Digital Active Vibration Control of a Cantilever Beam with Piezoelectric Actuators*

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A digital optimal regulator technique is introduced to the problem of vibration control of a cantilever beam. A pair of piezoelectric cells is adhered to the upper and lower surfaces of the beam and used as the actuator. Influences of the mass and the rigidity of the actuator are taken into account in the analysis, and the equations of motion for the beam-control system are derived by applying the transfer matrix method. Simulated transient response of the beam is compared with the experimental result. It is found that the effect of vibration suppression is obtained by means of the present technique which compensates for the delay arising from the computation time.

Key Words: Vibration Control, Cantilever Beam, Digital Optimal Regulator, Piezoelectric Ceramic Actuator

1. Introduction

In recent years, it has been planned to build large-scale multipurpose space stations. Since the large-scale spacecraft structure must be light weight, it will become flexible. When the structure begins to vibrate, its vibration will continue for a long time at a lower natural frequency. In order that the position of the antenna and telescope is assured precisely, it is necessary to control the vibration. Numerous papers have reported studies of the active vibration control of the flexible structure^{(1)~(11)}. Some reported research^{(1)~(5)} used piezoelectric devices as the actuators. However, there are few studies in which vibration was suppressed by the digital optimal regulator method. Hence, it

is indispensable to confirm the utility of suppressing the vibration with the digital optimal control and the piezoelectric actuator for practical use in the future.

In this paper, a vibration control problem of the cantilever beam is presented, and a pair of piezoelectric cells adhered to both sides of the beam is used as the actuator. The bending vibration of the cantilever beam can be suppressed by a moment generating from the piezoelectric actuator. The results of the vibration suppression are discussed for both the simulation and the experiment. In the theoretical analysis, the transfer matrix method is used to lead to a state equation of the system. An optimal feedback coefficient matrix, in which the time delay arising from the computation time is compensated, is designed by using the optimal regulator theory. In order to realize the state feedback control, a minimal-order observer is introduced since not all of the state variable vectors can be measured by the sensor. The experiment was conducted to confirm the control effect.

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2. Formulation

2.1 Fundamental equation

The subject of this research is the vibration control of a cantilever beam as shown schematically in Fig. 1. The length of the beam is L_b , Young's modulus is E_b , the moment of the inertia of the area is I_b , the density is ρ_b , the cross-sectional area is A_b and the internal damping coefficient is c_b .

A pair of piezoelectric devices is adhered to both sides of the beam at a distance x_1 from the clamped end. The length of the piezoelectric devices is L_p , Young's modulus is E_p , the moment of the inertia of the area is I_p , the density is ρ_p , the cross-sectional area is A_p and the internal damping coefficient of the piezoelectric device is c_p . A direct current with the same voltage is supplied on electrode formed at the cells. The induced bending moment versus the supplied voltage is given by

$$M(t) = kd_{31} V(t)/h_p. \quad (1)$$

Here, d_{31} is a constant of the piezoelectric effect, and k is a constant which depends on Young's modulus and the sectional size of the cantilever beam. The absolute displacement of the beam, $W(x, t)$, is the function of position x on the beam and time t . Under the assumption that the beam vibrates only in the x - y plane and that the internal damping of the beam is in proportion to its velocity, the partial differential equation describing the vibration of the beam is given by

$$\begin{aligned} \frac{\partial^2}{\partial x^2} \left[\{E_b I_b + E_p I_p [H(x-x_1) - H(x-x_2)]\} \frac{\partial^2 W(x, t)}{\partial x^2} \right] \\ + \frac{\partial^2}{\partial x^2} \left[\{c_b I_b + c_p I_p [H(x-x_1) - H(x-x_2)]\} \frac{\partial^3 W(x, t)}{\partial x^2 \partial t} \right] \\ + \{\rho_b A_b + \rho_p A_p [H(x-x_1) - H(x-x_2)]\} \frac{\partial^2 W(x, t)}{\partial t^2} \\ = M(t) [\delta'(x-x_1) - \delta'(x-x_2)]. \end{aligned} \quad (2)$$

For the cantilever beam as shown in Fig. 1, the appropriate boundary conditions are given by

Fig. 1 Analytical model and coordinate system

$$\left. \begin{aligned} W(x, t) \Big|_{x=0} = \frac{\partial W(x, t)}{\partial x} \Big|_{x=0} &= 0, \\ \{E_b I_b + E_p I_p [H(x-x_1) - H(x-x_2)]\} \\ \times \frac{\partial^2 W(x, t)}{\partial x^2} \Big|_{x=L_b} &= 0, \\ \frac{\partial}{\partial x} \left[\{E_b I_b + E_p I_p [H(x-x_1) - H(x-x_2)]\} \right. \\ \times \frac{\partial^2 W(x, t)}{\partial x^2} \Big|_{x=L_b} &= 0. \end{aligned} \right\} \quad (3)$$

Here, $c_b I_b = c E_b I_b$, $c_p I_p = c E_p I_p$, and $x_2 = x_1 + L_p$, while $H(x)$ is a unit step function, $\delta(x)$ is Dirac's delta function, $(\cdot)'$ is the differentiation with respect to the coordinate x , and c is a constant.

When rectangular sections of the beam and the piezoelectric devices have width of B_b , and B_p and thickness of h_b , and h_p , respectively, the following equations are obtained:

$$\begin{aligned} A_b &= B_b h_b, \quad A_p = B_p h_p, \\ I_b &= B_p h_b^3 / 12, \quad I_p = B_p [(2h_p + h_b)^3 - h_b^3] / 12. \end{aligned}$$

2.2 Formulation with the modal coordinate

Solutions to Eqs. (2) and (3) can be separated into components of space and time as follows:

$$W(x, t) = \sum_{j=1}^n \phi_j(x) q_j(t). \quad (4)$$

where

$$\phi_j(x) = d [H_1(x) w_1^j(x) + H_2(x) w_2^j(x) + H_3(x) w_3^j(x)]. \quad (5)$$

$$\left. \begin{aligned} H_1(x) &= 1 - H(x-x_1), \\ H_2(x) &= H(x-x_1) - H(x-x_2), \\ H_3(x) &= H(x-x_2). \end{aligned} \right\} \quad (6)$$

Here, w_i^j is the j th vibration mode of each element, and is sought by using the transfer matrix method⁽¹²⁾, while q_j is the unknown function of time and ϕ_j is the natural vibration modes of the beam.

Substituting Eq. (4) into Eq. (2) and rearranging, Eq. (2) becomes

$$\begin{aligned} \sum_{j=1}^n \{ [1 + \zeta H_2(x)] \phi_j''(x) \}'' q_j(t) \\ + c \{ [1 + \zeta H_2(x)] \phi_j''(x) \}'' \dot{q}_j(t) \\ + \rho_b A_b / E_b I_b [1 + \mu H_2(x)] \phi_j(x) \ddot{q}_j(t) \\ = M(t) \delta'(x) / E_b I_b. \end{aligned} \quad (7)$$

Here, $(\cdot)$ is the differentiation with respect to time.

Multiplying this equation by $\phi_k(x)$ and integrating over the length of the beam gives

$$\begin{aligned} \sum_{j=1}^n \left[\frac{\rho_b A_b}{E_b I_b} \ddot{q}_j(t) \int_0^{L_b} \phi_k(x) [1 + \mu H_2(x)] \phi_j(x) dx \right. \\ \left. + c \dot{q}_j(t) \int_0^{L_b} \phi_k(x) \{ [1 + \zeta H_2(x)] \phi_j''(x) \}'' dx \right. \\ \left. + q_j(t) \int_0^{L_b} \phi_k(x) \{ [1 + \zeta H_2(x)] \phi_j''(x) \}'' dx \right] \\ = \frac{\phi_k'(x_2) - \phi_k'(x_1)}{E_b I_b} M(t). \end{aligned} \quad (8)$$

Since function $\phi_j(x)$ is the solution to the following equation

$$\{E_b I_b [1 + \zeta H_2(x)] \phi_j''(x)\}''$$

$$-\rho_b A_b \omega_j^2 [1 + \mu H_2(x)] \phi_j(x) = 0, \quad (9)$$

these solutions have orthogonality with the weight $\rho_b A_b [1 + \mu H_2(x)]$ as follows:

$$\int_0^{L_b} \phi_k(x) \rho_b A_b [1 + \mu H_2(x)] \phi_j(x) dx = L_b \delta_{kj}, \quad (10)$$

where δ_{kj} is Kroneka's delta function. Using Eqs. (9) and (10), Eq. (8) becomes noncoupling as follows:

$$\ddot{q}_k(t) + c\omega_k^2 \dot{q}_k(t) + \omega_k^2 q_k(t) = \phi_k M(t) \quad (k=1, 2, \dots, n), \quad (11 \cdot a)$$

where

$$\phi_k = \frac{\phi'_k(x_2) - \phi'_k(x_1)}{L_b} \quad (11 \cdot b)$$

Rewriting Eq. (11) into a matrix form gives

$$[I]\{\ddot{q}(t)\} + c[\Omega]\{\dot{q}(t)\} + [\Omega]\{q(t)\} = [\Phi]M(t), \quad (12)$$

where $[I]$ is a unit matrix and $[\Omega]$ is a diagonal matrix as follows:

$$[\Omega] = \begin{pmatrix} \omega_1^2 & 0 & \dots & 0 \\ 0 & \omega_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \omega_n^2 \end{pmatrix}$$

$$[\Phi] = \frac{1}{L_b} \begin{pmatrix} \phi'_1(x_2) - \phi'_1(x_1) \\ \phi'_2(x_2) - \phi'_2(x_1) \\ \vdots \\ \phi'_n(x_2) - \phi'_n(x_1) \end{pmatrix}$$

$$\{q(t)\} = \{q_1(t), q_2(t), \dots, q_n(t)\}^T. \quad (13)$$

2.3 Control theory

2.3.1 State equation of the control plant and the discrete-time systems In order to establish the state equations, a state variable vector is introduced as

$$X_c(t) = [q_1(t), q_2(t), \dots, q_n(t), \dot{q}_1(t), \dot{q}_2(t), \dots, \dot{q}_n(t)]^T, \quad (14)$$

where superscript T indicates the transposed matrix.

With Eq. (14), the state equation of the continuous time system of the control plant can be given by

$$\dot{X}_c(t) = A_c X_c(t) + B_c u_c(t), \quad (15 \cdot a)$$

where

$$A_c = \begin{bmatrix} 0 & I \\ -\Omega & -c\Omega \end{bmatrix} : (N \times N),$$

$$B_c = \begin{pmatrix} 0 \\ \Phi \end{pmatrix} : (N \times m),$$

$$u_c(t) = M(t). \quad (15 \cdot b)$$

When the non-contact sensor is placed at the free end of the cantilever beam and $Y_c(t)$ is used to show the output of the sensor, we can obtain the output equation of this control system as

$$Y_c(t) = C_c X_c(t), \quad (16 \cdot a)$$

where

$$C_c = [\phi_1(x) \phi_2(x) \dots \phi_n(x) 0 \dots 0]_{x=L_b} : (l \times N),$$

$$N = 2n. \quad (16 \cdot b)$$

When a continuous time system is controlled with a digital computer, it is necessary to transform the

continuous time system into a discrete time system. Discretizing Eq. (15) with the sampling period ΔT , the difference equations of the discrete time system are given by

$$X[(i+1)\Delta T] = AX(i\Delta T) + Bu(i\Delta T), \quad (17)$$

$$y(i\Delta T) = CX(i\Delta T), \quad (18)$$

where

$$A = \exp[A_c \Delta T]$$

$$B = (\exp[A_c \Delta T] - I)^{-1} A_c B_c$$

$$C = C_c.$$

2.3.2 Digital optimal regulator In order to realize the digital optimal control, a performance index should be introduced as follows:

$$J = \frac{1}{2} \sum_{i=0}^{\infty} [X(i)^T Q X(i) + u(i)^T R u(i)], \quad (19)$$

where R and Q are weighting matrices about the control input and the state variable, respectively. Minimizing the performance index J , the optimal control of systems (17) and (18) is sought as follows:

$$u(i\Delta T) = -[R + B^T P B]^{-1} B^T P A X(i\Delta T) \\ \triangleq -F^0 X(i\Delta T), \quad (20)$$

where P is the positive definite matrix and a unique solution for the following Riccati matrix equation:

$$P = Q + A^T P A - A^T P B [R + B^T P B]^{-1} B^T P A. \quad (21)$$

The solution of this equation can be sought with Kucera's method⁽¹³⁾.

2.3.3 Minimal order observer When the state feedback control is performed according to Eq. (20), it is necessary to measure directly all of the state variables with the sensor. However, not all of the states can be detected directly with the sensor. Therefore, an observer must be designed to estimate those states not detected directly, with the sensor. Here, a minimal-order observer is designed with help of Goppinas's method⁽¹³⁾.

For Eq. (17), we have

$$S = \begin{pmatrix} C \\ G \end{pmatrix}, \quad G : (N-l \times N). \quad (22)$$

Here, we select a proper matrix G so that the determinant of the matrix S is not equal to zero: that is, $|S| \neq 0$. Hence, the matrices A and B can be transformed into

$$SAS^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad SB = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}. \quad (23)$$

The minimal-order optimal observer is sought as follows:

$$\hat{X}(i) = D\omega(i) + HY(i), \quad (24)$$

$$\omega(i) = \hat{A}\omega(i) + \hat{K}Y(i) + \hat{B}u(i), \quad (25)$$

where each coefficient is given by

$$\hat{K} = \hat{A}F_0 + A_{21} - F_0 A_{11} : (N-l \times l), \quad (26)$$

$$\hat{B} = B_2 - F_0 B_1 : (N-l \times m). \quad (27)$$

$$D = S^{-1} \begin{pmatrix} 0 \\ I_{N-l} \end{pmatrix}; (N \times N-l). \quad (28)$$

$$H = S^{-1} \begin{pmatrix} I_l \\ F_0 \end{pmatrix}; (N \times l). \quad (29)$$

$$F_0 = A_{22} \Lambda A_{12}^T (I_l + A_{12} \Lambda A_{12}^T)^{-1}. \quad (30)$$

$$\Lambda = I_{N-l} + A_{22} \Lambda A_{12}^T + A_{22} \Lambda A_{12}^T (I_l + A_{12} \Lambda A_{12}^T)^{-1} A_{12} \Lambda A_{12}^T. \quad (31)$$

Here Λ is a solution of the Riccati matrix equation (31) for the problem of estimating the states. Therefore, the control input will become

$$u(i) = -F^0 \hat{X}(i). \quad (32)$$

2.3.4 Optimal feedback control with time-delay compensation It must be possible to compute the control input at once, when the state feedback control $u = F^0 X$ is carried out. However, when the dimensions of the state equations are large and the time constant of the control plant is small, it may be impossible to control the system with a digital microprocessor. Therefore, the computing time must be considered.

Using Eq. (17) as the predicting equation and substituting this equation into Eq. (20), we obtain

$$u(i) = -F^0 X(i) = -F^0 A X(i-1) - F^0 B u(i-1). \quad (33)$$

Figure 2 shows the optimal control systems with the minimal-order observer, in which the computing time is taken into account.

Finally, we obtain

$$u(i) = -F^0 A \hat{X}(i-1) - F^0 B u(i-1). \quad (34 \cdot a)$$

$$\hat{X}(i-1) = D \omega(i-1) + H Y(i-1). \quad (34 \cdot b)$$

$$\omega(i) = \hat{A} \omega(i-1) + \hat{K} Y(i-1) + \hat{B} u(i-1), \quad (34 \cdot c)$$

where $n=2$, $2n=4$, and $m=l=1$, for the reason shown later.

3. Experiment

3.1 Experimental devices

The equipment used in the experiment is depicted schematically in Fig. 3. The mechanism was constructed with a cantilever beam and a pair of piezoelectric cells adhered to the beam as the active actuators. The beam was made of hard, saline vinyl. The actuator, which was called the multilayer pi-

ezoelectric device, was made of pseudoternary solid-solution ceramics. A gap sensor which was placed without contact at the free end of the beam was used to measure the displacements of the beam. The piezoelectric actuator was driven by a power amplifier. The control input was calculated by a digital computer. The results of the experiment were recorded by an X-Y plotter.

To transform the sensor output into D.C. voltage within ± 5 V, we used a preamplifier. The output of the preamplifier was converted into a digital voltage with a 12-bit A-D converter and then sent to the digital computer. The output of the computer was converted into analogue voltage with a 16-bit D-A converter and then supplied to the piezoelectric actuator after the voltage coming from the computer was amplified. Only the positive voltage was supplied to the electrodes of the piezoelectric device in order to reduce the effect of the piezoelectric hysteresis.

Young's modulus and the damping coefficients of

Table 1 Characteristic parameters of beam and piezoelectric actuator

	Cantilever Beam (units)	Piezoelectric Device (units)
Length	250 (mm)	50 (mm)
Thickness	0.97 (mm)	0.91 (mm)
Width	26.0 (mm)	13.31 (mm)
Young's modulus	3.82 (GPa)	50.12 (GPa)
Density	1.37×10^3 (kg/m ³)	6.14×10^3 (kg/m ³)
Damping coefficients	0.41×10^{-3} (Pa.s)

Fig. 2 Optimal regulator with compensation of time delay

Fig. 3 Schematic diagram of the experimental equipment

the beam and the piezoelectric actuator are shown in

3.2 Method of experiment

The vibration control was carried out according to the algorithm designed above. First, we gave an initial displacement at the free end of the beam and made it vibrate freely. The vibration of the beam was measured by using the sensor. Its output was then converted into a digital signal which was sent into the computer for sampling intervals of ΔT seconds. According to the control regulation of Eq. (34), the information sent from the sensor was processed on line by the computer. The control input was converted into an analogue voltage, and then supplied to the piezoelectric actuator to drive the actuator. The sampling period was the time that the control input was calculated by the computer based on the control algorithm of Eq. (34). On the other hand, the optimal feedback gain matrix F^0 and the observer gain matrix F_0 were sought previously in off-line and then stored in the memory of the computer for later use on line. Besides, the times for A-D and D-A conversion were much shorter than the processing times of the computer. Hence, they were neglected when the experiment was carried out.

4. Results of the Experiments and the Simulations, and Discussion

4.1 Effect of the piezoelectric actuators on the cantilever beam

First, we investigated the effect of the rigidity and the mass of the piezoelectric devices on the bending vibration of the cantilever beam before the control was carried out. Figures 4 (a) and (b) show the time responses of the free vibration of the cantilever beam with and without the actuators, respectively. The bold and fine lines are the experimental and theoretical results, respectively. In this case, the value of 0.2 mm was selected as the initial displacement at the free end of the beam. The natural frequencies of the cantilever beam are shown in Table 2.

For the fundamental mode, it can be seen that the natural frequency of the beam with the actuator is about 50 percent greater than that without the actuators. This is regarded as the effect of the rigidity of the actuator on the beam. The effect of the mass of the actuator on the natural frequency of the beam is smaller than that of the rigidity because of its tendency to increase the frequency. Besides, the difference between the measurement result and the theoretical one of the natural frequencies of the beam is due to a measurement error in the characteristic values of the piezoelectric actuators as well as the effect of an adhesive agent between the beam and the actuator.

4.2 Optimal control and the control effect

Next, the active vibration control was carried out under the initial displacement of 0.2 mm at the free end of the beam. In this case, the fundamental mode played an important part in the free vibration of the beam and there were few components of more than the second mode. Therefore, the components of the first vibration mode were controlled chiefly and those of the higher mode were suppressed by using the internal damping of the beam. The following values were selected as the weighting matrix :

$$Q = \begin{cases} 100, & (i=j=1, 3); \\ 1 \times 10^{-8}, & (i=j=2, 4); \\ 0, & (i \neq j). \end{cases}$$

Moreover, the smaller the weighting matrix R is,

Table 2 Natural frequencies of beam

Mode	Frequency (Unit)		Error(%)
	Theory	Experiment	
1st	6.278 (Hz)	6.0 (Hz)	4.6
2nd	32.657 (Hz)	33.75 (Hz)	3.2

(a) Without the piezoelectric actuator

(b) With the piezoelectric actuator

Fig. 4 Effect of the piezoelectric actuator on the free vibration of beam

the greater the control input is. That is, much more control energy is supplied. This means that the vibration will be suppressed quickly when R is small. However, there is a limitation in the moment generated by the piezoelectric actuators. Therefore, even if the control input exceeds the limit, the moment generated by the actuator cannot become greater than the limit. In this paper, since the limitation in the moment was $|M_{\max}|=0.01$ Nm, the weighting matrix R was set at the value of 0.1 ($R=0.1$). The responses of the displacement at the free end of the beam and the control input are shown in Figs. 5 (a) and (b), respectively. On the other hand, we selected the weighting matrix as $R=10$ so that the maximum peak of the control input could not reach the limit, and the weighting matrix Q was the same as stated above. In this situation, the controlled response is shown in Fig. 6. However, the initial displacement of $W(L_b, 0)=0.05$ mm was chosen. It can be seen that the control plant with the natural frequency of 6 Hz was controlled and a good effect of vibration suppression was achieved under the sampling frequency of 33.3 Hz. Although the vibration of the beam gradually becomes small when the R has a large value, it is found that the vibration is suppressed sufficiently, and

(a) Response at the free end of beam

(b) Time history of control input

Fig. 5 Suppression effect of the control input u with limitation [$W(L_b, 0)=0.2$ mm, $R=0.1$, $|u_{\max}|=0.001$ Nm]

(a) Response at the Free end of beam

(b) Time history of controll input

Fig. 6 Suppression effect of the control input u without limitation [$W(L_b, 0)=0.05$ mm, $R=10$]

(a) Without control

(b) Under control

Fig. 7 Reproducibility of the minimal-order observer

(a) Without time delay compensation

(b) Under time delay compensation

Fig. 8 Effect of time delay on vibration suppression
 $[W(L_b, 0)=0.2 \text{ mm}, R=0.1, |u_{\max}|=0.01 \text{ Nm}]$

also the experimental values agree with the theoretical ones, irrespective of whether the value of R is small or large. Figure 7 shows the reproducibility of the observer. As an object of estimation, it is taken into account in the design of the observer until the first two modes. Figures 7 (a) and (b) show the cases without and with control, respectively. It is found that the values $\hat{X}$ estimating the states have converged to the states X about two sampling periods later.

There is a time delay arising from the digital computer. Therefore, it is necessary to compensate the time delay. A controller with the delay of one sampling period was compensated and was designed according to the optimal control theory. The results are shown in Figs. 8 and 9. In Figs. 8 and 9, (a) shows the response of the displacement and the control input, the time delay of which was not compensated, and (b) shows the results compensated. A good result could be achieved when the time delay was compensated.

5. Conclusions

A digital vibration control of the cantilever beam was performed by using a piezoelectric actuator. The digital optimal regulator theory was used to deter-

(a) Without time delay compensation

(b) Under time delay compensation

Fig. 9 Effect of time delay on vibration suppression
 $[W(L_b, 0)=0.05 \text{ mm}, R=10]$

mine the control input and to design a minimal-order observer. The efficiency of this control method was investigated from the experiment and the simulation. The following conclusions were obtained.

(1) The experimental results are in good agreement with the theoretical one. It is possible and effective for the digital optimal regulator to control the bending vibration of the cantilever beam.

(2) Even if there is a limit in the control input, good control effects can be achieved when the weighting matrices are selected properly for a control plant.

(3) The time delay arising from the computation time has influence upon the control. However, a good result of controlling the vibration can be achieved if the time delay is compensated.

(4) The rigidity of the piezoelectric actuator increases the natural frequencies of the beam.

(5) For the bending vibration of the cantilever beam, it is possible to design a control system in which the components of the fundamental vibrating mode are controlled chiefly, and the other components of the higher modes can be suppressed by the internal damping of the beam.

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