. Based on these results, finally, the authors will confirm the effectiveness of the evaluation method using the sampling coefficient.

2. Evaluation Method for the Sampling Interval

2.1 Setting limits of the sampling interval

The sampling theorem in time domain signals, as is well known, ensures that if $f_{\max}$ denotes the maximum frequency included in a time function $f(t)$, then $f(t)$ is completely determined by the values at each instance specified by a uniform time interval Δt , shorter than $1/2 f_{\max}^{(7)}$. In other words, the following relation can be written :

$$\Delta t < \frac{1}{2f_{\max}} \quad (1)$$

It can be proved that the process for deriving this sampling theorem is applicable to a function in spatial domain, which is generally represented by a one-dimensional spatial coordinate⁽⁸⁾. In this case, a relation similar to Eq.(1) can be written between a spatial sampling interval and the maximum spatial frequency. The authors will then consider an application of the sampling theorem to a problem of sound field in spatial domain. Now, let $p(x)$ denote the sound pressure distribution along the direction of an arbitrary straight line expressed by a one-dimensional spatial coordinate x in the line direction. Also let $F_{\max}$ denote the maximum value of the spatial frequency F included in $p(x)$; then $p(x)$ will be completely determined by the values at each instance specified by a uniform sampling interval ΔL , smaller than $1/2 F_{\max}$, i.e., the following relation can be written :

$$\Delta L < \frac{1}{2F_{\max}} \quad (2)$$

Since a spatial frequency is the inverse of the corresponding wavelength, equation $F_{\max} = 1/\lambda_{\min}$ can be written, where $\lambda_{\min}$ denotes the minimum wavelength, included in $p(x)$. Consequently, Eq.(2) can be rewritten as follows :

$$\Delta L < \frac{\lambda_{\min}}{2} \quad (3)$$

Equation (3) states that a spatial sound field

such as sound pressure distribution should be measured with a sampling interval smaller than $\lambda_{\min}/2$, and there is no doubt about its usefulness as a criterion. It is necessary, however, to bear sufficiently in mind that Eq.(3) is, as is mentioned above, derived as a relation in one-dimensional spatial domain based on the theorem for time domain signals. To what degree Eq.(3) is effective for the measurement and computation of an actual three-dimensional sound field will be discussed in section 4 using a method with the sampling coefficient to be described in the following section.

2.2 Introduction of the sampling coefficient

As a criterion for numerically evaluating the appropriateness of a specified value of sampling interval, the authors define the sampling coefficient α as the ratio of a sampling interval ΔL to the half-wavelength $\lambda/2$ of sound in question. Thus, the relation of Eq.(3) can be written for a sound wave of an arbitrary frequency as follows :

$$\Delta L = \alpha \frac{\lambda}{2}, 0 < \alpha < 1 \quad (4)$$

If the sound in question contains several frequency components, substitute the wavelength λ in Eq.(4) for $\lambda_{\min}$, corresponding to the maximum frequency. In the measurement of a sound field, cases are frequently found where the same sampling interval has been employed. In such case, let α_1 and α_2 denote the values of sampling coefficients corresponding to different frequencies f_1 and f_2 ($f_1 < f_2$), respectively ; from Eq.(4), the following relation can be written :

$$0 < \alpha_1 < \alpha_2 < 1 \quad (5)$$

If the sampling interval is consequently set so that Eq.(4) holds for a higher frequency, it is ensured that Eq.(4) is valid at a lower frequency as a matter of course. By means of the sampling coefficient, in this way, it is possible to accurately evaluate the value of the sampling interval to be set. While the sampling coefficient is to be set at a value less than 1, it is difficult to determine the value of the sampling coefficient that is the most suitable for each frequency. When the value of the sampling coefficient is made smaller, a higher accuracy of results can be achieved but the measurement will require more time and man-power. As for calculation, in addition, the requirement of computer capacity and computing time may be considerably large. It appears that the value of sampling coefficient should be determined according to the accuracy required for the results. Such determination will require a different, practical standard indicating the accuracy of results obtained with the change in the value of sampling coefficient. This standard will be discussed in section 4.

3. Experimental Method

The authors investigate the influence of the sampling coefficient through measuring experimentally the sound pressure distribution around each of the three types of sound source models. The experiment comprises the spatial sampling measurement of sound pressure at lattice points of regular intervals in the sound field and the continuous visualization measurement of sound pressure distribution by a sound visualization technique using three color-light-emitting diodes. The visualization measurement is made for the purpose of comparing the results of computation and of the sampling measurement. Because of the principle of sound visualization in section 3.3, the visualization measurement makes it possible to obtain detailed results. The sound source models and each measurement method are described in the following.

3.1 Sound source models

In the experiment, the following three types of sound source models are used: Models 1 and 2 shown in Fig. 1, that is, two enclosure models having a sound-insulating enclosure with apertures for air supply, exhaust, and heat radiation, while in Model 3, the front plate with the round aperture of Model 1 is replaced with a diaphragm.

Models 1 and 2 are made of veneer board and are internally equipped with speakers. The object of measurement is to determine the sound field, formed outside each model with the sound radiated from each round aperture. Figure 2 shows the structure of Model 2. Except for one speaker used as an internal sound source, the structure of Model 1 is approximately the same as that of Model 2. Model 3 is equipped with a 3-mm-thick diaphragm bolted to its enclosure and is air-excited by its inner speaker. The object of measurement is also to determine the sound field radiated from the diaphragm. For the purpose of experimentation, three sound-source models are installed on a frame and placed on a turntable.

3.2 Measurement of spatial sampling

Sound pressure distribution is measured on the horizontal plane passing through the center of each enclosure. For transferring of the microphone, a microphone-moving apparatus is used by means of computer control. Since the microphone can be moved for each sampling interval, sound pressure can be measured at each measuring point. Based on the measured value thus obtained at each measuring point, primary interpolation makes it possible to depict the sound pressure distribution in terms of an equi-sound-pressure level diagram with 5-dB interval.

3.3 Visualization measurement

Sound pressure distribution is measured in detail by a visualization technique⁽⁹⁾ of sound field with light-emitting diodes of three colors: red, yellow and green. This technique is based on the following measuring principle: With a scanning microphone, three color-light-emitting diodes fixed to the microphone automatically and continuously travel on a measuring plane, and the respective light-emitting diodes are repeatedly switched on or off at each instance when the sound pressure level changes by 5 dB. If the stream of colored lights emitted from the diodes is recorded on a color film, it is possible to quantitatively visualize the sound pressure distribution. To transfer the microphone, use is made of a microphone-moving apparatus by means of computer control, as in the case of sampling measurement. All the experiments have been carried out in a hemianechoic room.

4. Results and Discussion

4.1 Analysis of sampling coefficient by means of computation

The sound field around the enclosure models was calculated by a computational method of diffracted sound waves with multiple edges, the method being suitable to the computation of diffracted sound waves around such a box-type structure as this. Since this computational method is described in detail in the references^{(4),(5)}, the complicated calculation formula is not shown here.

The calculation of a spatial sound pressure distribution depends, as in the case of its measurement, on how the sampling interval is selected. To determine the sound pressure distribution around Model 1 shown in Fig. 1, the authors carried out, with various values of the sampling coefficient, the calculation and the visualization measurement of 1/3-octave-band noise, with center frequency f_c ranging from 315 to 6 300 Hz. As an example, Figs. 3 and 4 show the results of calculation and visualization, respectively, at center frequency 3 150 Hz. The calculation was carried out within a range of 1 600 × 1 600 mm from the model. In the calculation, is used the experimental value 342

Fig. 1 Enclosure models provided with round apertures

m/s of sound velocity, and further, the value of the sound pressure level measured at 20 mm in front of the round aperture of the model is used as a reference one.

From the results of Fig. 3, it can be seen that, with decreasing the sampling coefficient α , the calculated result of sound pressure converges at a constant value, tending to coincide well with the visualization result (Fig. 4). Also, from the results, it can be seen that the sound pressure distribution is considerably complicated in the rear domain of the enclosure. Since the center frequency was used in the calculation (in other words, the calculation was carried out on the assumption that the sound was pure), the sound pressure distribution is more finely depicted than that of the visualization result. If the value of α exceeds 1, as in Figs. 3(a) and (b), the contours of the sound pressure level are too sparse, so it is impossible to obtain an accurate result. In Figs. 3(c) and (d), where the value of α ranges from 1 to 0.5, the calculated results of sound pressure distribution in the rear domain are considerably improved. It still may not be said that the result of Fig. 3(c), where the value of α can be approximately 1, is sufficient. On the other hand, the

result of Fig. 3(d), where the value of α is 0.74, is sufficiently accurate from the practical standpoint of sound field prediction. Finally, the results of Figs. 3(e) and (f), where the value of α is less than 0.5, are sufficiently satisfactory from the standpoint of accuracy. Also, when the frequency has values other than those shown in Fig. 3, it is possible to obtain a result with a tendency similar to that of Fig. 3. In the case of lower frequencies, however, the sampling interval becomes considerably wide even if the value of α is taken to be about 0.7. If, to cope with this, the calculation domain is made smaller, the number of calculation points is reduced, causing some difficulty for the determination of sound pressure distribution. In such case, the situation can be improved by widening the calculation domain or sufficiently decreasing the value of α . When calculation is carried out at 315 Hz, for example, the sampling interval is about 100 mm, even for $\alpha=0.1$, and it is thus possible to obtain a highly accurate result with a sampling interval much wider than that in the case of high frequency.

The above-mentioned results suggest that the relation of Eq.(4) is valid also for actual three-dimensional sound fields such as these.

4.2 Analysis of sampling coefficient by means of experimental measurement

Figures 5 and 6 show the results of measurement for different sampling intervals and visualization measurement of the sound pressure distribution around Model 2 shown in Fig. 1, respectively. The 1/3-octave band noise at the center frequency of 6300 Hz was input in phase to the two speakers. Since the cutoff frequency of this band noise ranges from 5600

Fig. 2 Structure of Enclosure Model 2

Fig. 4 Visualization of the sound pressure distribution around Model 1

(a) $\alpha = 2.96$, $\Delta L = 160$ mm

(b) $\alpha = 1.48$, $\Delta L = 80$ mm

(c) $\alpha = 0.98$, $\Delta L = 54$ mm

(d) $\alpha = 0.74$, $\Delta L = 40$ mm

(e) $\alpha = 0.50$, $\Delta L = 27$ mm

(f) $\alpha = 0.37$, $\Delta L = 20$ mm

Fig. 3 Influence of sampling coefficient on the calculations of the sound pressure distribution around Model 1

to 7100 Hz, the sampling coefficient is evaluated at 7100 Hz as the upper limit frequency $f_{\max}$.

No clear sound pressure distribution was obtained as shown in Fig. 5(a), which shows the result of measurement with the sampling interval ΔL of 60 mm. In this case, the wavelength $\lambda_{\min}$, corresponding to $f_{\max}$, is equal to 48.2 mm and the value of α considerably exceeds 1, that is, 2.49. For this reason, a sampling interval of even 60 mm is too wide for higher frequency. In Fig. 5(b), where the sampling interval is reduced by half to 30 mm, the sound pressure distribution is, as can be seen from the comparison to the visualization result in Fig. 6, considerably improved as a whole. As far as some areas of complicated sound pressure distribution are concerned, however, it is still impossible to consider the results sufficient. The value of sampling coefficient in this case is 1.25, to some degree larger than 1. It appears after all that the

(a) $\alpha = 2.49$, $\Delta L = 60$ mm

(b) $\alpha = 1.25$, $\Delta L = 30$ mm

Fig. 5 Sampling measurement of the sound pressure distribution around Model 2

sampling interval should be selected so that the value of α is less than 1. Even in this case of Fig. 5(b) the number of measuring points is 1612, and if the sampling interval is reduced by half to 15 mm, the value of α becomes 0.63, whereas the number of measuring points is markedly increased to 6222. For cases in which the sampling coefficient was lower than 1, analysis by means of calculation was carried out. As an example, Figure 7 shows the calculated result with sampling interval 20 mm, corresponding to the value of α equal to 0.74, and the obtained sound pressure distribution coincides well with the visualization result in Fig. 6.

Next, Figs. 8 and 9 show the results obtained through sampling and visualization measurements,

Fig. 6 Visualization of the sound pressure distribution around Model 2

Fig. 7 Calculations of the sound pressure distribution around Model 2

Fig. 8 Sampling measurements of the sound field radiated from diaphragm

respectively, of the pressure distribution of the sound field radiated from the diaphragm of Model 3, as described in section 3.1. During these measurements, the diaphragm was in resonance at frequency 2 479 Hz and the mode of vibration (5, 3). Measurement was carried out on the horizontal plane, passing through the center of the diaphragm.

Figure 8(a) corresponds to the sampling interval 60 mm and although the value of α is equal to 0.87, less than 1 to some degree, the obtained sound pressure distribution is not clear. The actual sound pressure distribution in this case is, as can be seen from the visualization result shown in Fig. 9, considerably complicated over the entire measuring plane. If such a complicated sound pressure distribution as this is expected, it can be recommended (as can be supposed from the calculated results described in section 4.1 on Model 1 and from the above-mentioned results on Model 2) that the sampling coefficient be set to a value considerably less than 1 (about 0.7 at most), if possible, about 0.5. Figure 8(b) shows the measurement results obtained by reducing the sampling interval by half to 30 mm. Since in this case the value of α becomes sufficiently small, equal to 0.43, the obtained sound pressure distribution coincides well with the visualization result (Fig. 9).

Fig. 9 Visualization of the sound field radiated from diaphragm

5. Conclusions

The authors have proposed an evaluation method for spatial sampling intervals in the measurement or the calculation of sound pressure distribution. In this method, a sampling coefficient was newly defined. This method makes it possible to evaluate the appropriateness of a sampling interval to be set for measuring or computing a spatial sound field. The authors investigated the effectiveness of this evaluation method, utilizing some sound source models to experimentally measure and calculate the sound pressure distribution around these models. The obtained results can be summarized as follows :

(1) The sampling coefficient was defined as the ratio of a spatial sampling interval to the half-wavelength of the sound wave in question. On the assumption that sound pressure distribution can be represented in terms of a one-dimensional spatial coordinate, it can be theoretically shown that the sampling interval should be set such that $0 < \alpha < 1$.

(2) From the calculation and measurement results, obtained on the sound source models, it could be seen that the evaluation method by means of sampling coefficient is very effective and also very applicable to an actual three-dimensional sound field. Although, with decreasing value of sampling coefficient, the accuracy of the obtained result becomes higher, the computation and measurement will require more time.

(3) By computing and measuring a sound pressure distribution with different values of the sampling coefficient, the authors concretely showed the influence of the magnitude of the coefficient on the

accuracy of result to be obtained, as well as offered a preferable level, as a standard, of the coefficient when a sampling interval is to be set.

(4) In cases in which the sound pressure distribution belongs to a monotonous sound field, satisfactory results can be obtained even with a value of sampling coefficient of slightly less than 1. If the sound pressure distribution belongs to a complicated sound field, it is recommended that the sampling coefficient be set to a value of about 0.7 (at the highest) and, if possible, about 0.5.

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